Mathematics (Standard) – Theory

Answers

2019-2020

Time Allowed: 3 hours **Maximum Marks: 80**

Section A

1. **Correct Answer: B**

$P(x) =$ (divisor) $\times$ (quotient) $+$ Remainder

$= (x^2 - 4)x + 3$

$= x^3 - 4x + 3$

2. **Correct Answer: A**

Given that ACB is an isosceles triangle right angled at C.

Therefore, $AC = BC$

Using Pythagoras theorem in the given triangle,

We have

$AB^2 = AC^2 + BC^2$

$= AC^2 + AC^2$

$= 2AC^2$

3. **Correct Answer: D**

The required point and the given points as well lie on the x-axis.

The required point $(x, 0)$ is the mid-point of the line joining points $(-4, 0)$ and $(10, 0)$.

So, $x = \dfrac{(-4+10)}{2}$

$=\frac{6}{2}$

$=3$

Required point $=(x,0)$

$=(3,0)$

OR

Correct Answer: C

The centre of a circle is the mid-point of its diameter.

End points of the diameter are: $(-6,3)$ and $(6,4)$

Coordinates of the centre $=\left(\frac{(-6+6)}{2},\frac{(3+4)}{2}\right)$

$=\left(0,\frac{7}{2}\right)$

4. **Correct Answer: B**

The given equation is:

$2x^2+kx+2=0$

Discriminant $=b^2-4ac$

Here, $b=k, a=2,$ and $c=2$

So, Discriminant $=k^2-4\times2\times2$

$=k^2-16$

A quadratic equation has equal roots if its discriminant is zero.

$k^2-16=0$

$k^2=16$

$k=\pm4$

5. **Correct Answer: C**

$\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \dots$

$\frac{7}{3} - \frac{4}{3} = \frac{7-4}{3}$

$= \frac{3}{3}$

$= 1$

$\frac{9}{3} - \frac{7}{3} = \frac{9-7}{3}$

$= \frac{2}{3}$

$\Rightarrow \frac{3}{3} \neq \frac{2}{3}$

Difference between consecutive terms is not same. So, this is not an A.P.

6. **Correct Answer: B**

$\frac{3x}{2} + \frac{5y}{3} = 7$

$\frac{9x + 10y}{6} = 7$

$9x + 10y = 42$ ....(1)

$9x + 10y = 14$ ....(2)

Ratios of coefficients of x and that of y are

$\frac{9}{9} = \frac{10}{10} = \frac{1}{1}$

Ratio of constants $= \frac{42}{14} = \frac{3}{1} \neq \frac{1}{1}$

Ratios of coefficients of x and y are equal but they are not equal to the ratio of constants.

So, the given equations represent a pair of parallel lines and so they do not have a common solution.

7. **Correct Answer: A**

$OA = OB$ (radii)

So, $\angle OAB = \angle OBA$

$= \frac{(180° - 100°)}{2}$

$= 40°$

Now, a radius of a circle meets a tangent at $90°$.

So, $\angle ABP = \angle OBP - \angle OBA$

$= 90° - 40° = 50°$

8. **Correct Answer: C**

Volume of sphere $= \frac{4}{3}\pi r^3$

$12\pi = \frac{4}{3}\pi r^3$

$r^3 = 3^2$

$r = 3^{\frac{2}{3}}$

9. **Correct Answer: C**

Distance $= \sqrt{m - (-m)^2 + (-n - n)^2}$

$= \sqrt{(m + m)^2 + (-2n)^2}$

$= 2\sqrt{m^2 + n^2}$

10. **Correct Answer: B**

Tangents are drawn from an external point P.

So, line joining centre O and point P bisects $\angle PQR$.

OP bisects $\angle QPR = 90°$.

In ΔOQP

$\angle Q = 90°$ (radius meets tangent at $90°$)

$\angle QPO = 45° = \angle QOP$

Thus, $OQ = PQ = 4\,\text{cm}$

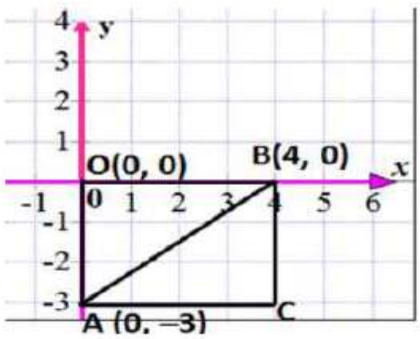
11. 1

12. $\cot^2 A$

$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\text{cosec}^2 A} = \frac{\sin^2 A}{\cos^2 A} = \cot^2 A$$

