# JEE Main - 2023

## Mathematics

## Answers

## Section A

**1. Correct Answer: C**

$x_1.x_2.x_3.x_4.x_5 = 420 = 2^2 \times 3 \times 5 \times 7$

Now,

$3, 5$ and $7$ can be considered as any of $x_1, x_2, x_3, x_4$, and $x_5$ places.

hence the number of ways are $5 \times 5 \times 5$

Also $2, 2$ can be distributed in $5$ places in

${}^{2+5-1}C_5 - 1 = {}^6C_4 = 15$ Ways.

So total no. of positive integral solutions of $420$ is $= 15 \times 5 \times 5 \times 5$

$= 1875$

**For Statement 2**: Given Number is $420$

Prime factorization of given number is $420 = 2^2 \times 3 \times 5 \times 7$

Use If $N = x^a y^b z^c$ then total divisors of $N = (a+1)(b+1)(c+1)$

Therefore total number of divisors of $420$ are $(2+1)(1+1)(1+1)(1+1)$

$= 3 \times 2 \times 2 \times 2 = 24$

**2. Correct Answer: A**

First we find out the probability of drawing a green ball

Given data

Green Ball $= 12$

Yellow Ball $=8$

Total Ball $=12+8=20$

Let $P(G)=$ Probability of drawing a green ball

$$\text{Probability}=\frac{^{12}C_1}{^{20}C_1}\quad\left[\because {}^nC_1=\frac{n\times(n-1)!}{(n-1)!\times 1!}=n\right]$$

$$=\frac{12}{20}$$

$$=\frac{3}{5}$$

As we know,

$$P(G)+P(G')=1$$

$$\frac{3}{5}+P(G')=1$$

$$P(G')=\frac{2}{5}$$

**Step 2**: Find out the variance

Variance $(\sigma^2)=10\times\frac{3}{5}\times\frac{2}{5}$ ..........Using variance $=npq$ here $p=\frac{3}{5}$ and $q=\frac{2}{5}$

Variance $(\sigma^2)=\frac{12}{5}$

**3. Correct Answer: B**

Given:

$$|z-i\omega|=|z-i\overline{\omega}|=3$$

$$\Rightarrow |z-i\omega|^2=9$$

$$\Rightarrow (z-i\omega)(\overline{z}+i\overline{\omega})=9$$

$$\Rightarrow z\overline{z}+iz\overline{\omega}-i\omega\overline{z}+\omega\overline{\omega}=9 \qquad \text{.........(1)}$$

Similarly,

$$\Rightarrow \left|z - i\overline{\omega}\right|^2 = 9$$

$$\Rightarrow \left(z - i\overline{\omega}\right)\left(\overline{z} + i\omega\right) = 9$$

$$\Rightarrow z\overline{z} + iz\omega - i\overline{\omega}\overline{z} + \omega\overline{\omega} = 9 \qquad \text{........(2)}$$

From (2) – (1), we get

$$z\left(\omega - \overline{\omega}\right) - \overline{z}\left(\overline{\omega} - \omega\right) = 0$$

$$\Rightarrow \left(z + \overline{z}\right)\left(\omega - \overline{\omega}\right) = 0$$

**Case I**

$$\omega - \overline{\omega} = 0$$

$\Rightarrow \omega$ is an purely real number

**Case II:**

$$z + \overline{z} = 0$$

$\Rightarrow z$ is an purely imaginary number

Now,

$$\Rightarrow \left|z - i\omega\right| \le |z| + |i\omega| \le 2$$

Because, it is given that $|z|, |\omega| \le 1$

But, given is $|z - i\omega| = 2$

Which is only possible when

$$|z| = |\omega| = 1$$

Since, $z$ is an purely imaginary number and its modulus is $1$.

Therefore, $z$ is either $i$ or $-i$

**4. Correct Answer: C**

Let

$$L = \lim_{n\to\infty} n\left[\left(\frac{n}{n+1}\right)^{\alpha} + \sin\left(\frac{1}{n}\right) - 1\right]$$

Put $\frac{1}{n} = x$ as $n \to \infty \therefore x \to 0$

$$\Rightarrow \lim_{x\to 0} \frac{1}{x}\left[\left(\frac{1}{1+x}\right)^{\alpha} + \sin x - 1\right]$$

$$\Rightarrow \lim_{x\to 0}\left[\frac{(1+x)^{-\alpha} + \sin x - 1}{x}\right]$$ : by L-Hospital rule

$$\Rightarrow \lim_{x\to 0}\left[\frac{(-\alpha)(1+x)^{-\alpha-1} + \cos x}{1}\right]$$

$L \Rightarrow 1-\alpha.............\cos 0 = 1$

$\therefore$ solution of given example is $e^{1-\alpha}$

**5. Correct Answer: C**

$4f(\sin x) + f(\cos x) = 10x$ ........(1)

Replace $x$ by $\frac{\pi}{2} - x$

we get $4f(\cos x) + f(\sin x) = 10\left[\frac{\pi}{2} - x\right]$ ........(2)

Multiply eq. (1) by $4$ and subtract eq. (2) from it

Solving we get

$15f(\sin x) = 50x - 5\pi$

$\therefore f(x) = \frac{10}{3}\sin^{-1} x - \frac{\pi}{3}$

$\therefore \frac{d}{dx} f(x) = \frac{10}{3}\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx} f(\sin x) = \frac{10}{3}\frac{1}{\sqrt{1-\sin^2 x}}$

$$\frac{d}{dx}f\left(\sin x\right)=\frac{10}{3}\sec x$$

**6. Correct Answer: A**

Using partial fractions

$$\int\frac{x^3+x}{x^4-9}dx=\int\frac{x^3}{x^4-9}dx+\int\frac{x}{x^4-9}dx$$

Putting $\left(x^4-9\right)=t$

So, $x^4-9=t$ (differentiating)

