CBSE 12th – 2022-23

Applied Mathematics

Answers

Section A

1. **Correct Answer: C**

$x \equiv 27 \pmod 4$

$\Rightarrow x - 27 = 4k$, for some integer k

$\Rightarrow x = 31$ as $27 < x \le 36$

2. **Correct Answer: D**

3. **Correct Answer: B**

$n = 26 \Rightarrow |t| = 3.07 > t_{25}(0.05) = 2.06$

4. **Correct Answer: B**

$n = 34 \Rightarrow v = 34 - 1 = 33$

5. **Correct Answer: B**

Speed of boat downstream $= u = 10\,\text{km/h}$

And, speed of boat upstream $= v = 6\ \text{km/h}$

$\Rightarrow$ Speed of stream $= \frac{1}{2}(u - v) = 2\,\text{km/h}$

6. **Correct Answer: C**

7. **Correct Answer: C**

Truck A carries water $=100-\left(\frac{20\times1,500}{1,000}\right)=70\,l$

Truck B carries water $=80-\left(\frac{20\times1,000}{1,000}\right)=60\,l$

8. **Correct Answer: D**

Let the face value of the bond $=x$

Then, $\frac{10}{200}x=1,800\Rightarrow x=36,000$

9. **Correct Answer: C**

10. **Correct Answer: D**

11. **Correct Answer: B**

$$D=\frac{C-S}{n}=\frac{4,80,000-25,000}{10}=45,500$$

12. **Correct Answer: A**

13. **Correct Answer: B**

$$\int\frac{dy}{y\log y}=\int\frac{dx}{x}$$

$$\Rightarrow\log(\log y)=\log|x|+\log|C|$$

$$\Rightarrow\log(\log y)=\log|Cx|$$

$$\Rightarrow y=e^{|Cx|}$$

14. **Correct Answer: C**

$$\left[\left(\frac{60,000}{10,000}\right)^{\frac{1}{4}}-1\right]\times 100=\left[\sqrt[4]{6}-1\right]\times 100$$

15. **Correct Answer: C**

$\Rightarrow 180:300=3:5$

16. **Correct Answer: D**

17. **Correct Answer: C**

18. **Correct Answer: B**

For questions 19 and 20, two statements are given – one labelled Assertion(A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (i), (ii), (iii) and (iv) as given below:

19. **Correct Answer: A**

P (Win in one game) $=P$ (Lose in one game) $=\frac{1}{2}$

$\Rightarrow P$ (Beena to win in 3 out of 4 games) $={}^4C_3\left(\frac{1}{2}\right)^3\cdot\left(\frac{1}{2}\right)=\frac{1}{4}=25\%$

Assertion is correct and Reason is the correct explanation for it

20. **Correct Answer: B**

Effective rate of interest = Nominal rate – inflation rate $=12.5-2=10.5\%$

Assertion is correct

Reason is true but not supportive of assertion

Section B

21. $P = 2,50,000, R = 7,500, i = \frac{r}{400}$

$\Rightarrow 2,50,000 = \frac{7,500 \times 400}{r} \Rightarrow r = 12$

22. $a - 8 = 1 \Rightarrow a = 9$

$3b = -2 \Rightarrow b = -\frac{2}{3}$

$-c + 2 = -28 \Rightarrow c = 30$

$\Rightarrow 2a + 3b - c = -14$

OR

22. Expanding C_1, we get $\Delta = 1(2x^2 + 4) - 2(-4x - 20) = 86$

$\Rightarrow x^2 + 4x - 21 = 0$

$\therefore x = 3, -7$

23. Let the number of hardcopy and paperback copies be x and y respectively

$\Rightarrow$ Maximum profit $Z = (72x + 40y) - (9600 + 56x + 28y) = 16x + 12y - 9600$

Subject to constraints:

$x + y \leq 960$

$5x + y \leq 2400$

$x, y \geq 0$

24. Speed of boat in still waters $= x$ km/h

Speed of stream $= y$ km/h

Distance travelled $= d$ km

Time taken to travel downstream $= \frac{d}{x+y}$

Time taken to travel upstream $= \frac{d}{x-y}$

Then, $\frac{2d}{x+y} = \frac{d}{x-y} \Rightarrow x:y = 3:1$

OR

24. Param runs $5\,\text{m}$ in 3 seconds

$\Rightarrow$ time taken to run $200\,\text{m} = \frac{3}{5} \times 200 = 120$ seconds