13. In right –angled triangle AOB,

$$AB = \sqrt{OA^2 + OB^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$



14. $\frac{x_i - a}{h}$

15. Similar

16. $1+2+3+\ldots 100$ is an $A.P.$

Here first term $a=1$

Common difference $d=1$

Sum of n terms of an A.P. $=\frac{n}{2}\left[2a+(n-1)d\right]$

The sum of first 100 natural numbers

$=\frac{100}{2}\left[2\times 1+(100-1)\times 1\right]$

$=\frac{100(101)}{2}$

$=50\times 101$

$=5050$

17. $\tan 30°=\frac{AB}{30}$

$\frac{1}{\sqrt{3}}=\frac{AB}{30}$

$AB=\frac{30}{\sqrt{3}}=10\sqrt{3}$

So, the height of the tower is $10\sqrt{3}\,\text{m}$.

18. $LCM\times HCF=$ Product of the two numbers

$182\times 13=26\times x$

$x=\frac{182\times 13}{26}=91$

So, the other number is 91.

19. $x^2 -$ (sum of zeroes) $x +$ product of zeroes

$= x^2 - (-3)x + 2$

$= x^2 + 3x + 2$

So, the required polynomial is $x^2 + 3x + 2$.

OR

When a polynomial $p(x)$ is divided by another polynomial $g(x)$, then the degree of remainder $r(x) <$ degree of $g(x)$

Therefore, for the given question $x^2 - 1$ cannot be a remainder while dividing $x^4 - 3x^2 + 5x - 9$ by $x^2 + 3$ because $\deg(x^2 - 1) = \deg(x^2 + 3)$.

20. $\frac{2\tan 45° \times \cos 60°}{\sin 30°}$

$= \frac{2 \times 1 \times \frac{1}{2}}{\frac{1}{2}}$

$= 2$

Section B

21. In $\Delta ABC, DE \parallel AC$

So, using basic proportionality theorem, we get

$$\frac{BD}{DA}=\frac{BE}{EC} \quad ....(1)$$

In $\Delta BAE, DF \parallel AE$

So, using basic proportionality theorem, we get

$$\frac{BD}{DA}=\frac{BF}{FE} \quad ....(2)$$

From (1) an (2), we get

$$\frac{BE}{EC}=\frac{BF}{FE}$$

22. Let us assume, to the contrary, that $5+2\sqrt{7}$ is rational.

That is, we can find coprime a and $b(b\neq 0)$ such that

$$5+2\sqrt{7}=\frac{a}{b}$$

$$\therefore 2\sqrt{7}=\frac{a}{b}-5$$

Rearranging this equation, we get $\sqrt{7}=\frac{1}{2}\left(\frac{a}{b}-5\right)=\frac{a-5b}{2b}$

Since, a and b are integers, we get $\frac{a-5b}{2b}$ is rational, and so $\sqrt{7}$ is a rational.

But this contradicts the fact that $\sqrt{7}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $5+2\sqrt{7}$ is rational.

So, we conclude that $5+2\sqrt{7}$ is irrational.

OR

If the number 12^n, for any n, were to end with the digit zero, then it would be divisible by 5.

That is, the prime factorisation of 12^n would contain the prime 5. This is not possible

$\because 12^n = (2\times2\times3)^n$

So, the prime numbers in the factorisation of 12^n are 2 and 3.

So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorisation of 12^n.

So, there is no natural number n for which 12^n ends with the digit zero.

23. Given that A, B and C are interior angles of a triangle ABC.

$\therefore A+B+C=180^\circ$

or $A=180^\circ-B-C$

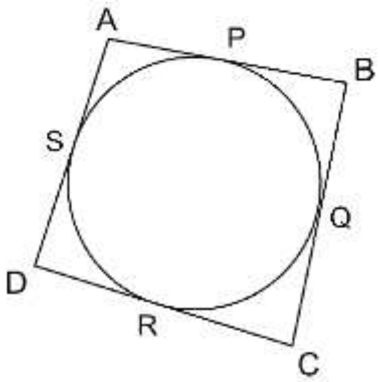
Now,

$$\cos\left(\frac{B+C}{2}\right)=\sin\left(90^\circ-\frac{B+C}{2}\right)$$

$$=\sin\left(\frac{180^\circ-B-C}{2}\right)$$

$$=\sin\left(\frac{A}{2}\right)$$

24.



We have to prove that

$AB+CD=BC+AD$

We know that lengths of tangents drawn from a point to a circle are equal.