$4x^3dx=dt \qquad =\int\frac{dt}{4\left(t\right)}+\int\frac{dt}{4t\sqrt{t+9}}$

Putting $t+9=u^2$

$dt=2u\,du$

$$\int\frac{dt}{4t}+\int\frac{du}{2\left(u^2-9\right)}$$

$$\frac{1}{4}\int\frac{dt}{t}+\frac{1}{2}\int\frac{du}{\left(u\right)^2-\left(3\right)^2}$$

We know,

$$\int\frac{dx}{x}=\ln\left|x\right|$$

Using $\int\frac{1}{x^2-a^2}=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$

$$=\frac{1}{4}\ln\left|t\right|+\frac{1}{2}\times\frac{1}{2\times3}\ln\left|\frac{u-3}{u+3}\right|+c$$

Again putting $u=\sqrt{t+9}$

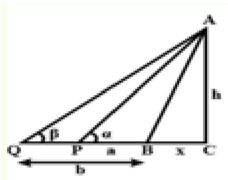
$$=\frac{1}{4}\ln\left|t\right|+\frac{1}{12}\ln\left|\frac{\sqrt{t+9}-3}{\sqrt{t+9}+3}\right|$$

Again putting $t = x^4 - 9$

$$= \frac{1}{4}\ln\left|x^4 - 9\right| + \frac{1}{12}\ln\left|\frac{x^2 - 3}{x^2 + 3}\right|$$

Therefore $A = 4, B = 12$ and $f(x) = \log\left|x^4 - 9\right|$

**7. Correct Answer: A**




Let $AB$ be the tree, $P$ and $Q$ be the points from where it is observed.

Draw $AC$ perpendicular to the ground.

Here, $BP = a, BQ = b$

Let $AC = h$ and $BC = x$

$$\Rightarrow \ln \Delta ACP, \tan\alpha = \frac{Ac}{PC}$$

$$\Rightarrow \tan\alpha = \frac{h}{x + a}$$

$$\therefore x + a = \frac{h}{\tan\alpha}$$

$$\therefore x = \frac{h}{\tan\alpha} - a \qquad .......(1)$$

Similarly, in $\Delta ACQ, \tan\beta = \frac{AC}{QC}$

$$\Rightarrow \tan\beta = \frac{h}{x + b}$$

$$\therefore x + b = \frac{h}{\tan\beta}$$

$\therefore x=\frac{h}{\tan\beta}-b$ .......(2)

$\Rightarrow \frac{h}{\tan\alpha}-a=\frac{h}{\tan\beta}-b$ [Comparing (1) and (2)]

$\therefore \frac{h}{\tan\alpha}-\frac{h}{\tan\beta}=-b+a$

$\therefore h\left(\frac{\tan\beta-\tan\alpha}{\tan\alpha\times\tan\beta}\right)=-(b-a)$

$\therefore h=-(b-a)\times\frac{\tan\alpha\times\tan\beta}{\tan\beta-\tan\alpha}$

$\therefore h=\frac{(b-a)\tan\alpha\times\tan\beta}{\tan\alpha-\tan\beta}$

**8. Correct Answer: D**

$$\sum_{r=0}^{n}\tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right)=\sum_{r=0}^{n}\tan^{-1}\left(\frac{2^r-2^{r-1}}{1+2^r.2^{r-1}}\right)$$

use $\tan^{-1}\left(\frac{x-y}{1+xy}\right)=\tan^{-1}x-\tan^{-1}y$

$$\sum_{r=0}^{n}\tan^{-1}\left(\frac{2^r-2^{r-1}}{1+2^r.2^{r-1}}\right)=\sum_{r=0}^{n}\left(\tan^{-1}2^r-\tan^{-1}2^{r-1}\right)$$

$$=\left[\tan^{-1}(2^0)-\tan^{-1}(2^{-1})\right]+\left[\tan^{-1}(2)-\tan^{-1}(2^0)\right]$$
$$+\left[\tan^{-1}(2^2)-\tan^{-1}(2)\right]+\left[\tan^{-1}(2^3)-\tan^{-1}(2^2)\right]$$

$$+\ldots\ldots\ldots\ldots+\left[\tan^{-1}(2^{n-1})-\tan^{-1}(2^{n-2})\right]+\left[\tan^{-1}(2^n)-\tan^{-1}(2^{n-1})\right]$$

$$=\tan^{-1}(2^n)-\tan^{-1}(2^{-1})$$

$$=\tan^{-1}\left(\frac{2^n-2^{-1}}{1+2^n.2^{-1}}\right)$$

$$= \tan^{-1}\left(\frac{2^{n+1}-1}{2+2^{n}}\right)$$

**9. Correct Answer: B**

From the equation $2x^2+6x+a=0$ we have

$$\alpha+\beta=\frac{-6}{2}=-3$$

$$\alpha\beta=\frac{a}{2}$$

Now, other given equation is

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}<2$$

$$\frac{\alpha^2+\beta^2}{\alpha\beta}<2$$

$$\frac{(\alpha+\beta)^2}{\alpha\beta}-\frac{2\cancel{\alpha\beta}}{\cancel{\alpha\beta}}<2$$

$$\frac{(\alpha+\beta)^2}{\alpha\beta}<4$$

Now, by putting values we get

$$\frac{(-3)^2}{\frac{a}{2}}<4$$

$$\frac{9\times\cancel{2}}{\cancel{4}_{2}}<a$$

$$4.5<a$$

$$\therefore\ a>4.5$$

Hence, $2a>9$

Therefore least Integral prime value of $2a$ is $11$

**10. Correct Answer: A**

A vector perpendicular to the plane containing the vectors $\vec{a}$ and $\vec{b}$ is $\vec{a}\times\vec{b}$

$$\therefore \vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 1\\ -2 & 1 & 3\end{vmatrix}=5\hat{i}-5\hat{j}+5\hat{k}$$

