

## Mathematics (Basic)

### Answers

2019-2020

Time Allowed : 3 hours

Maximum Marks : 80

### Section A

1. **Correct Answer: B**

intersecting or coincident If a pair of linear equations is consistent, then the lines represented by them are intersecting or coincident.

2. **Correct Answer: A**

$$\begin{aligned} \text{Distance} &= \sqrt{(3-(3))^2 + (-2-2)^2} \\ &= \sqrt{(3+3)^2 + (-4)^2} \\ &= \sqrt{36+16} \\ &= \sqrt{52} \end{aligned}$$

3. **Correct Answer: C**

$$\begin{aligned} &8 \cot^2 A - 8 \operatorname{cosec}^2 A \\ &= 8(\cot^2 A - \operatorname{cosec}^2 A) \\ &= 8 \times -1 \\ &= -8 \end{aligned}$$

4. **Correct Answer: C**

The total surface area of a frustum-shaped glass tumbler is  $\frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 r_2)$  where radii  $r_1 > r_2$ .

5. **Correct Answer: D**

$$\begin{aligned}120 &= 20 \times 6 \\ &= 5 \times 4 \times 2 \times 3 \\ &= 5 \times 2^3 \times 3\end{aligned}$$

**6. Correct Answer: D**

The given equation is:

$$4x^2 - 6x + 3 = 0$$

$$\text{Discriminant} = b^2 - 4ac$$

Here,  $b = -6$ ,  $a = 4$ , and  $c = 3$

$$\begin{aligned}\text{So, Discriminant} &= (-6)^2 - 4 \times 4 \times 3 \\ &= 36 - 48 = -12.\end{aligned}$$

**7. Correct Answer: C**

$(3, -6)$  is the mid-point of the line segment joining  $(0, 0)$  and  $(x, y)$ .

$$\text{So, } \frac{(0+x)}{2} = 3 \text{ or, } x = 6$$

$$\text{and } \frac{(0+y)}{2} = -6 \text{ or, } y = -12$$

**8. Correct Answer: D**

In the given figure, number of tangents parallel to tangent  $PQ$  is 1.

9. **Correct Answer: A**

Class	Frequency	Cumulative frequency
0–5	8	8
5–10	10	18
10–15	19	37
15–20	25	62
20–25	8	70
<b>Sum:</b>	70	

Sum of frequencies ( $n$ ) = 70

Middle observation =  $\left(\left(\frac{n}{2}\right) + 1\right)$ th observation

$$= \left(\frac{70}{2} + 1\right)\text{th observation}$$

$$= 36^{\text{th}} \text{ observation}$$

$36^{\text{th}}$  observation lies in class interval 10–15. So, median class is 10–15 and its upper limit is 15.

10. **Correct Answer: D**

The probability of an impossible event is 0.

**Fill in the blanks in question numbers 11 to 15.**

11. Secant

12. 1

$$a(2)^2 - 2 \times 2 = 0$$

$$\Rightarrow 4a - 4 = 0$$

$$\Rightarrow a = 1$$

13. similar

14. 8:27

$$r_1 : r_2 = 2 : 3$$

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{r_1^3}{r_2^3}$$

$$\frac{V_1}{V_2} = \frac{8}{27}$$

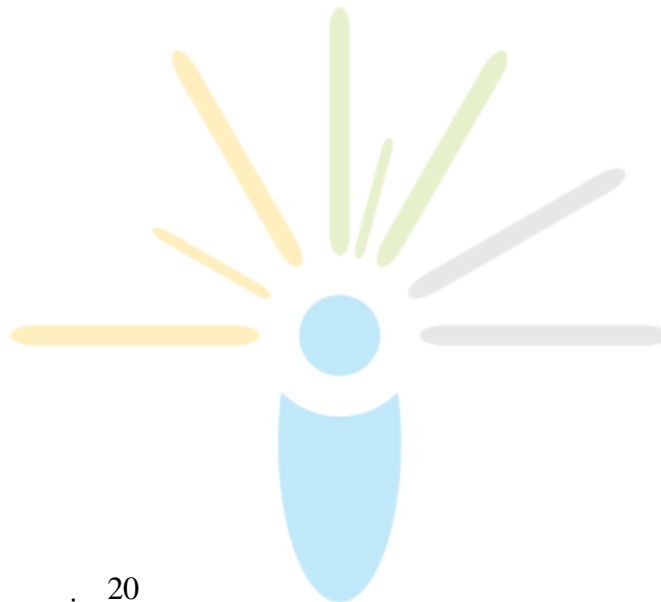
15. Collinear

$$16. \tan 60^\circ = \frac{20}{AB}$$

$$\sqrt{3} = \frac{20}{AB}$$

$$AB = \frac{20}{\sqrt{3}}$$

So, the required distance is  $\frac{20}{\sqrt{3}}$  m.



17.  $\tan 40^\circ \times \tan 50^\circ$

$$= \tan(90^\circ - 50^\circ) \times \tan 50^\circ$$

$$= \cot 50^\circ \times \tan 50^\circ$$

$$= 1 \quad (\because \tan \theta \cot \theta = 1)$$

OR

$$\cos A = \sin 42^\circ$$

$$\Rightarrow \cos A = \sin(90^\circ - 48^\circ)$$

$$\Rightarrow \cos A = \cos 48^\circ$$

$$\Rightarrow A = 48^\circ$$

18. All possible outcomes are  $HH, HT, TT, TH$  .

$$\text{Probability of an event} = \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$$

$$\text{Probability of getting head both the times} = \frac{1}{4}$$

$$19. \text{ Height of the cone} = \sqrt{13^2 - 5^2}$$

$$= \sqrt{169 - 25}$$

$$= \sqrt{144}$$

$$= 12$$

Therefore, the height of the cone is 12m.