Therefore, from figure, we have

$DR=DS, CR=CQ, AS=AP, BP=BQ$

Now,

$LHS=AB+CD=(AP+BP)+(CR+DR)$

$=(AS+BQ)+(CQ+DS)$

$=BQ+CQ+AS+DS$

$=BC+AD$

$=RHS$

OR

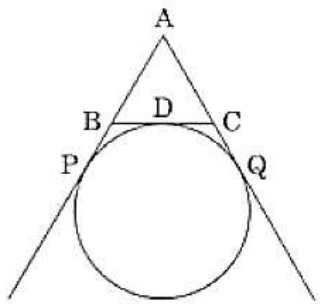


Figure-7

From the given figure, we have $AP=12\,\text{cm}$

Since AQ and AB are the tangent to the circle from a common point A, hence $AP=AQ=12$

Similarly, $PB=BD$ and $CD=CQ$

Also, $AP=AB+PB$ and $AQ=AC+CQ$

Perimeter of $ABC=AB+BD+CD+AC$

$= AB + PB + CQ + AC$
$(\text{since } PB = BM \text{ and } CM = CQ)$

$= (AB + PB) + (CQ + AC)$

$= AP + AQ$

$= 12 + 12$

$= 24 \text{ cm}$

Therefore, the perimeter of triangle $ABC = 24 \text{ cm}$

25.

Marks	$0-10$	$10-20$	$20-30$	$30-40$	$40-50$	$50-60$
Number of Students	4	6	7	12	5	6

From the given data, we have

$I = 30, f_1 = 12, f_0 = 7, f_2 = 5, h = 10$

Mode $= I + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 30 + \left(\dfrac{12 - 7}{2 \times 12 - 7 - 5}\right) \times 10$

$= 34.1667$

$\therefore$ Mode of the given data is 34.1667 .

26. Let the side of the old cube $= a$

The volume of the old cube $= 125 \text{ cm}^3$ (Given)

The volume of the cube $= a^3$

$a^3 = 125 \text{ cm}^3$

$a^3 = 5^3$

$a = 5\text{cm}$

The dimensions of the resulting cuboid are:

Length, $l = 10$ cm

Breadth, $b = 5\text{cm}$

Height, $h = 5$ cm

Total surface area of the resulting cuboid:

$= 2(lb + bh + hl)$

$= 2[10(5) + 5(5) + 5(10)]$

$= 2[50 + 25 + 50]$

$= 2[125]$

$= 250 \text{ cm}^2$

Section C

27. Let the numerator of the fraction be x and denominator be y.

Therefore, the fraction is $\frac{x}{y}$.

According to question,

$\frac{x-1}{y} = \frac{1}{3}$

$3x - 1 = y$

$3x - 3 = y$ ....(1)

And $\frac{x}{y+8} = \frac{1}{4}$

$4x = y + 8$

$4x - 8 = y$ ....(2)

From equations 1 and 2, we get

$3x-3=4x-8$

$4x-3x=8-3$

$x=5$

Putting $x=5$ in equation (1),

$3\times5-3=y$

$y=12$

So, the required fraction $=\frac{5}{12}$.

OR

Let the son's present age be x.

So, father's present age $=3x+3$

3 years later:

Son's age $=x+3$

Father's age $=3x+3+3=3x+6$

But, according to the given condition,

3 years later father's age $=2x+3+10$

$=2x+6+10$

$=2x+16$

So, we can write

$3x+6=2x+16$

$3x-2x=16-6$

$x=10$

So, son's present age $=10$ years

and father's present age $=10\times3+3$

$=33\text{years}$

28. Let a be a positive integer and $b=3$.

By Euclid's Algorithm,

$a=3\mathrm{m}+r$ for some integer $\mathrm{m}\geq 0$ and $0\leq r<3$

The possible remainders are $0,1$ and 2. Therefore, a can be $3\mathrm{m}$ or $3\mathrm{m}+1$ or $3\mathrm{m}+2$

Thus,

$a^2=9\mathrm{m}^2$ or $(3\mathrm{m}+1)^2$ or $(3\mathrm{m}+2)^2$

$=9\mathrm{m}^2 \text{ or} \left(9\mathrm{m}^2+6\mathrm{m}+1\right) \text{or} \left(9\mathrm{m}^2+12\mathrm{m}+4\right)$

$=3\times\left(3\mathrm{m}^2\right) \text{or } 3\left(3\mathrm{m}^2+2\mathrm{m}\right)+1 \text{or } 3\left(3\mathrm{m}^2+4\mathrm{m}+1\right)+1$

$=3k_1 \text{ or } 3k_2+1 \text{or } 3k_3+1$

where k_1,k_2 and k_3 are some positive integers.

Hence, square of any positive integer is either of the form $3q \text{ or } 3q+1$ for some integer q.

29. Let the ratio in which the line segment joining $A(6,-4)$

and $B(-2,-7)$ is divided by the $y-$axis be $k:1$.

Let the coordinate of point on $y-$axis be $(0,y)$.