Let $\theta$ be the angle between the vectors $5\hat{i}-5\hat{j}+5\hat{k}$ & given vector $\hat{i}+\hat{j}+\hat{k}$

$$\therefore \tan\theta=\frac{\left(5\hat{i}-5\hat{j}+5\hat{k}\right).\left(\hat{i}+\hat{j}+\hat{k}\right)}{\left|5\hat{i}-5\hat{j}+5\hat{k}\right|\left|\hat{i}+\hat{j}+\hat{k}\right|}$$

$$\theta=\tan^{-1}\left(\frac{5(1-1+1)}{5\sqrt{1+1+1}\sqrt{1+1+1}}\right)=\tan^{-1}\left(\frac{1}{3}\right)$$

**11. Correct Answer: C**

Given Curve is $y=x^2-x+1$

$\therefore \frac{dy}{dx}=2x-1,\ \therefore$ Slope of normal is $m_N=\frac{1}{1-2x}$

$m_{x1}=1$ point $(0,1)$

Hence equation of normal is

$y-1=1(x)$

$x-y+1=0$ ............(1)

Now

$m_{x2}=\frac{1}{3}$ point ,$(-1,3)$ hence equation of normal is

$y-3=\frac{1}{3}(x+1)\Rightarrow 3y-9=x+1$

$x-3y+10=0$ ............(2)

$x_3 = \frac{5}{2}, \left(\frac{5}{2}, \frac{19}{4}\right)$

And $m_{x3} = -\frac{1}{4}$

Hence equation of normal is $y - \frac{19}{4} = -\frac{1}{4}\left(x - \frac{5}{2}\right)$

$\Rightarrow x + 4y = \frac{43}{2}$ ...........(3)

Intersection point of (1) and (2) is $\left(\frac{7}{2}, \frac{9}{12}\right)$ and equation (3) passes through it

Hence normal are concurrent

**12. Correct Answer: B**

Let $P$ $\left(\mathrm{at}^2, 2\mathrm{at}\right)$ be any point on the parabola $y^2 = 4ax$

$\therefore$ The equation of tangent at $P$ is $ty = x + at^2$

Since the tangent meets the axis of parabola in $T$ and tangent at the vertex $O$ in $A$,

$\therefore$ Coordinates of $T$ and $A$, are $\left(-at^2, 0\right)$ and $\left(0, \mathrm{at}\right)$ respectively

Let the coordinate of $G$ be $\left(x_1, y_1\right)$

Since $T\ OA\ G$ is a rectangle

$\therefore$ midpoint of diagonals $TA$ and $GO$ is same

$\Rightarrow \frac{x_1 + 0}{2} = \frac{-\mathrm{at}^2 + 0}{2}$ and $\frac{y_1 + 0}{2} = \frac{0 + \mathrm{at}}{2}$

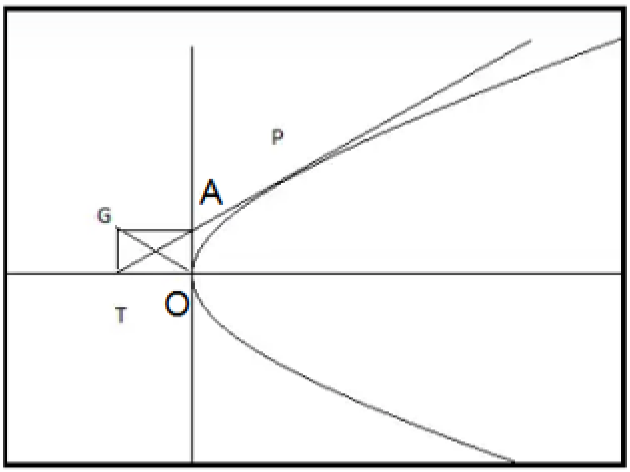
$\Rightarrow x_1 = -\mathrm{at}^2$ (1)

and $y_1 = \mathrm{at}$ (2)

Eliminating $(t)$ from (1) and (2), we get

$x_1 = -a\left(\frac{y_1}{a}\right)^2 \Rightarrow y_1^2 + ax_1 = 0$

$\therefore$ The Locus of $G(x_1, y_1)$ is $y^2 + ax = 0$




**13. Correct Answer: D**

If $P\ (a\sec\theta, b\tan\theta)$ and $Q\ (a\sec\phi, b\tan\phi)$, be two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

then

Normal at $\theta, \phi$ are

$$\begin{cases} ax\cos\theta + by\cot\theta = a^2 + b^2 \\ ax\cos\phi + by\cot\phi = a^2 + b^2 \end{cases}$$