Anuj's time $= 120 - 3 = 117$ seconds

25. $V_f = 4,37,500, V_i = 3,50,000$

Nominal rate $= \frac{V_f - V_i}{V_i} \times 100$

$= \frac{4,37,500 - 3,50,000}{3,50,000} \times 100 = 25\%$

Section C

26. $f'(x) = x^3 - 6x^2 + 11x - 6 = (x-1)(x-2)(x-3)$

$\Rightarrow x = 1, 2, 3$

Strictly increasing in $(1,2) \cup (3,\infty)$

Strictly decreasing in $(-\infty,1) \cup (2,3)$

27. Daily diet of team $A=\begin{bmatrix}2 & 3 & 1\end{bmatrix}\begin{bmatrix}2,500 & 65\\ 1,900 & 50\\ 2,000 & 54\end{bmatrix}=\begin{bmatrix}12,700\\ 334\end{bmatrix}$

Team A consumes $12,700$ calories and 334 g vitamin

Daily diet of team $B=\begin{bmatrix}1 & 2 & 2\end{bmatrix}\begin{bmatrix}2,500 & 65\\ 1,900 & 50\\ 2,000 & 54\end{bmatrix}=\begin{bmatrix}10,300\\ 273\end{bmatrix}$

Team B consumes $10,300$ calories and 273 g vitamin

28. $\int\frac{dx}{\left(1+e^{x}\right)\left(1+e^{-x}\right)}$

$=\int\frac{e^{x}dx}{\left(1+e^{x}\right)^{2}}$

$=\int\frac{dt}{t^{2}}$, where $t=e^{x}+1$ and $dt=e^{x}dx$

$=\frac{-1}{t}+C$

$=\frac{-1}{1+e^{x}}+C$

OR

28. $\int \underset{II}{x}\,\underset{I}{\log\left(1+x^{2}\right)}dx$, Integration by parts

$=\log\left(1+x^{2}\right).\int x\,dx-\int\left[\frac{d}{dx}\log\left(1+x^{2}\right).\int x\,dx\right]dx$

$=\frac{x^{2}}{2}\log\left(1+x^{2}\right)-\int\left[\frac{2x}{1+x^{2}}.\frac{x^{2}}{2}\right]dx$

$=\frac{x^{2}}{2}\log\left(1+x^{2}\right)-\int\frac{x^{3}}{1+x^{2}}dx$

$$=\frac{x^2}{2}\log\left(1+x^2\right)-\int\left[x-\frac{x}{1+x^2}\right]dx$$

$$=\frac{x^2}{2}\log\left(1+x^2\right)-\frac{x^2}{2}+\frac{1}{2}\log\left(1+x^2\right)+C$$

$$=\frac{1}{2}\left[\left(1+x^2\right)\log\left(1+x^2\right)-x^2\right]+C$$

29. Under pure competition, $p_d = p_s$

$$\Rightarrow \frac{8}{x+1}-2=\frac{x+3}{2}$$

$$\Rightarrow x^2+8x-9=0$$

$$\Rightarrow x=-9,1$$

$$\therefore x=1$$

When $x_0=1 \Rightarrow p_0=2$

$\therefore$ Produce surplus $=2-\int_0^1 \frac{x+3}{2}dx=2-\left[\frac{x^2}{4}+\frac{3x}{2}\right]_0^1=\frac{1}{4}$

OR

29. $p=274-x^2$

$$\Rightarrow R=px=274x-x^3$$

$$\frac{dR}{dx}=274-3x^2$$

Given $MR=4+3x$

In profit monopolist market,

$$MR=\frac{dR}{dx}\Rightarrow 4+3x=274-3x^2$$

$$\Rightarrow x^2+x-90=0$$

$$\Rightarrow x=-10,9$$

$\therefore x = 9$

When $x_0 = 9 \Rightarrow p_0 = 193$

$\therefore$ Consumer surplus $= \int_0^9 \left(274 - x^2\right) dx - 193 \times 9$

$$= \left[274x - \frac{x^3}{3}\right]_0^9$$

$$= 486$$

30. Purchase = ₹ $40,00,000$

Down payment $= x$

Balance $= 40,00,000 - x$

$i = \dfrac{9}{1,200} = 0.0075,\ n = 25 \times 12 = 300$

$E =$ ₹ $30,000$

$$\Rightarrow 30,000 = \frac{(4000000 - x) \times 0.0075}{1 - \ (1.0075)^{-300}}$$

$$\Rightarrow 30000 = \frac{(4000000 - x) \times 0.0075}{1 - \ 0.1062}$$

$\Rightarrow x = 4,24,800$

Down payment = ₹ $4,24,800$

31. $n = 10 \times 2 = 20, S = 10,21,760, i = \dfrac{5}{200} = 0.025, R = ?$

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$\Rightarrow 10,21,760 = R\left[\frac{(1+0.025)^{20}-1}{0.025}\right]$