20.  $-6, x, 8$  are in A.P.

$$\Rightarrow 2x = -6 + 8$$

$$\Rightarrow 2x = 2$$

$$\Rightarrow x = 1$$

**OR**

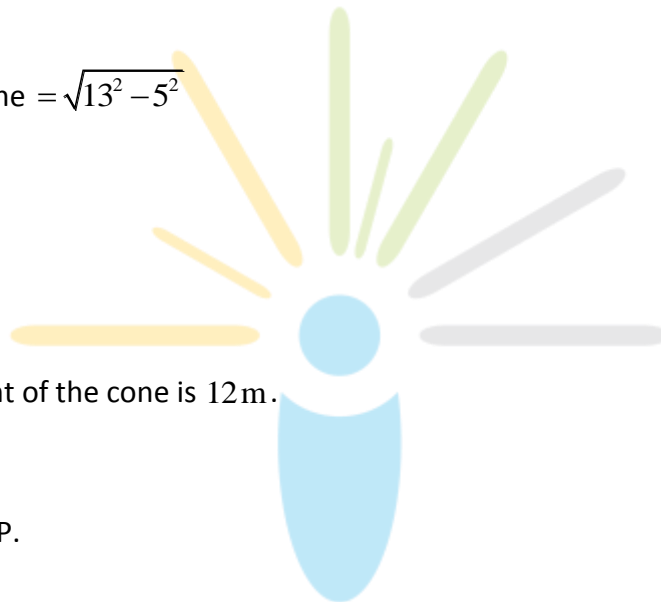
$$-27, -22, -17, -12, \dots$$

$$a_n = a + (n-1)d$$

$$a_{11} = -27 + (11-1) \times 5$$

$$= -27 + 50$$

$$= 23$$



## Section B

$$\begin{aligned}
 21. \quad 3x^2 - 4\sqrt{3x} + 4 &= 3x^2 - 2\sqrt{3x} - 2\sqrt{3x} + 4 \\
 &= \sqrt{3x}(\sqrt{3x} - 2) - 2(\sqrt{3x} - 2) \\
 &= (\sqrt{3x} - 2)(\sqrt{3x} - 2)
 \end{aligned}$$

So, the roots of the equation are the values of  $x$  for which

$$(\sqrt{3x} - 2)(\sqrt{3x} - 2) = 0$$

$$\text{Now, } \sqrt{3x} - 2 = 0 \text{ for } x = \frac{2}{\sqrt{3}}$$

So, this root is repeated twice, one for each repeated factor  $\sqrt{3x} - 2$ .

Therefore, the roots of  $3x^2 - 4\sqrt{3x} + 4$  are  $\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$ .

22. If the number  $6^n$ , for any  $n$ , were to end with the digit zero, then it would be divisible by 5.

That is, the prime factorisation of  $6^n$  would contain the prime 5. This is not possible

$$\therefore 6^n = (2 \times 3)^n$$

So, the prime numbers in the factorisation of  $6^n$  are 2 and 3.

So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorisation of  $6^n$ .

So, there is no natural number  $n$  for which  $6^n$  ends with the digit zero.

**OR**

We have,

$$150 = 5^2 \times 3 \times 2$$

$$\text{and, } 200 = 5^2 \times 2^3$$

Here,  $2^3$ ,  $3^1$  and  $5^2$  are the greatest powers of the prime factors 2, 3 and 5 respectively involved in the two numbers.

$$\text{So, LCM}(150, 200) = 2^3 \times 3^1 \times 5^2 = 600$$

23. We have

$$\tan(A+B) = \sqrt{3}$$

$$\text{or } \tan(A+B) = \tan 60^\circ$$

$$\text{or } A+B = 60^\circ \quad \dots(1)$$

Again, we have

$$\tan(A-B) = \frac{1}{\sqrt{3}}$$

$$\text{or } \tan(A-B) = \tan 30^\circ$$

$$\text{or } A-B = 30^\circ \quad \dots(2)$$

On adding equations (1) and (2), we get

$$2A = 90^\circ$$

$$\text{or } A = 45^\circ$$

On putting this value of  $A$  in equation (1), we get

$$B = 15^\circ$$

24. In  $\triangle ABC$  and  $\triangle XZY$ ,

$$\frac{BC}{ZY} = \frac{6}{12}$$

$$= \frac{1}{2}$$

$$\frac{AC}{XY} = \frac{2\sqrt{3}}{4\sqrt{3}}$$

$$= \frac{1}{2}$$

$$\frac{AB}{XZ} = \frac{3}{6}$$

$$= \frac{1}{2}$$

Ratios of the corresponding sides of the given pair of triangles are equal.

$$\text{i.e., } \frac{BC}{YZ} = \frac{AC}{XY} = \frac{AB}{XZ} = \frac{1}{2}$$

Therefore, by SSS similarity criterion,  $\Delta ABC \sim \Delta XZY$ .

The corresponding angles are equal in  $\Delta ABC$

and  $\Delta XZY$ . i.e.,

$$\angle A = \angle X = 80^\circ$$

$$\angle B = \angle Z = 60^\circ$$

and

$$\angle C = \angle Y$$

In  $\Delta ABC$ ,

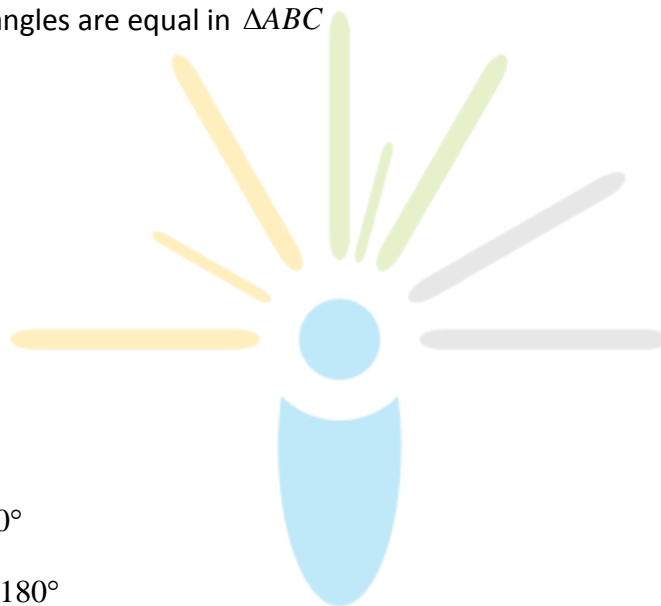
$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 80^\circ + 60^\circ + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 180^\circ - 140^\circ$$

$$\Rightarrow \angle C = 40^\circ$$

$$\Rightarrow \angle Y = 40^\circ$$





25. Number of defective bulbs = 14

Number of good bulbs = 98

Total number of outcomes =  $98 + 14 = 112$

Probability of getting a good bulbs

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{98}{112}$$

$$= \frac{7}{8}$$

26.