Where $\phi = \frac{\pi}{2} - \theta$ and these passes through $(h, k)$

$\therefore ah\cos\theta + bk\cot\theta = a^2 + b^2$ ..............(i)

$ah\sin\theta + bk\tan\theta = a^2 + b^2$ ..............(ii)

Multiply (i) by $\sin\theta$ & (ii) by $\cos\theta$ & subtract them,

We get

$\Rightarrow (bk + a^2 + b^2)(\sin\theta - \cos\theta) = 0$

$k = -\left(\dfrac{a^2+b^2}{b}\right)$

Here $a = 4$ and $b = 3$

Therefore $k = \dfrac{-25}{3}$

**14. Correct Answer: C**

Let $A-D, A, A+D, A+2D$ be the distinct terms in $A.P.$

$\Rightarrow A+2D = (A-D)^2 + A^2 + (A+D)^2$

$\Rightarrow A+2D = A^2 - 2AD + D^2 + A^2 + A^2 + 2AD + D^2$

$\Rightarrow A+2D = 3A^2 + 2D^2$

$\Rightarrow 3A^2 + 2D^2 - A - 2D = 0$

$\Rightarrow 2D^2 - 2D + (3A^2 - A) = 0$

$\Rightarrow D = \dfrac{\left[2 \pm \sqrt{\left(4 - 8\left(3A^2 - A\right)\right)}\right]}{4}$

$= \dfrac{1 \pm \sqrt{\left(1 - 2\left(3A^2 - A\right)\right)}}{2}$

$= \dfrac{\left[1 \pm \sqrt{\left(1 - 6A^2 + 2A\right)}\right]}{2}$ ..............(i)

Since $D$ should be an integer,

$1 - 6A^2 + 2A = m^2$ multiply by $6$

$6\left(1 - 6A^2 + 2A\right) = 6m^2$

$\Rightarrow (6A-1)^2 = 7 - 6m^2$

LHS is positive, so $7 - 6m^2 > 0$

$\Rightarrow m = 0$ or $\pm 1$

$p + q + r + s = A - D + A + A + D + A + 2D$

$= 4A + 2D$ ..............(ii)

$\Rightarrow D = \frac{(1 \pm m)}{2}$

Where $m = 0$

$D = \frac{(1+0)}{2}$

$= \frac{1}{2}$

Where $m = 1$

$D = \frac{(1+1)}{2}$

$D$ should be an integer

So $D = 1$ or $0$

Since $p, q, r, s$ are distinct integers, $D$ cannot be $0$

So $D = 1$

Substitute $D$ in $(i)$, we get

$1 = \frac{\left[1 \pm \sqrt{(1 - 6A^2 + 2A)}\right]}{2}$

$\Rightarrow 2 = 1 \pm \sqrt{(1 - 6A^2 + 2A)}$

$\Rightarrow 1 = 1 - 6A^2 + 2A$

$\Rightarrow 6A^2 - 2A = 0$

$\Rightarrow 2A(3A - 1) = 0$

$\Rightarrow A = 0$ or $A = \frac{1}{3}$

But $A$ cannot be $\frac{1}{3}$

Put $A=0$ and $D=1$ in $(\text{ii})$, we get

$$p+q+r+s=4A+2D$$
$$=0+2$$
$$=2$$

Hence, option 'C' is correct.

**15. Correct Answer: A**

Given Equation of line is

$$\left(x^2+y^2\right)\sin^2\alpha=\left(x\cos\theta-y\sin\theta\right)^2$$

$$\Rightarrow x^2\sin^2\alpha+y^2\sin^2\alpha=x^2\cos^2\theta+y^2\sin^2\theta-2xy\cos\theta\sin\theta$$

$$\Rightarrow x^2\left(\sin^2\alpha-\cos^2\theta\right)+y^2\left(\sin^2\alpha-\sin^2\theta\right)+2xy\cos\theta\sin\theta=0$$

Using $\tan\phi=\left|2\frac{\sqrt{h^2-ab}}{a+b}\right|$

$$\tan\phi=\left|\frac{\sqrt[2]{\cos^2\theta\sin^2\theta-\left(-\sin^2\theta\sin^2\alpha+\sin^4\alpha-\cos^2\theta\sin^2\alpha+\cos^2\theta\sin^2\theta\right)}}{\sin^2\alpha-\cos^2\theta+\sin^2\alpha-\sin^2\theta}\right|$$

$$\tan\phi=\left|\frac{\sqrt[2]{\cos^2\theta\sin^2\theta+\sin^2\theta+\sin^2\alpha-\sin^4\alpha+\cos^2\theta\sin^2\theta-\cos^2\theta\sin^2\theta}}{2\sin^2\alpha-\left(\cos^2\theta+\sin^2\theta\right)}\right|$$

$$\tan\phi=\left|\frac{\sqrt[2]{-\sin^4\alpha+\sin^2\alpha}}{2\sin^2\alpha-1}\right|=\left|\frac{\sqrt[2]{\sin^2\alpha\left(1-\sin^2\alpha\right)}}{\cos 2\alpha}\right|$$

$$\tan\phi=\left|\frac{2\sin\alpha\cos\alpha}{\cos 2\alpha}\right|=\frac{\sin 2\alpha}{\cos 2\alpha}=\tan 2\alpha$$

$$\therefore \phi=2\alpha$$

**16. Correct Answer: D**

$s_1 : x^2 + y^2 + 4x = 0$

Centre $C_1 \equiv (-2, 0)$, radius $= 2$

$s_2 : x^2 + y^2 + 4y = 0$

Centre $C_2 \equiv (0, -2)$, radius $= 2$

$P$ is the mid -point of $C_1$ and $C_2$ becomes radius of both circles are equal