$\Rightarrow 10,21,760 = R\left[\frac{1.6386-1}{0.025}\right]$

$\Rightarrow R = \left[\frac{10,21,760 \times 0.025}{0.6386}\right]$

$\Rightarrow R =$ ₹ $40,000$

Mr Mehra set aside an amount of ₹ $40,000$ at the end of every six months

Section D

32. Probability of defective bucket $= 0.03$

$n = 100$

$m = np = 100 \times 0.03 = 3$

Let X = number of defective buckets in a sample of 100

$P(X = r) = \frac{m^r e^{-m}}{r!}, r = 0,1,2,3,\ldots$

(i) P (no defective bucket) $= P(r=0) = \frac{3^0 e^{-3}}{0!} = 0.049$

(ii) P (at most one defective bucket) $= P(r = 0,1)$

$= \frac{3^0 e^{-3}}{0!} + \frac{3^1 e^{-3}}{1!}$

$= 0.049 + 0.147$

$= 0.196$

OR

32. $X =$ scores of students, $\mu = 45, \sigma = 5$

$$\therefore Z = \frac{X-\mu}{\sigma} = \frac{X-45}{5}$$

(i) When $X = 45, Z = 0$

$P(X > 45) = P(Z > 0) = 0.5$

$\Rightarrow 50\%$ students scored more than the mean score

(ii) When $X = 30, Z = -3$ and when $X = 50, Z = 1$

$P(30 < X < 50) = P(-3 < Z < 1) = P(-3 < Z \le 1)$

$= P(-3 < Z \le 0) + P(0 \le Z < 1)$

$= P(0 \le Z < 3) + P(0 \le Z < 1)$

$= 0.4987 + 0.3413 = 0.84$

$\Rightarrow 84\%$ students scored between 30 and 50 marks

33. Let x be the number of guests for the booking

Clearly, $x > 100$ to avail discount

$\therefore$ Profit, $P = \left[4800 - \frac{200}{10}(x-100)\right]x = 6,800x - 20x^2$

$\Rightarrow \frac{dP}{dx} = 6,800 - 40x \Rightarrow x = 170$

As $\frac{d^2p}{dx^2} = -40 < 0, \forall x$

A booking for 170 guests will maximise the profit of the company And, Profit = ₹ $5,78,000$

OR

33. $P(x) = R(x) - C(x)$

$= 5x - (100 + 0.025x^2)$

$\Rightarrow P'(x) = 5 - 0.05\,x \Rightarrow x = 100$

As $P''(x) = -0.05 < 0, \forall x$

$\therefore$ Manufacturing 100 dolls will maximise the profit of the company And, Profit $=$ ₹ $1,50,000$

34. Let the number of tables and chairs be x and y respectively

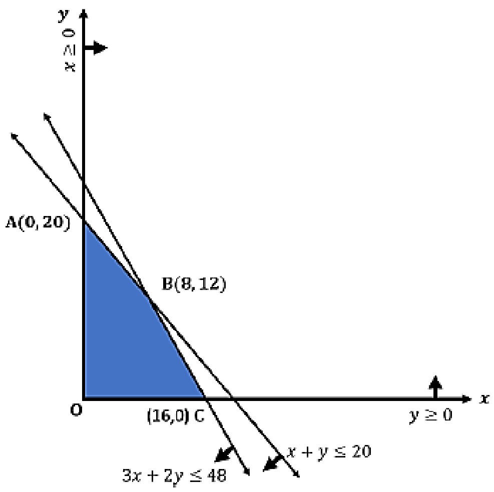
(Max profit) $Z = 22x + 18y$

Subject to constraints:

$x + y \leq 20$

$3x + 2y \leq 48$

$x, y \geq 0$



The feasible region $OABCA$ is closed (bounded)

Corner points	$Z = 22x + 18y$
$O(0,0)$	0
$A(0,20)$	360
$B(8,12)$	392
$C(16,0)$	352

Buying 8 tables and 12 chairs will maximise the profit

35. $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 2 \\ 2 & 3 & 2 \end{bmatrix}$

$\Rightarrow |A| = 9 \Rightarrow A^{-1}$ exists

And $A^{-1} = \frac{1}{9}\begin{bmatrix} -2 & 5 & -2 \\ -2 & -4 & 7 \\ 5 & 1 & -4 \end{bmatrix}$

$AX = B \Rightarrow X = A^{-1}B$

$$\Rightarrow X = \frac{1}{9}\begin{bmatrix} -2 & 5 & -2 \\ -2 & -4 & 7 \\ 5 & 1 & -4 \end{bmatrix}\begin{bmatrix} 85 \\ 105 \\ 110 \end{bmatrix} = \begin{bmatrix} 15 \\ 20 \\ 10 \end{bmatrix}$$

$\Rightarrow p_1 = 15, p_2 = 20, p_3 = 10$

Section E

CASE STUDY – I

36. a). Pipe C empties 1 tank in $20\text{ h} \Rightarrow \frac{2}{5}\text{th}$ tank in $\frac{2}{5} \times 20 = 8$ hours

b). Part of tank filled in 1 hour $= \frac{1}{15} + \frac{1}{12} - \frac{1}{20} = \frac{1}{10}\text{th}$