Therefore,

$0=\dfrac{-2k+6}{k+1}$ and $y=\dfrac{-7k-4}{k+1}$

Now,

$0=\dfrac{-2k+6}{k+1}$

Or $0=-2k+6$

Or $k=3$

Therefore, the required ratio is $3:1$.

Also,

$$y=\frac{-7k-4}{k+1}$$

$$=\frac{-7\times3-4}{3+1}$$

$$=\frac{-25}{4}$$

Therefore, the given line segment is divided by the point $\left(0,\frac{-25}{4}\right)$ in the ratio $3:1$.

OR

Let the given points are $P(7,10), Q(-2,5)$ and $R(3,4)$.

Now, using distance formula we find distance between these points i.e., PQ, QR and PR.

Distance between points $P(7,10)$ and $Q(-2,5)$,

$$PQ=\sqrt{(-2-7)^2+(5-10)^2}$$

$$=\sqrt{81+25}$$

$$=\sqrt{106}$$

Distance between points $Q(-2,5)$ and $R(3,4)$,

$$QR=\sqrt{(3+2)^2+(-4-5)^2}$$

$$=\sqrt{25+81}$$

$$=\sqrt{106}$$

Distance between points $P(7,10)$ and $R(3,4)$,

$$PR=\sqrt{(3-7)^2+(-4-10)^2}$$

$$=\sqrt{16+196}$$

$$\sqrt{212}$$

Now,

$PQ^2 + QR^2 = 106 + 106$

$= 212 = PR^2$

i.e., $PQ^2 + QR^2 = PR^2$

Therefore, points $P(5,-2), Q(6,4)$ and $R(7,-2)$ form an isosceles right triangle because sides PQ and QR are equal.

30. $\text{LHS} = \sqrt{\frac{1+\sin A}{1-\sin A}}$

$= \sqrt{\frac{1+\sin A}{1-\sin A}} \times \sqrt{\frac{1+\sin A}{1+\sin A}}$

$= (1+\sin A)\sqrt{\frac{1}{1+\sin^2 A}}$

$= \frac{1+\sin A}{\sqrt{\cos^2 A}}$

$= \frac{1+\sin A}{\cos A}$

$= \frac{\sin A}{\cos A} + \frac{1}{\cos A}$

$= \tan A + \sec A = RHS$

31. Here, $a = 5, d = 3, a_n = 50$

We need to find S_n

Firstly, we will find the value of n.

We know that

$a_n = a + (n-1)d$

So, $50 = 5 + (n-1)3$

Or $50 - 5 = (n-1)3$

Or $\frac{45}{3} + 1 = n$

Or $n = 16$

We know that sum of first n terms of an AP is given by

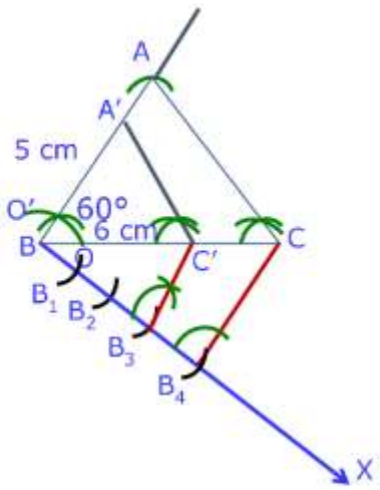
$S_n = \frac{n}{2}(a + a_n)$

So, $S_{16} = \frac{16}{2}(5+50)$

$= 8 \times 55$

Or $S_{16} = 440$

32.



Steps of Construction:

Step 1: Draw a ΔABC with sides $AB = 5\text{cm}, BC = 6\text{cm}$ and $\angle ABC = 60°$.

Step 2: Draw a ray BX making an acute angle with line BC on the opposite side of vertex A.

Step 3: Locate 4 points B_1, B_2, B_3, B_4 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

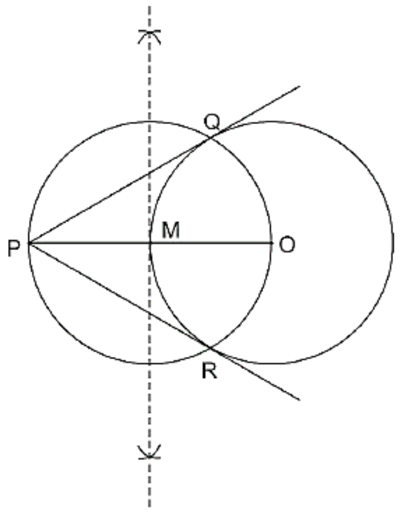
Step 4: join the points C and B_4.

Step 5: through the point B_3, draw a line parallel to CB_4 intersecting line segment BC at point C'

Step 6: Draw a line through C' parallel to the line AC to intersect line segment AB at A'.