$\therefore P \equiv (-1, -1)$

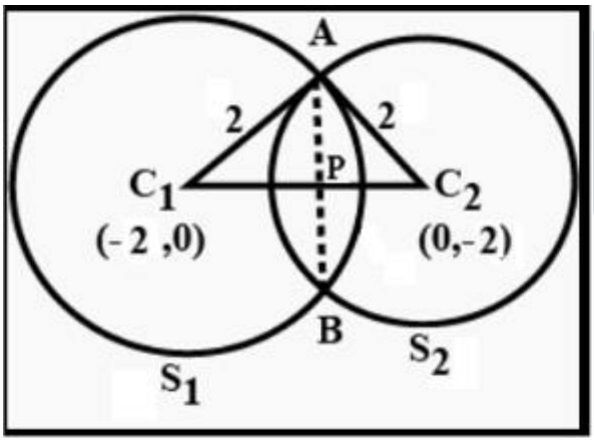
And $AP = \sqrt{c_1A^2 - c_1p^2} = \sqrt{4-2}$

$AP = \sqrt{2}$

$\therefore$ Equation of circle having $AB$ as diameter is

$(x+1)^2 + (y+1)^2 = 2$

$\Rightarrow x^2 + y^2 + 2x + 2y = 0$




**17. Correct Answer: B**

From given Equations

$(2, -1, \sqrt{\lambda})$ are the direction ratios perpendicular to the plane.

and,

$(1,2,2)$ is the direction ratio of the line.

Then,

$$\vec{l}=\hat{i}+2j+2k$$

$$\vec{p}=2i-j+\sqrt{\lambda}k$$

Since,

$$\because \cos\theta'=\frac{\bar{l}.\bar{p}}{|\bar{l}|.|p|}=\frac{2-2+2\sqrt{\lambda}}{3\times\sqrt{5+\lambda}}$$

$$\Rightarrow \cos\theta'=\frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}}=\frac{2}{3}\sqrt{\frac{\lambda}{5+\lambda}}$$

$$\Rightarrow \theta'=\cos^{-1}\frac{2}{3}\sqrt{\frac{\lambda}{5+\lambda}}$$

$$\Rightarrow \theta=\frac{\pi}{2}-\theta'=\sin^{-1}\frac{2}{3}\sqrt{\frac{\lambda}{5+\lambda}}$$

$$\sin\theta=\frac{2}{3}\sqrt{\frac{\lambda}{5+\lambda}}.......(1)$$

But from the question,

$$\sin\theta=\frac{1}{3}.....(2)$$

Equation (1) and (2)

$$\frac{1}{3}=\frac{2}{3}\sqrt{\frac{\lambda}{5+\lambda}}$$

$$\Rightarrow \frac{1}{2}=\sqrt{\frac{\lambda}{5+\lambda}}$$

$$\Rightarrow \frac{1}{4}=\frac{\lambda}{5+\lambda}$$

$$5+\lambda=4\lambda$$

$5 = 3\lambda$

$\lambda = \dfrac{5}{3}$

**18. Correct Answers: D**

Given, there are $11$ terms

We know that, for $n$ observations and if $n$ is odd, then median is $\left(\dfrac{n+1}{2}\right)^{\text{th}}$ term

$\therefore$ Median is $\dfrac{(11+1)^{\text{th}}}{2}$ term

$\Rightarrow$Median is $6\text{th}$ term

Now,

Given, largest $5$ observations are increased by $3$

$\Rightarrow$ Last $5$ terms are increased by $3$

$\Rightarrow 7\text{th}, 8\text{th}, 9\text{th}, 10\text{th}$ and $11\text{th}$ term are increased by $3$

Therefore $6\text{th}$ term remains unchanged

$\Rightarrow$ Median remains unchanged

Thus, the median of new set remains same as that of the original set.

**19. Correct Answer: A**

Let $S = t_r(A) + t_r\left(\dfrac{ABC}{2}\right) + t_r\left(\dfrac{A(BC)^2}{4}\right) + t_r\left(\dfrac{A(BC)^3}{8}\right) + \ldots\ldots + \infty$

$$BC = \begin{bmatrix} -3 & -4 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} \frac{5}{9} & -\frac{4}{9} \\ -\frac{2}{3} & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore S = t_r(A) + t_r\left(\dfrac{A}{2}\right) + t_r\left(\dfrac{A}{4}\right) + \ldots\ldots\infty$

$t_r(A) = 2 + 1 = 3$

$S = 3 + \frac{3}{2} + \frac{3}{4} + \ldots\ldots\infty$

The sum of infinite terms of a GP series is $S_\infty = \frac{a}{(1-r)}$

$S = 3\left(\frac{1}{1-\frac{1}{2}}\right) = 6$

**20. Correct Answer: B**

Given proposition, $(p \Rightarrow -p) \wedge (-p \Rightarrow p) \Leftrightarrow F$

Make a truth table

| $p$ | $-p$ | $p \Rightarrow -p$ | $-p \Rightarrow p$ | $(p \Rightarrow -p) \wedge (-p \Rightarrow p)$ | $(p \Rightarrow -p) \wedge (-p \Rightarrow p) \Leftrightarrow F$ |
|---|---|---|---|---|---|
| $T$ | $F$ | $F$ | $T$ | $F$ | $T$ |
| $F$ | $T$ | $T$ | $F$ | $F$ | $T$ |

$(p \Rightarrow -p) \wedge (-p \Rightarrow p) \Leftrightarrow F$ is a Tautology

Hence, option B is the correct answer.