<b>Classes</b>	5 – 15	15 – 25	25 – 35	35 – 45
<b>Frequency</b>	2	4	3	1

Here, we observe that class marks and frequencies are small quantities.

So, we use direct method to compute the mean and proceed as below.

<b>Classes</b>	<b>Frequency (<math>f_i</math>)</b>	$x_i$	$f_i x_i$
5 – 15	2	10	20
15 – 25	4	20	80
25 – 35	3	30	90
35 – 45	1	40	40
<b>Total</b>	10		230

$$\text{Mean, } \bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{230}{10}$$

$$= 23$$

Therefore, mean for the following distribution is 23.

**OR**

Expenditure	200 – 400	400 – 600	600 – 800	800 – 1000	1000 – 1200
Number of employees	21	25	19	23	12

From the given data, we have

$$l = 400, f_1 = 25, f_0 = 21, f_2 = 19, h = 200$$

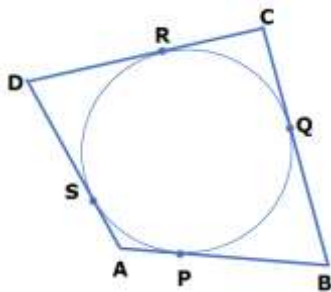
$$\text{Mode} = l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 400 + \left( \frac{25 - 21}{2 \times 25 - 21 - 19} \right) \times 200$$

$$= 480$$

∴ Mode of the given data is 480.

27.



Section C

In the given figure, quadrilateral  $ABCD$  is circumscribing the given circle and its sides are touching the circle at  $P, Q, R$  and  $S$ .

We have to prove that

$$AB + CD = AD + BC$$

We know that lengths of tangents drawn from a point to a circle are equal.

Therefore, from figure, we have

$$DR = DS, CR = CQ, AS = AP, BP = BQ$$

Now,

$$\begin{aligned}
 \text{L.H.S.} &= AB + CD = (AP + BP) + (CR + DR) \\
 &= (AS + BQ) + (CQ + DS) \\
 &= AS + DS + BQ + CQ \\
 &= AD + BC \\
 &= \text{R.H.S}
 \end{aligned}$$

28. Let the larger number be  $y$  and the smaller number be  $x$ .

According to question,

$$y - x = 26 \quad \dots(1)$$

$$\text{and } y = 3x + 4 \quad \dots(2)$$

Substituting the value of  $y$  from equation (2) in equation (1), we get

$$3x + 4 - x = 26$$

$$\text{or } 2x = 26 - 4$$

$$\text{or } 2x = 22$$

$$\text{or } x = 11$$

Putting this value of  $x$  in equation (1), we get

$$y - 11 = 26$$

$$\text{or } y = 26 + 11 = 37$$

Hence, the numbers are 11 and 37.

**OR**

$$\frac{2}{x} + \frac{3}{y} = 13 \quad \text{and} \quad \frac{5}{x} - \frac{4}{y} = -2$$

$$\text{Let } \frac{1}{x} = p \quad \text{and} \quad \frac{1}{y} = q$$

then given equations can be written as:

$$2p + 3q = 13$$

$$2p + 3q - 13 = 0 \quad \dots(1)$$

and

$$5p - 4q = -2$$

$$5p - 4q + 2 = 0 \quad \dots(2)$$

Using cross-multiplication method, we get

$$\frac{p}{6-52} = \frac{q}{-65-4} = \frac{1}{-8-15}$$

$$\Rightarrow \frac{p}{-46} = \frac{q}{-69} = \frac{1}{-23}$$

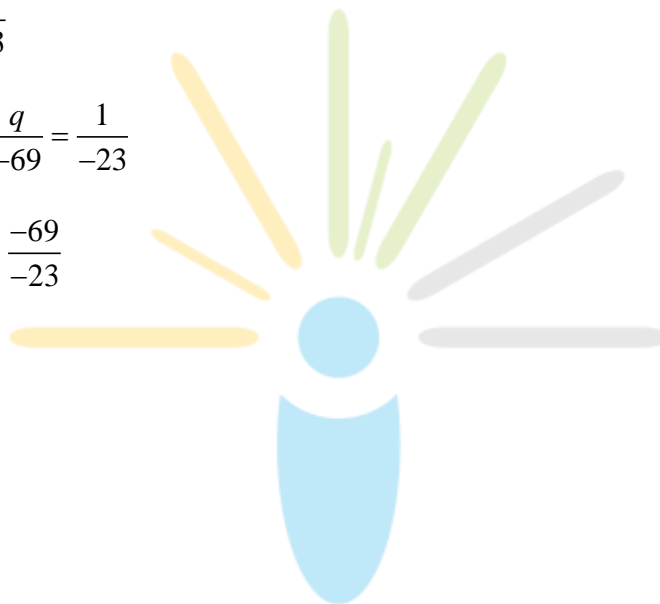
$$\Rightarrow \frac{p}{-46} = \frac{1}{-23} \text{ and } \frac{q}{-69} = \frac{1}{-23}$$

$$\Rightarrow p = \frac{-46}{-23} \text{ and } q = \frac{-69}{-23}$$

$$\Rightarrow p = 2 \text{ and } q = 3$$

$$\Rightarrow \frac{1}{x} = 2 \text{ and } \frac{1}{y} = 3$$

$$\Rightarrow x = \frac{1}{2} \text{ and } y = \frac{1}{3}$$



29. Let us assume that  $\sqrt{3}$  is rational.

So we can find integers  $r$  and  $s (\neq 0)$  such that  $\sqrt{3} = \frac{r}{s}$ .