The required triangle is $\Delta A'BC'$

OR



Steps of Construction:

Step 1: Draw a circle of radius 3.5cm with centre at point O. Locate a point P, at a distance of 7cm from O, and join O and P.

Step 2: Bisect OP. Let M be the mid – point of OP.

Step 3: Draw a circle with centre at M and MO as radius. Q and R are points of intersections of this circle with the circle having centre at O.

Step 4: Join PQ and PR

PQ and PR are the required tangents.

33. Numbers on spinner $=1,2,4,6,8,10$

Even numbers on spinner $=2,4,6,8,10$

Shweta will pick black marble, if spinner stops on even number.

Therefore,

n (Even number) $=5$

n (Possible number) $=6$

(i) P (Shweta allowed to pick a marble)

$=P$ (Even number)

$$=\frac{n(\text{Even number})}{n(\text{Possible number})}$$

$$=\frac{5}{6}$$

Therefore, the probability of allowing Shweta to pick a marble is $\frac{5}{6}$

(ii). Since, prizes are given, when a black marble is picked.

Number of black marbles $=6$

Total number of marbles $=20$

Therefore, P (getting a prize) $=P$ (a black marble)

$$=\frac{n(\text{Black marbles})}{n(\text{Total marbles})}$$

$$=\frac{6}{20}$$

$$=\frac{3}{10}$$

Therefore, the probability of getting prize is $\frac{3}{10}$.

34. Given that, $OQ = 6\sqrt{2}\,\text{cm}$

$OPQR$ is a square.

Let the side of square $= a$

The diagonal of square $= a\sqrt{2}$

Here, OQ is diagonal of square.

$\Rightarrow a\sqrt{2} = 6\sqrt{2}$

$\Rightarrow a = 6\,\text{cm}$

Area of square $OPQR = 6^2$

$= 36\,\text{cm}^2$

Radius of the quadrant $OAQB =$ Diagonal of the square $OPQR$

$= 6\sqrt{2}\,\text{cm}$

Area of the quadrant $OAQB = \frac{90°}{360°} \times \frac{22}{7} \times \left(6\sqrt{2}\right)^2$

$= \frac{396}{7}\,\text{cm}^2$

Area of shaded region = Area of the quadrant $OAQB$ – Area of square $OPQR$

$= \frac{396}{7} - 36$

$= \frac{144}{7}$

$= 22.6\,\text{cm}^2$

Section D

35. The given polynomial is $p(x)=2x^4-x^3-11x^2+5x+5$

The two zeroes of $p(x)$ are $\sqrt{5}$ and $-\sqrt{5}$.

Therefore, $(x-\sqrt{5})$ and $(x+\sqrt{5})$ are factors of $p(x)$.

Also, $(x-\sqrt{5})(x+\sqrt{5})=x^2-5$

and so x^2-5 is a factor of $p(x)$.

Now,

$$
\begin{array}{r|l}
 & 2x^2-x-1 \\
\hline
x^2-5 & 2x^4-x^3-11x^2+5x+5 \\
 & 2x^4 \quad -10x^2 \\
 & - \qquad + \\
\hline
 & -x^3-x^2+5x+5 \\
 & -x^3 \quad +5x \\
 & + \qquad - \\
\hline
 & -x^2 \quad +5 \\
 & -x^2 \quad +5 \\
 & + \qquad - \\
\hline
 & 0
\end{array}
$$

$2x^4-x^3-11x^2+5x+5=(x^2-5)(2x^2-x-1)$

$=(x^2-5)(2x^2-2x+x-1)$

$=(x^2-5)(2x+1)(x-1)$

Equating $(x^2-5)(2x+1)(x-1)$ to zero, we get the zeroes of the given polynomial.

Hence, the zeroes of the given polynomial are:

$\sqrt{5},-\sqrt{5},-\frac{1}{2}$ and 1.

OR

The given polynomial is $2x^3 - 3x^2 + 6x + 7$.

Here, divisor is $x^2 - 4x + 8$.