## Section-B

**21. Answer: 100**

First we find the total number of Indians at the conference.

Now , Total Indians $=$ Indian women $+$ Indian men.

$$=39+43$$

$$=82$$

Then we Find the number of foreigners at the conference.

As, total number of people at the conference $=200$.

$\therefore$ Number of foreigners $=$ Total people $-$ Number of total Indians.

$$=200-82$$

$$=118$$

Therefore $2A=118$........(1)

Now we Find the number of women doctors at the conference.

Number of doctors $=6$

Given, either Indian men or doctor, i.e. (Indian men $\cup$ Doctor) $=45$

$\because n(A\cup B)=n(A)+nB-n(A\cap B)$

$\therefore n$ (Indian men $\cup$ Doctor)$=n$ (Indian men)$+n$ (Doctor)$-n$ (Indian men $\cap$ Doctor)

$\Rightarrow 45=43+6-n$ (Indian men $\cap$ Doctor)

$\Rightarrow n$ (Indian men $\cap$ Doctor)$=4$

As, there are no foreign doctors so, all doctors are Indian.

$\Rightarrow$ Men doctors $=4$

Women doctors $=$ Total doctors $-$ Men doctors

$$=6-4$$

$$=2$$

Therefore $B=2$

Now the value of $2A-32B$ is $=118-2(9)=118-18=100$

**22. Answer: 2**

From Binomial theorem using $n^{\text{th}}$ term of the given expansion we write

$t_3 = t_{2+1} = {}^5C_2.(x)^{5-2}.(x^p)^2$

$= 10(x)^3(x)^{2p}$

$t_3 = 10x^{3+2p}$

Given

$t_3 = 10^6$

$\therefore 10^6 = 10\,x^{3+2p}$

$10^5 = x^{3+2p}$

Taking $\log_{10}$ on both side

$\log_{10}(10)^5 = \log_{10}\left(x^{3+2p}\right)$

$5 = (3+2p)(p)..............p = \log_{10} x$

$\therefore 2p^2 + 3p - 5 = 0$

$\therefore (2p+5)(p-1) = 0$

$\therefore p = \frac{-5}{2}$ or $p = 1$

$\therefore \log_{10} x = \frac{-5}{2}$ & $\log_{10} x = 1$

$\therefore x = 10^{\frac{-5}{2}}$ & $x = (10)$

$x = \frac{1}{\sqrt{10^5}}$

or

$x = 10$

$\therefore 2$ values are possible for $x$.

**23. Answer: 7**

Let I be the integral

$$\therefore I=\int_0^1 \frac{\log(1+x)}{1+x^2}dx$$

Substitute $x=\tan t \Rightarrow dx=\sec^2 t dt$ as $x\ =0 \therefore t=0$ and $x=1 \therefore t=\frac{\pi}{4}$

$$\therefore I=\int_0^{\frac{\pi}{4}} \frac{\log(1+\tan t)}{\sec^2 t}\sec^2 t dt$$

$$\therefore I=\int_0^{\frac{\pi}{4}} \log(1+\tan t)dt \quad \text{.................(1)}$$

Now, as $\int_0^a f(x)dx=\int_0^a f(a-x)dx$, we have

$$I=\int_0^{\frac{\pi}{4}} \log\left(1+\tan\left(\frac{\pi}{4}-t\right)\right)dt$$

$$I=\int_0^{\frac{\pi}{4}} \log\left(1+\frac{1-\tan t}{1+\tan t}\right)dt \quad \left(\text{using} \tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}\right)$$

$$\therefore I=\int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+\tan t}\right)dt$$

$$\therefore I=\int_0^{\frac{\pi}{4}} \left[\log(2)-\log(1+\tan t)\right]dt$$

$$\therefore I=\int_0^{\frac{\pi}{4}} \log(2)dt-I \quad \text{..............from (1)}$$

$$\therefore 2I=\int_0^{\frac{\pi}{4}} \log(2)dt$$

$$\therefore 2I=\log(2)[t]_0^{\frac{\pi}{4}}$$

$$\therefore 2I=\frac{\pi}{4}\log(2)$$

$$I=\frac{\pi}{8}\log(2)$$

$A = 1$

Therefore $B = 8$

$C = 2$

$\therefore A + B - C = 1 + 8 - 2 = 7$

**24. Answer: 9**

Volume of $A = (V_A) = 2\times$ Volume of $B(V_B)$

Let water present at any time in Tank $A \& B$ be $x \& y$ respectively.

$$\frac{-dx}{dt} = K_1 x; \quad \frac{-dy}{dt} = K_2 y$$

$$x = C_1 e^{-k_1 t}; y = C_2 e^{-k_2 t}$$

At $t = 0; \quad x = 2y$

$$\Rightarrow C_1 = 2C_2$$

At $t = 1; \quad x = \frac{3}{2} y$

$$\Rightarrow C_1 e^{-k_1} = \frac{3}{2} C_2 e^{-k_2}$$

$$\Rightarrow 2C_2 e^{-k_1} = \frac{3}{2} C_2 e^{k_2} \quad .............C_1 = 2C_2$$

$$\Rightarrow e^{-k_1} = \frac{3}{4} e^{-k_2}$$

$$\Rightarrow e^{k_1 - k_2} = \frac{4}{3}$$

$$\Rightarrow k_1 - k_2 = \ln\left(\frac{4}{3}\right) ........(1)$$

When $x = y$

$$C_1 e^{k_1 t} = C_2 e^{k_2 t}$$

$$\Rightarrow 2C_2 e^{-k_1 t} = C_2 e^{-k_2 t} \quad .............C_1 = 2C_2$$

$\Rightarrow 2e^{-k_1t} = e^{-k_2t}$

$\Rightarrow e^{k_1-k_2t} = 2$

$\Rightarrow t = \dfrac{\ln 2}{k_1 - k_2}$

$\Rightarrow T = \dfrac{\ln 2}{\ln\left(\dfrac{4}{3}\right)}$ ............................from (1)