Suppose  $r$  and  $s$  have a common factor other than 1.

Then we divide  $r$  and  $s$  by the common factor and get  $\sqrt{3} = \frac{a}{b}$

where  $a$  and  $b$  are coprime.

So,  $\sqrt{3}b = a$

Squaring on both sides, we get

$$3b^2 = a^2$$

Therefore,

$a^2$  is divisible by 3, and so is also divisible by 3.

So, we can write  $a = 3c$  for some integer  $c$ .

Now,

$$3b^2 = a^2$$

$$\Rightarrow 3b^2 = 9c^2$$

$$\Rightarrow b^2 = 3c^2$$

This means that  $b^2$  is divisible by 3, and so  $b$  is also divisible by 3.

Therefore,

$a$  and  $b$  have at least 3 as a common factor.

But this contradicts the fact that  $a$  and  $b$  are coprime. So, our assumption that  $\sqrt{3}$  is a rational is wrong.

Hence,  $\sqrt{3}$  is an irrational number.

30. (i) From the given figure, the coordinates of points  $A, B, C$  and  $D$  can be written as below:

$A(2,2), B(5,4), C(7,7)$  and  $D(4,5)$ .

(ii) We know that a quadrilateral is a parallelogram if its opposite sides are equal.

Now, using distance formula, we will find the length of each side of the quadrilateral  $ABCD$ .

$$AB = \sqrt{(5-2)^2 + (4-2)^2} = \sqrt{9+4} = \sqrt{13}$$

$$BC = \sqrt{(7-5)^2 + (7-4)^2} = \sqrt{4+9} = \sqrt{13}$$

$$CD = \sqrt{(4-7)^2 + (5-7)^2} = \sqrt{9+4} = \sqrt{13}$$

$$DA = \sqrt{(2-4)^2 + (2-5)^2} = \sqrt{4+9} = \sqrt{13}$$

We see that sides  $AB, BC, CD$  and  $DA$  are equal in lengths, Therefore, quadrilateral  $ABCD$  is a parallelogram.

31. Here,  $S_{14} = 1050, a = 10$

We have to find  $a_{21}$ .

We know that sum of first  $n$  terms of an AP is given by

$$S_n = \frac{n}{2}[2a + n - 1d]$$

$$\text{So, } S_{14} = \frac{14}{2} 2 \times 10 + 13 \times d$$

$$1050 = 7(20 + 13d)$$

$$\text{or } d = 10$$

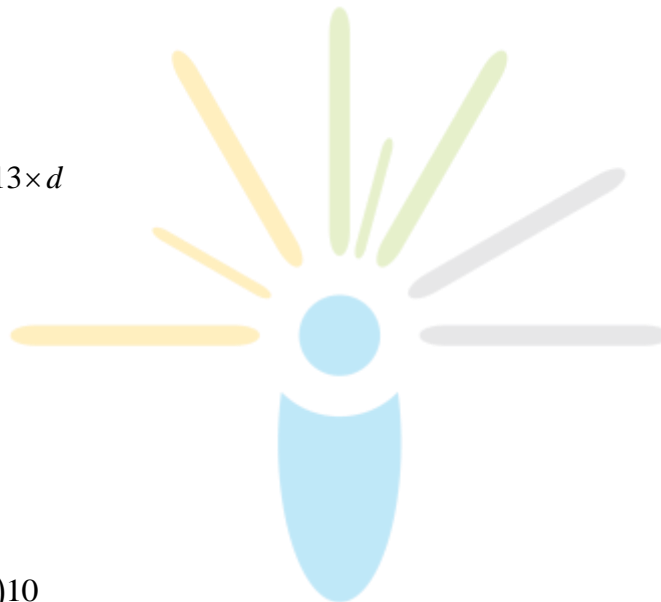
We know that

$$a_n = a + (n-1)d$$

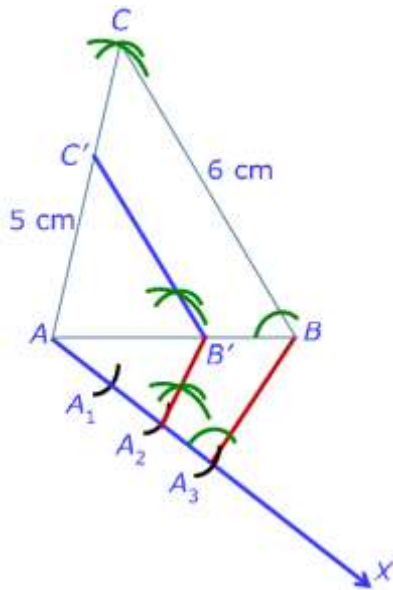
$$\text{So, } a_{21} = 10 + (21-1)10$$

$$= 10 + 20 \times 10$$

$$= 210$$



32.



**Step 1:** Draw a line segment  $AB = 4\text{ cm}$ . Taking point  $A$  as centre, draw an arc of  $5\text{ cm}$  radius. Again, taking point  $B$  as centre, draw an arc of  $6\text{ cm}$ . These arcs intersect each other at point  $C$ . So, we have  $AC = 5\text{ cm}$  and  $BC = 6\text{ cm}$ .  $\triangle ABC$  is the required triangle.