Divide $2x^3 - 3x^2 + 6x + 7$ by $x^2 - 4x + 8$ and find the remainder.

$$
\begin{array}{r|l}
 & 2x + 5 \\
x^2 - 4x + 8 & 2x^3 - 3x^2 + 6x + 7 \\
 & 2x^3 - 8x^2 + 16x \\
 & - \quad + \quad - \\
\hline
 & \quad 5x^2 - 10x + 7 \\
 & \quad 5x^2 - 20x + 40 \\
 & \quad - \quad + \quad - \\
\hline
 & \qquad 10x - 33
\end{array}
$$

Remainder $= 10x - 33$

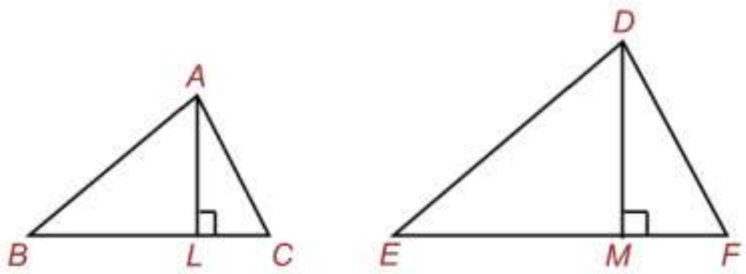
Therefore, we should add $-(10x - 33)$ to make it exactly divisible by $x^2 - 4x + 8$

Thus, we should add $-10x + 33$ to $2x^3 - 3x^2 + 6x + 7$

36. Given: $\Delta ABC \sim \Delta DEF$

To prove: $\frac{\text{Area } \Delta ABC}{\text{Area } \Delta DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DE}\right)^2$

Construction: Draw $AL \perp BC$ and $DM \perp EF$



Proof: Here $\frac{\text{Area } \Delta ABC}{\text{Area } \Delta DEF} = \frac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times EF \times DM} = \frac{BC \times AL}{EF \times DM}$ 1

In ΔALB and ΔDME

$\angle ALB = \angle DME$ Each $90°$

And $\angle B = \angle E$ Since $\Delta ABC \sim \Delta DEF$

So, $\Delta ALB \sim \Delta DME$ AA similarity criterion

$\Rightarrow \frac{AL}{DM} = \frac{AB}{DE}$

But $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ since $\Delta ABC \sim \Delta DEF$

Therefore, $\frac{AL}{DM} = \frac{BC}{EF}$ ....(2)

From (1) and (2) we have

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{BC}{EF} \times \frac{AL}{DM} = \frac{BC}{EF} \times \frac{BC}{EF} = \left(\frac{BC}{EF}\right)^2$$

But $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ Since $\Delta ABC \sim \Delta DEF$

This implies that,

$$\frac{\text{Area } \Delta ABC}{\text{Area } \Delta DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DE}\right)^2$$

37. Let the sides of first and second square be x any y. Then,

Area of first square $= x^2$

And,

Area of second square $= y^2$

According to the question,

$x^2 + y^2 = 544$ ....(1)

Now,

Perimeter of first square $= 4x$

And,

Perimeter of second square $= 4y$

According to the question,

$4x - 4y = 32$ ....(2)

From equation (2), we get

$4(x - y) = 32$

Or, $x - y = \frac{32}{4}$

Or, $x - y = 8$

Or, $x = 8 + y$ ....(3)

Substituting this value of x in equation (1), we get

$x^2 + y^2 = 544$

Or , $(8 + y)^2 + y^2 = 544$

Or, $64 + y^2 + 16y + y^2 = 544$

Or, $2y^2 + 16y + 64 = 544$

Or, $2y^2 + 16y + 64 - 544 = 0$

Or, $2y^2 + 16y - 480 = 0$

Or, $2(y^2 + 8y - 240) = 0$

Or, $y^2 + 8y - 240 = 0$

Or, $y^2 + 20y - 12y - 240 = 0$

Or, $y(y + 20) - 12(y + 20) = 0$

Or, $(y + 20)(y - 12) = 0$

$\Rightarrow y + 20 = 0 \text{ or } y - 12 = 0$

$\Rightarrow y = -20 \text{ or } y = 12$

Since side of a square cannot be negative, therefore $y = 12$.

Substituting $y = 12$ in equation (3), we get

$x = 8 + y = 8 + 12 = 20$

Therefore,

Side of first square $= x = 20\,\text{cm}$

And,

Side of second square $= y = 12\,\text{cm}$

OR

Let the speed of the stream be $x\,\text{km/h}$.

Therefore, speed of the boat upstream $= (18 - x)\,\text{km/h}$

And the speed of the boat downstream $= (18 + x)\,\text{km/h}$.