Therefore $P = 2, Q = 4, R = 3$

$P + Q + R = 2 + 4 + 3 = 9$

**25. Answer: 7**

Use $|x-a| = \begin{cases} x-a, x \geq a \\ a-x, x < a \end{cases}$

$\& |x| = \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases}$

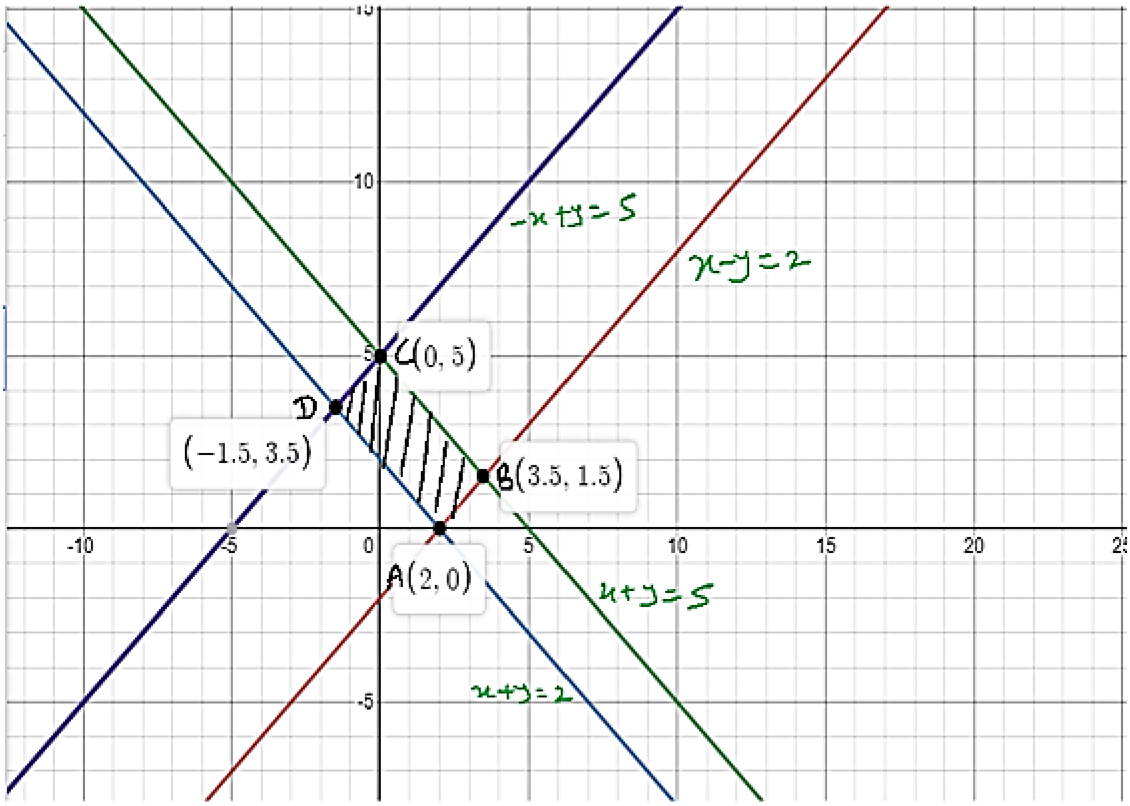
$\therefore$ given equation of curves becomes

For $y = |x-2|$ and For $y = 5 - |x|$

$\therefore\ y = x-2, \quad y = 2-x$ and $\therefore\ y = 5-x \quad y = 5+x$

$\therefore\ x-y = 2, \quad x+y = 2$ and $\therefore\ x+y = 5, \quad -x+y = 5$

We draw the graph for above equations




$\therefore$ Required Area $= AB \times AD$

$= \left(\sqrt{(3.5-2)^2}\right)\left(\sqrt{(2+1.5)^2+(3.5)^2}\right)$

$= \sqrt{2}(1.5)+\sqrt{2}(3.5)$

$= \sqrt{2}[5]$

$= 5\sqrt{2}\,\text{sq.unit}$

$\approx 7.07$

$\therefore$ Integral value of required area $= 7$.

**26. Correct Answer: 6**

$\Rightarrow$ Let $e^{\sin^2 x} = t$ $\qquad \therefore e^{\cos^2 x} = e^{1-\sin^2 x}$

$$= \frac{e}{e^{\sin^2 x}}$$

$$\therefore e^{\cos^2 x} = \frac{e}{t}$$

$\therefore f(t) = 5t - \frac{e}{t} + 2$ for $t \in [1, e]$

$\therefore f'(t) = 5 + \frac{e}{t^2} > 0 \forall t \in R$

Hence $f$ is increasing function.