$\Rightarrow$ time taken to fill tank completely $=10$ hours

c). At 5am,

Let the tank be completely filled in 't' hours

$\Rightarrow$ pipe A is opened for 't' hours

pipe B is opened for '$t-3$' hours

And, pipe C is opened for '$t-4$' hours

$\Rightarrow$ In one hour,

part of tank filled by pipe $A = \dfrac{t}{15}\text{th}$

part of tank filled by pipe $B = \dfrac{t-3}{15}\text{ th}$

and, part of tank emptied by pipe $C = \dfrac{t-4}{15}\text{th}$

Therefore $\dfrac{t}{15} + \dfrac{t-3}{12} - \dfrac{t-4}{20} = 1$

$\Rightarrow t = 10.5$

Total time to fill the tank $=10$ hours 30 minutes

OR

36. 6 am, pipe C is opened to empty $\dfrac{1}{2}$ filled tank

Time to empty $=10$ hours

Time for cleaning $=1$ hour

Part of tank filled by pipes A and B in 1 hour $= \dfrac{1}{15} + \dfrac{1}{12} = \dfrac{3}{20}\text{th}$ tank

$\Rightarrow$ time taken to fill the tank completely $= \dfrac{20}{3}$ hours

Total time taken in the process $= 10 + 1 + \dfrac{20}{3} = 17$ hour 40 minutes

CASE STUDY – II

37. a)

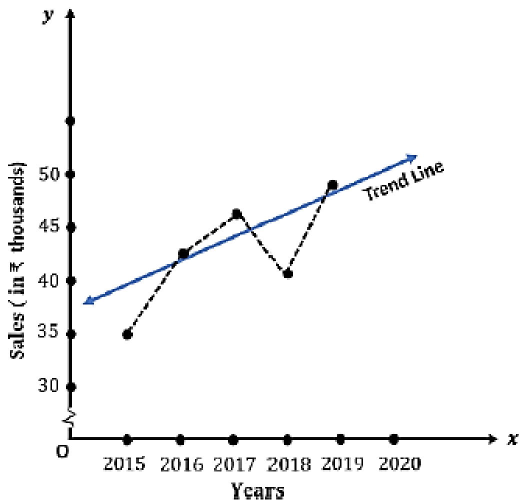
Year	Y	X	X^2	XY
2015	35	−2	4	−70
2016	42	−1	1	−42
2017	46	0	0	0
2018	41	1	1	41
2019	48	2	4	96
	212		10	25

$a = \frac{\sum Y}{n} = \frac{212}{5} = 42.4$ and $b = \frac{\sum XY}{\sum X^2} = \frac{25}{10} = 2.5$

$$Y_C = 42.4 + 2.5X$$

OR

Year	Y	3-year moving average
2015	35	
2016	42	41
2017	46	43
2018	41	45
2019	48	-



b) For year 2022,

$Y_{2022} = 42.4 + 2.5(2022 - 2017) = 54.9$

$\Rightarrow$ the estimated sales for year 2022 $=$ ₹ $54,900$

c) $Y_C = 42.4 + 2.5X$

$\Rightarrow 67.4 = 42.4 + 2.5X$

$\Rightarrow X = 10$

Sales will be ₹ $67,400$ in year $(2017+10) =$ year 2027

CASE STUDY – III

38. a) $\frac{k}{6} + \frac{2k}{6} + \frac{3(1-k)}{6} + \frac{4k}{2} = 1 \Rightarrow k = \frac{1}{4}$

b) P (getting admission on applying at least 2 weeks ahead of application deadline)

$= P(X = 2, 3, 4)$

$= \frac{1}{12} + \frac{3}{8} + \frac{1}{2} = \frac{23}{24}$

[alternated method: $1 - P(X = 1) = 1 - \frac{1}{24} = \frac{23}{24}$]

c) $X =$ week applied ahead of application deadline

X	1	2	3	4
$P(X)$	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{3}{8}$	$\frac{1}{2}$
$XP(X)$	$\frac{1}{24}$	$\frac{1}{6}$	$\frac{9}{8}$	2

$\therefore E(X) = \frac{80}{24} = 3\frac{1}{3}$ weeks

OR

$X =$ Scholarship money awarded for the week applied in, before the deadline

Week applied in	1	2	3	4
X	$9,600$	$12,000$	$20,000$	$50,000$
$P(X)$	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{3}{8}$	$\frac{1}{2}$
$XP(X)$	$\frac{9,600}{24}$	$\frac{12,000}{12}$	$\frac{60,000}{8}$	$\frac{50,000}{2}$

$\therefore E(X) =$ ₹ $33,900$