The time taken to go upstream $= \dfrac{\text{distance}}{\text{speed}}$

$$= \frac{24}{18 - x}\,\text{hours}$$

Similarly, the time taken to go downstream $= \dfrac{24}{18 + x}$ hours

According to the question,

$\dfrac{24}{18 - x} - \dfrac{24}{18 + x} = 1$

Or, $\dfrac{24(18 + x) - 24(18 - x)}{(18 + x)(18 - x)} = 1$

Or, $24(18 + x) - 24(18 - x) = (18 + x)(18 - x)$

Or, $432 + 24x - 432 + 24x = 324 - x^2$

Or, $x^2 + 48x - 324 = 0$

Using the quadratic formula, we get

$$x=\frac{-48\pm\sqrt{48^2-4(1)(-324)}}{2}$$

$$=\frac{-48\pm\sqrt{2304+1296}}{2}$$

$$=\frac{-48\pm\sqrt{3600}}{2}$$

$$=\frac{-48\pm 60}{2}$$

Therefore, $x=\frac{-48+60}{2}$ or $x=\frac{-48-60}{2}$

$\Rightarrow x=\frac{12}{2}$ or $x=\frac{-108}{2}$

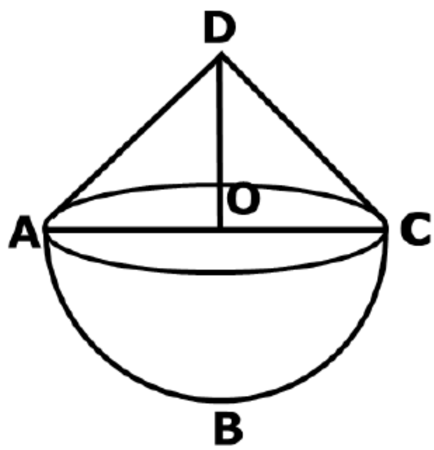
$\Rightarrow x=6$ or $x=-54$

Since x is the speed of the stream, it cannot be negative.

So, we ignore the root $x=-54$. Therefore, $x=6$ gives

The speed of the stream as $6\,\text{km/h}$.

38.



Let ABC be the hemisphere and ADC be the cone standing on the base of the hemisphere.

Height of the cone $\left(h_1\right)=10\,cm$ (Given)

Radius of the cone $\left(r_1\right)=7\,cm$ (Given)

Since the hemisphere is surmounted by the right circular cone of same radius, therefore

Radius of the hemisphere $\left(r_2\right)=7\,cm$

So,

Volume of the toy

$=$ Volume of the cone $+$ Volume of the hemisphere

$$=\frac{1}{3}\pi r_1^2 h_1+\frac{2}{3}\pi r_2^3$$

$$=\left[\left(\frac{1}{3}\times\frac{22}{7}\times7\times7\times10\right)+\left(\frac{2}{3}\times\frac{22}{7}\times7\times7\times7\right)\right]cm^3$$

$$=\left[\frac{1540}{3}+\frac{2156}{3}\right]cm^3$$

$$=\frac{3696}{3}\,cm^3$$

$$=1232\,cm^3$$

Area of the coloured sheet required to cover the toy

$= CSA$ of hemisphere $+ CSA$ of cone

$$=2\pi r_2^2+\pi r\ell$$

Where ℓ is the slant height of the cone

$$\ell=\sqrt{r_1^2+h_1^2}$$

$$=\sqrt{7^2+10^2}$$

$$=\sqrt{49+100}$$

$$=\sqrt{149}$$

$$=12.2\,cm$$

So,

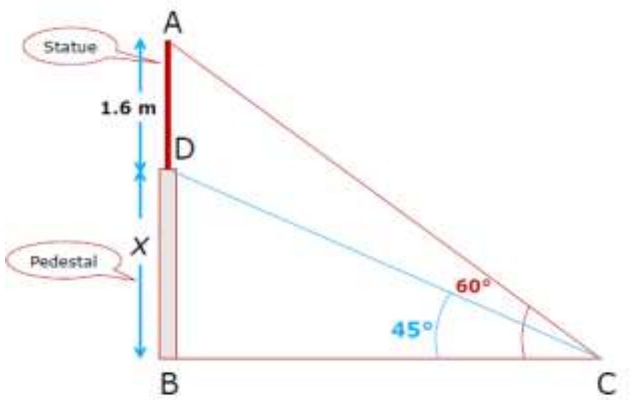
Area of the coloured sheet required to cover the toy

$$=\left[\left(2\times\frac{22}{7}\times7\times7\right)+\left(\frac{22}{7}\times7\times12.2\right)\right]\text{cm}^2$$

$$=(308+268.4)\text{cm}^2$$

$$=576.4\,\text{cm}^2$$

39.



Let BD be a pedestal of height $x\,\text{m}$ and AD be a statue of height $1.6\,\text{m}$. The angle of elevation of the top of pedestal from a point C is $45°$ and that of point statue from C is $60°$.