$\therefore f_{\min} = f(1) = 5 - e + 2 = 7 - e$

and

$f_{\max} = f(e) = 5e - 1 + 2 = 5e + 1$

$\therefore \sqrt{5f_{\min} + f_{\max}} = \sqrt{5(7-e) + 5e + 1}$

$= \sqrt{36}$

$= 6$

**27. Correct Answer: 2**

Suppose the line $lx + my + n = 0$ cuts the ellipse at $P(a\cos\theta, b\sin\theta)$ and $\left(a\cos\left(\frac{\pi}{2}+\theta\right), b\sin\left(\frac{\pi}{2}+\theta\right)\right)$

Then these two points $P, Q$ lie on the line then

$la\cos\theta + mb\sin\theta + n = 0$ and $-la\sin\theta + mb\cos\theta + n = 0$

$\Rightarrow la\cos\theta + mb\sin\theta = -n$ .......(1)

and $-la\sin\theta + mb\cos\theta = -n$ .......(2)

Square and add the equation (1) & (2)

$(la\cos\theta + mb\sin\theta)^2 + (-la\sin\theta + mb\cos\theta)^2 = n^2 + n^2$

$l^2a^2(\cos^2\theta + \sin^2\theta) + m^2b^2(\sin^2\theta + \cos^2\theta) = 2n^2$

$l^2a^2 + m^2b^2 = 2n^2$

$\frac{l^2b^2 + m^2b^2}{2} = 2$

**28. Correct Answer: 53**

Given

$AGP$ is $3 + 5 \times \frac{1}{4} + 7 \times \frac{1}{4^2}$

Now,

$3, 5, 7$ .......are in $AP$

$a = 3$

$\rightarrow d = 5 - 3$

$\rightarrow d = 2$

$1, \frac{1}{4}, \frac{1}{4^2}$ .........are in $GP$

$r = \frac{\left(\frac{1}{4}\right)}{1}$

$\rightarrow r = \frac{1}{4}$

We know that,

Sum of an infinite $AGP$ $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

Hence,

$\Rightarrow S_\infty = \frac{3}{1 - \frac{1}{4}} + \frac{2 \times \frac{1}{4}}{\left(1 - \frac{1}{4}\right)^2}$

$$\Rightarrow S_\infty = \frac{3}{\frac{4-1}{4}} + \frac{\frac{1}{2}}{\left(\frac{4-1}{4}\right)^2}$$

$$\Rightarrow S_\infty = \frac{3}{3}\times 4 + \frac{1}{2}\times\frac{4^2}{3^2}$$

$$\Rightarrow S_\infty = 4 + \frac{8}{9}$$

$$\Rightarrow S_\infty = \frac{36+8}{9}$$

$$\Rightarrow S_\infty = \frac{44}{9}$$

But given $S_\infty = \frac{P}{Q}$

Hence $P = 44$ and $Q = 9$

Therefore $P + Q = 44 + 9 = 53$

**29. Correct Answer: 7**

We have, $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$

Let point $(1,2,3)$ be $P$ and the point through which the line passes be $Q(6,7,7)$.

Also, the line is parallel to the vector $\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$

Now,

$\overrightarrow{PQ} = 5\hat{i} + 5\hat{j} + 4\hat{k}$

$$\therefore \vec{b}\times\overrightarrow{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -2 \\ 5 & 5 & 4 \end{vmatrix}$$

$$= \hat{i}(8+10) - \hat{j}(12+10) + \hat{k}(15-10)$$

$$= 18\hat{i} - 22\hat{j} + 5\hat{k}$$

$$\Rightarrow \left|\vec{b} \times \overrightarrow{PQ}\right| = \sqrt{(18)^2 + (-22)^2 + (5)^2}$$

$$= \sqrt{324 + 484 + 25}$$

$$= \sqrt{833}$$

Using Formula

$$\Rightarrow d = \frac{\left|\vec{b} \times \overrightarrow{PQ}\right|}{\left|\vec{b}\right|}$$

$$= \frac{\left|\sqrt{833}\right|}{\left|\sqrt{(3)^2 + (2)^2 + (2)^2}\right|}$$

$$= \frac{\sqrt{833}}{\sqrt{17}}$$

$$= \sqrt{49}$$

$$= 7$$

**30. Answer: 24**

Given

$$D = \begin{vmatrix} x! & (x+1)! & (x+2)! \\ (x+1)! & (x+2)! & (x+3)! \\ (x+2)! & (x+3)! & (x+4)! \end{vmatrix}$$

$$D = \begin{vmatrix} x! & (x+1)x! & (x+2)(x+1)x! \\ (x+1)! & (x+2)(x+1)! & (x+3)(x+2)(x+1)! \\ (x+2)! & (x+3)(x+2)! & (x+4)(x+3)(x+2)! \end{vmatrix}$$

$$= x!(x+1)!(x+2)! \begin{vmatrix} 1 & (x+1) & (x+2)(x+1) \\ 1 & (x+2) & (x+3)(x+2) \\ 1 & (x+3) & (x+4)(x+3) \end{vmatrix}$$

By $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$

$$= x!(x+1)!(x+2)!\begin{vmatrix} 0 & -1 & -2(x+2) \\ 0 & -1 & -2(x+3) \\ 1 & (x+3) & (x+4)(x+3) \end{vmatrix}$$

$D = 2x!(x+1)!(x+2)!$ the value of $D$ at $x = 1$ is $= 2(1!)(2!)(3!)$

$$= 2(2)(6)$$

$$= 24$$