**Step 2:** Draw a ray  $AX$  making an acute angle with line  $AB$  on the opposite side of vertex  $C$ .

**Step 3:** Locate 3 points  $A_1, A_2, A_3$  on  $AX$  such that

$$AA_1 = A_1A_2 = A_2A_3.$$

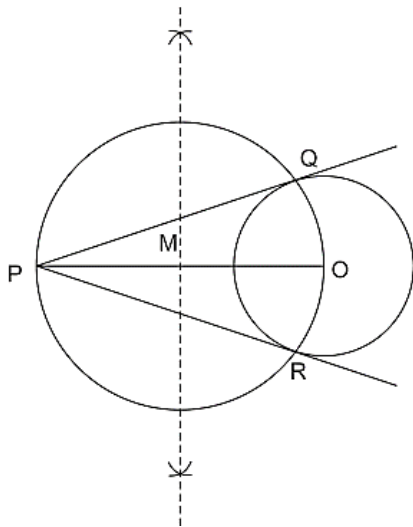
**Step 4:** Join the points  $B$  and  $A_3$ .

**Step 5:** Through the point  $A_2$ , draw a line parallel to  $BA_3$  intersecting  $AB$  at point  $B'$ .

**Step 6:** Draw a line through  $B'$  parallel to the line  $BC$  to intersect  $AC$  at  $C'$ .

The required triangle is  $\triangle AB'C'$ .

OR



**Steps of Construction :**

**Step 1:** Draw a circle of radius 2.5cm with centre at point  $O$ .

Locate a point  $P$ , at a distance of 8cm from  $O$ , and join  $O$  and  $P$ .

**Step 2:** Bisect  $OP$ . Let  $M$  be the mid-point of  $OP$ .

**Step 3:** Draw a circle with centre at  $M$  and  $MO$  as radius.

$Q$  and  $R$  are points of intersections of this circle with the circle having centre at  $O$ .

**Step 4:** Join  $PQ$  and  $PR$ .

$PQ$  and  $PR$  are the required tangents.

$$33. \text{LHS} = (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$$

$$= \left( \frac{1}{\sin A} - \sin A \right) \left( \frac{1}{\cos A} - \cos A \right)$$

$$= \left( \frac{1 - \sin^2 A}{\sin A} \right) \left( \frac{1 - \cos^2 A}{\cos A} \right)$$

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A}$$

$$= \frac{\sin A \cos A}{1}$$



$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$$

$$= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\tan A + \cot A}$$

= RHS

34. For bigger circle,  $OA = 7$  cm

Diameter of the smaller circle = 7 cm

Radius of the smaller circle =  $\frac{7}{2}$  cm

Area of the smaller circle =  $\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$$= \frac{77}{2} \text{ cm}^2$$

Area of shaded region

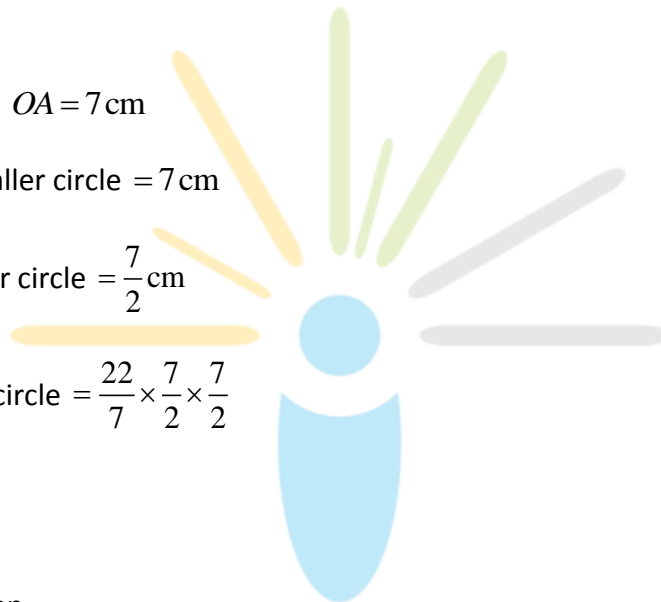
= Area of the smaller circle + 2 × Area of segment  $OCB$

$$= \frac{77}{2} \text{ cm}^2 + 2 \times (\text{Area of quadrant} - \text{Area } \triangle ABC)$$

$$= \frac{77}{2} \text{ cm}^2 + 2 \times \left( \frac{1}{4} \times \frac{22}{7} \times 7^2 - \frac{1}{2} \times 7 \times 7 \right) \text{ cm}^2$$

$$= \frac{77}{2} \text{ cm}^2 + 49 \left( \frac{11}{7} - 1 \right) \text{ cm}^2$$

$$= \frac{77}{2} \text{ cm}^2 + 49 \left( \frac{4}{7} \right) \text{ cm}^2$$



$$= \frac{77}{2} \text{cm}^2 + 28 \text{cm}^2$$

$$= \frac{77 + 56}{2} \text{cm}^2$$

$$= \frac{133}{2} \text{cm}^2$$

$$= 66.5 \text{cm}^2$$

**OR**

$ABCD$  is a square with side 7 cm . Then,

Length of the diagonal of square =  $7\sqrt{2}$  cm

Diameter of circle = Diagonal of square

$$\Rightarrow BD = 7\sqrt{2} \text{ cm}$$

$$\text{Radius of circle} = \frac{BD}{2}$$

$$= \frac{7\sqrt{2}}{2} \text{ cm}$$

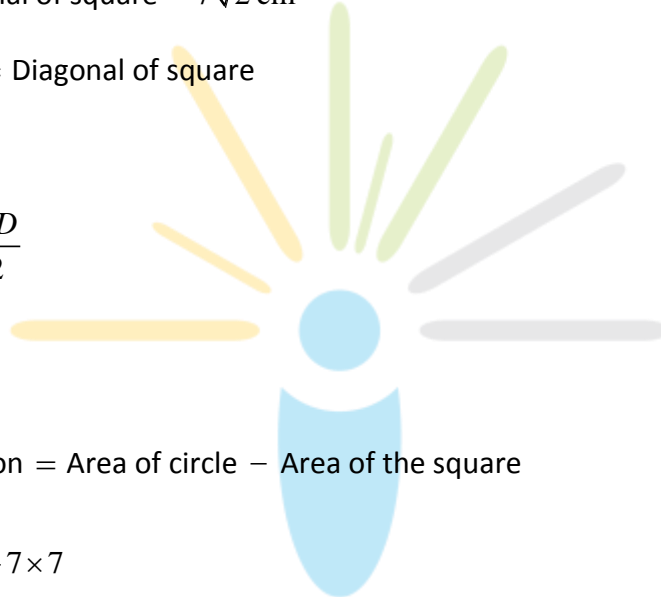
Area of shaded region = Area of circle – Area of the square