In the triangle ABC :

$$\frac{AB}{BC}=\tan 60°$$

$$\frac{1.6+x}{BC}=\sqrt{3}$$

Or, $BC=\dfrac{1.6+x}{\sqrt{3}}$ 1

In the triangle DBC :

$$\frac{DB}{BC}=\tan 45°$$

Or, $\frac{x}{BC} = 1$

Or, $x = BC$ 2

By equations 1 and 2, we get

$$x = \frac{1.6 + x}{\sqrt{3}}$$

Or, $\sqrt{3x} = 1.6 + x$

$\sqrt{3} - 1x = 1.6$

Or, $x = \frac{1.6}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$

$= \frac{1.61.73 + 1}{3 - 1}$

$= \frac{1.6 \times 2.73}{2}$

$= 2.184\,\text{m}$

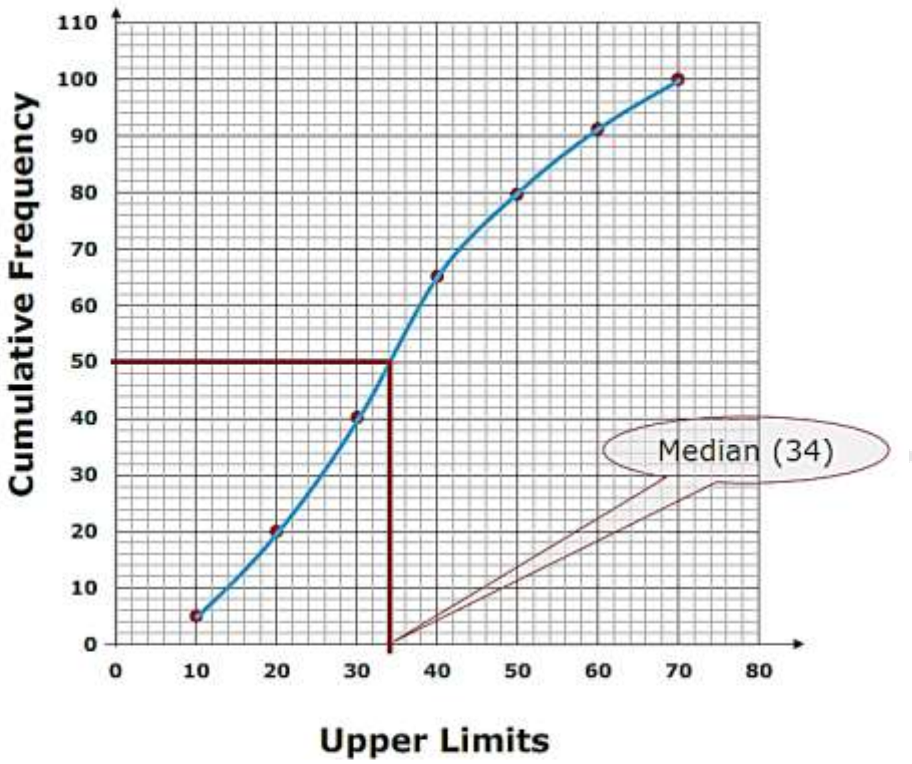
Therefore, the height of the pedestal is $2.184\,\text{m}$.

40.

Age	**Number of Persons (Cumulative frequency)**
Less than 10	5
Less than 20	$5 + 15 = 20$
Less than 30	$20 + 20 = 40$
Less than 40	$40 + 25 = 65$
Less than 50	$65 + 15 = 80$
Less than 60	$80 + 11 = 91$
Less than 70	$91 + 9 = 100$

Age	No. of Persons (f)	Cumulative frequency (cf)
$0-10$	5	5
$10-20$	15	20
$20-30$	20	40
$30-40$	25	65
$40-50$	15	80
$50-60$	11	91
$60-70$	9	100

Plot the points $(10,5),(20,20),\ldots,(70,100)$ on a graph paper.



OR

Class interval	No. of bowlers f_i	Class mark x_i	$f_i x_i$
$20-60$	7	40	280
$60-100$	5	80	400
$100-140$	16	120	1920
$140-180$	12	160	1920
$180-220$	2	200	400
$220-260$	3	240	720
Total	$\sum f_i = 45$		$\sum f_i x_i = 5640$

$$\bar{x}=\frac{\sum f_i x_i}{\sum f_i}=\frac{5640}{45}=125.33$$

Number of wickets	Number of bowlers	Cumulative Frequency
$20-60$	7	7
$60-100$	5	12
$100-140$	16	28
$140-180$	12	40
$180-220$	2	42
$220-260$	3	45

$n=45$

$$\Rightarrow \frac{n}{2}=\frac{45}{2}=22.5$$

Median class $=100-140$

Median $=I+\dfrac{\left(\dfrac{n}{2}-cf\right)}{f}\times h$

$I=100, \frac{n}{2}=22.5, cf=12, f=16, h=40$

Median $=100+\dfrac{22.5-12}{16}\times 40$

$=100+26.25$

$=126.25$