$$= \frac{22}{7} \times \frac{7\sqrt{2}}{2} \times \frac{7\sqrt{2}}{2} - 7 \times 7$$

$$= 77 - 49$$

$$= 28 \text{cm}^2$$

Therefore, the area of the shaded region is  $28 \text{cm}^2$  .



### Section D

35. The given polynomial is  $p(x) = 3x^4 - 4x^3 - 10x^2 + 8x + 8$

The two zeroes of  $p(x)$  are  $\sqrt{2}$  and  $-\sqrt{2}$ .

Therefore,  $(x - \sqrt{2})$  and  $(x + \sqrt{2})$  are factors of  $p(x)$ .

Also,  $(x - \sqrt{2})(x + \sqrt{2}) = x^2 - 2$

and so  $x^2 - 2$  is a factor of  $p(x)$ .

Now,

$$\begin{array}{r}
 \phantom{x^2 - 2} \overline{3x^2 - 4x - 4} \\
 x^2 - 2 \overline{) 3x^4 - 4x^3 - 10x^2 + 8x + 8} \\
 \underline{3x^4 \phantom{- 4x^3} - 6x^2} \phantom{+ 8x + 8} \\
 -4x^3 - 4x^2 + 8x + 8 \\
 \underline{-4x^3 \phantom{- 4x^2} + 8x} \phantom{+ 8} \\
 + \phantom{- 4x^2} - 4x^2 + 8 \\
 \phantom{+} \underline{-4x^2 \phantom{+ 8}} \\
 \phantom{+} \phantom{- 4x^2} + 8 \\
 \phantom{+} \phantom{- 4x^2} \underline{+ \phantom{- 8}} \\
 \phantom{+} \phantom{- 4x^2} \phantom{+ 8} 0
 \end{array}$$



$$3x^4 - 4x^3 - 10x^2 + 8x + 8 = (x^2 - 2)(3x^2 - 4x - 4)$$

$$= (x^2 - 2)(3x^2 - 6x + 2x - 4)$$

$$= (x^2 - 2)(3x + 2)(x - 2)$$

Equating  $(x^2 - 2)(3x + 2)(x - 2)$  to zero, we get the zeroes of the given polynomial.

Hence, the zeroes of the given polynomial are:

$$\sqrt{2}, -\sqrt{2}, -\frac{2}{3} \text{ and } 2.$$

OR

The given polynomial is  $g(x) = x^3 - 3x^2 + x + 2$

Here, divisor is  $x^2 - 2x + 1$ .

Divide  $g(x) = x^3 - 3x^2 + x + 2$  by  $x^2 - 2x + 1$  and find the remainder.

$$\begin{array}{r}
 \phantom{x^2 - 2x + 1} \overline{) x^3 - 3x^2 + x + 2} \\
 \underline{x^3 - 2x^2 + x} \phantom{+ 2} \\
 -x^2 \phantom{+ x} + 2 \\
 \underline{-x^2 + 2x - 1} \\
 +x - 3 \\
 \underline{+x - 2x + 3} \\
 -2x + 3
 \end{array}$$

So, Quotient =  $x - 1$  and Remainder =  $-2x + 3$ .

The division algorithm states that

Dividend = Divisor  $\times$  Quotient + Remainder

RHS = Divisor  $\times$  Quotient + Remainder

$$= (x^2 - 2x + 1)(x - 1) - 2x + 3$$

$$= x^3 - 2x^2 + x - x^2 + 2x - 1 - 2x + 3$$

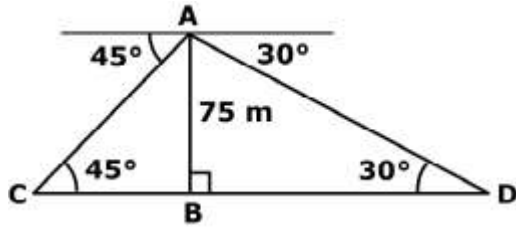
$$= x^3 - 3x^2 + x + 2$$

= LHS

Thus, the division algorithm is verified.

36. Let  $AB$  be a lighthouse and ships be at points  $C$  and  $D$ .

It is given that  $AB = 75\text{ m}$ . We have to find the distance  $CD$ .



In  $\triangle ABC$ , we have

$$\tan 45^\circ = \frac{AB}{BC}$$

$$\text{or } 1 = \frac{AB}{BC}$$

$$\text{or } BC = AB = 75 \quad \dots(1)$$

Now,

In  $\triangle ABD$ , we have

$$\tan 30^\circ = \frac{AB}{BD}$$

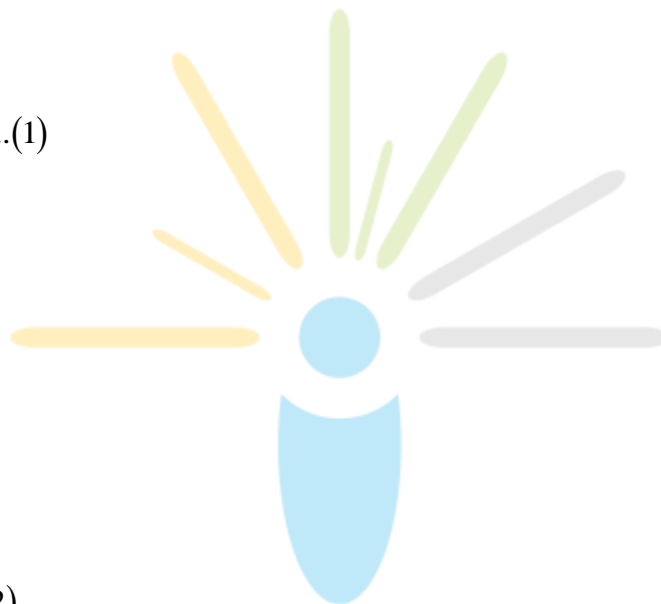
$$\text{or } \frac{1}{\sqrt{3}} = \frac{75}{BD}$$

$$\text{or } BD = 75\sqrt{3} \quad \dots(2)$$

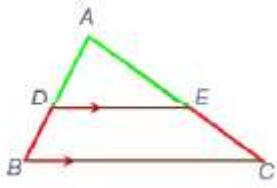
From (1) and (2), we get

$$CD = BC + BD = 75 + 75\sqrt{3} = 75(1 + \sqrt{3})$$

Therefore, the distance between the two ships is  $75(1 + \sqrt{3})\text{ m}$ .



37.



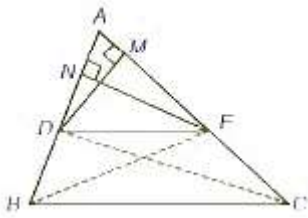
Given In  $\triangle ABC = DE \parallel BC$ .

To Prove:  $\frac{AD}{DB} = \frac{AE}{EC}$ .

Construction:

i. Join  $BE$  and  $CD$ .

ii Draw  $DM \perp AC$  and  $EN \perp AB$ .



Proof:

$$\text{area}(\triangle ADE) = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times AD \times EN$$

and

$$\text{area}(\triangle BDE) = \frac{1}{2} \times BD \times EN$$

Therefore,

$$\frac{\text{area} \triangle ADE}{\text{area} \triangle BDE} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times BD \times EN} = \frac{AD}{BD} \dots(1)$$

Similarly,

$$\frac{\text{area } \triangle ADE}{\text{area } \triangle DEC} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad \dots(2)$$

$$\text{But area}(\triangle BDE) = \text{area}(\triangle DEC) \quad \dots(3)$$

Triangles on the same base and between the same parallels are equal in area.

Therefore, from (1), (2) and (3), we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence, proved.

OR

$ABC$  is an equilateral triangle.

Therefore,  $AB = BC = AC$

Now,

$$BD = DC = \frac{BC}{2} = \frac{AB}{2} \quad (D \text{ is the midpoint of } BC \text{ and } AB = BC)$$

Using Pythagoras theorem in  $\triangle ADC$ , we get

$$AC^2 = AD^2 + DC^2$$

$$AB^2 = AD^2 + \left(\frac{AB}{2}\right)^2 \quad \left(AC = AB \text{ and } DC = \frac{AB}{2}\right)$$

$$AB^2 = AD^2 + \frac{AB^2}{4}$$

$$AB^2 - \frac{AB^2}{4} = AD^2$$

$$\frac{3AB^2}{4} = AD^2$$

$$3AB^2 = 4AD^2 \quad \dots(1)$$

Similarly, using Pythagoras theorem in  $\triangle AEB$ , we get

$$3AB^2 = 4BE^2 \quad \dots(2)$$

Again, using Pythagoras theorem in  $\triangle AFC$ , we get

$$3AB^2 = 4CF^2 \quad \dots(3)$$

On adding equations (1), (2) and (3), we get

$$3AB^2 + 3AB^2 + 3AB^2 = 4AD^2 + 4BE^2 + 4CF^2$$

$$\text{or, } 9AB^2 = 4AD^2 + BE^2 + CF^2$$

$$\text{or, } 4AD^2 + BE^2 + CF^2 = 9AB^2$$

Hence, proved

38. Height of the frustum =  $h = 14\text{cm}$

Radius of upper end of the frustum =  $r_1 = 20\text{cm}$

Radius of lower end of the frustum =  $r_2 = 8\text{cm}$

Capacity of container = Volume of the frustum

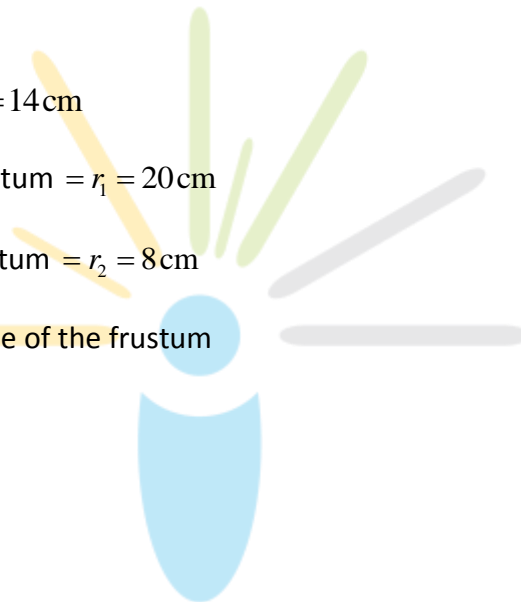
$$= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1r_2)$$

$$= \frac{14}{3} \times \frac{22}{7} (20^2 + 8^2 + 20 \times 8)$$

$$= \frac{44}{3} (400 + 64 + 160)$$

$$= \frac{44}{3} \times 624$$

$$= 9152\text{cm}^3$$





39. Let  $A$  and  $B$  be the time taken by the smaller and the larger taps respectively to fill the tank.

Since both the taps together can fill the tank in  $9\frac{3}{8}$  hours =  $\frac{75}{8}$  hours.

$$\text{So, } \frac{1}{A} + \frac{1}{B} = \frac{1}{\frac{75}{8}}$$

$$\text{Or, } \frac{1}{A} + \frac{1}{B} = \frac{8}{75} \quad \dots 1$$

Tap with larger diameter takes 10 hours less than smaller one to fill the tank.

$$\text{So, } A - 10 = B$$

$$\text{Or, } \frac{1}{B} = \frac{1}{A - 10} \quad \dots (2)$$

By placing the value of  $\frac{1}{B}$  from 2 in to 1, we get

$$\frac{1}{A} + \frac{1}{A - 10} = \frac{8}{75} \quad \dots 1$$

$$\text{Or, } \frac{A - 10 + A}{A^2 - 10A} = \frac{8}{75}$$

$$\text{Or, } \frac{A - 5}{A^2 - 10A} = \frac{4}{75}$$

$$\text{Or, } 75A - 375 = 4A^2 - 40A$$

$$\text{Or, } 4A^2 - 40A - 75A + 375 = 0$$

$$\text{Or, } 4A^2 - 115A + 375 = 0$$

$$\text{Or, } 4A^2 - 100A - 15A + 375 = 0$$

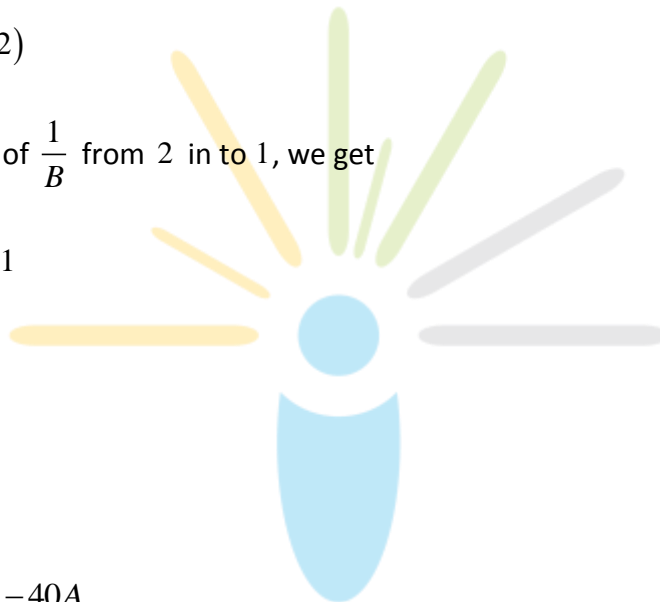
$$\text{Or, } 4A(A - 25) - 15(A - 25) = 0$$

$$\text{Or, } (A - 25)(A - 15) = 0$$

$$A = 25 \text{ Hours}$$

$$\left( A \neq \frac{15}{4} \text{ hours, because } B \text{ becomes negative.} \right)$$

$$\text{So, } B = 25 - 10 = 15 \text{ hours}$$



OR

Let  $L$  be the length of the rectangle.

So, breadth of the rectangle =  $L - 3$

Area of the rectangle =  $LL - 3$  ...1

Base of the isosceles triangle =  $L - 3$

Altitude of the isosceles triangle = 12m

Area of the isosceles triangle =  $\frac{1}{2}(12)(-3)$  ... (2)

Given that

$$L(-3) = \frac{1}{2}(12)(-3) - 4$$

$$\text{Or, } L^2 - 3L = 6L - 18 + 4$$

$$\text{Or, } L^2 - 9L + 14 = 0$$

$$\text{Or, } L^2 - 7L - 2L + 14 = 0$$

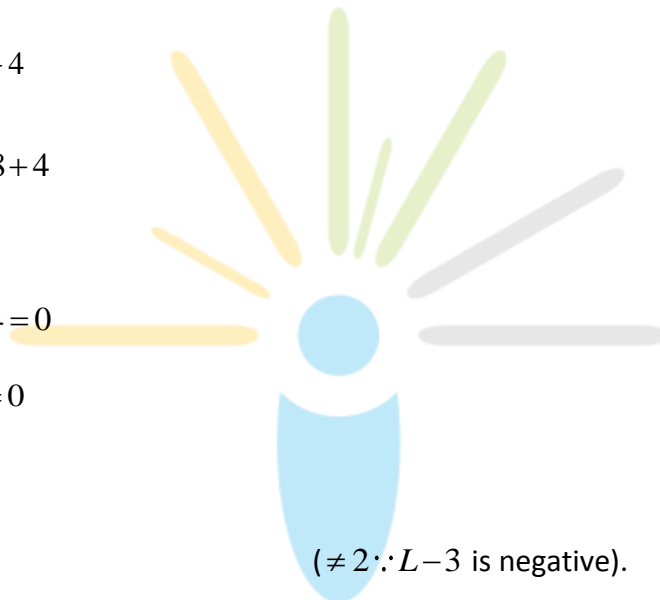
$$\text{Or, } L(-7) - 2(-7) = 0$$

$$\text{Or, } (-7)(-2) = 0$$

$$\text{So, } L = 7 \text{ m}$$

$$\text{Breadth} = 7 \text{ m} - 3 \text{ m} = 4 \text{ m}$$

Length and breadth of the rectangle are 7 m and 4 m respectively.



( $\neq 2$   $\because L - 3$  is negative).

40.

Marks	Cumulative frequency
Less than 10	7
Less than 20	$7 + 14 = 21$
Less than 30	$21 + 13 = 34$
Less than 40	$34 + 12 = 46$
Less than 50	$46 + 20 = 66$
Less than 60	$66 + 11 = 77$
Less than 70	$77 + 15 = 92$
Less than 80	$92 + 8 = 100$

Marks	Frequency ( $f$ )	Cumulative frequency ( $cf$ )
0–10	7	7
10–20	14	21
20–30	13	34
30–40	12	46
40–50	20	66
50–60	11	77
60–70	15	92
70–80	8	100

Now, plot  $(10, 7), (20, 21), \dots, (80, 100)$  on the graph.

