

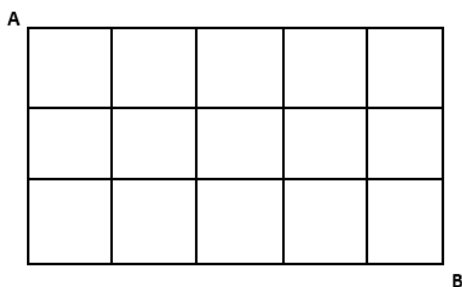
JEE Main - 2023

Mathematics

Section A

This Section A contains 20 multiple choice questions from 1 to 20. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which Only One is correct.

- Find the number of divisors of the number $N = 2^3 \cdot 3^5 \cdot 5^7 \cdot 7^7$ which are perfect square.
 - 60
 - 120
 - 96
 - 100
- $$f(x) = \begin{cases} -1, & -2 \leq x < 0 \\ x^2 - 1, & 0 \leq x \leq 2 \end{cases}$$
 & $g(x) = |f(x) + f(|x|)|$. then, in the interval $(-2, 2)$, g is
 - differentiable at all points
 - not differentiable at two points
 - not continuous
 - not differentiable at one point
- If an insect wants to travel from A to B on the net in the shortest possible path, then how many paths are possible? (All grid sizes are of equal width and height)



- 26
- 56
- 845
- None of the Above

4. Let $f : \left[0, \frac{\pi}{2}\right] \rightarrow \mathbb{R}$ be a function defined by $f(x) = \max\left\{\sin x, \cos x, \frac{3}{4}\right\}$, then the number of points where $f(x)$ is non-differentiable.

- (A) 1
- (B) 2
- (C) 0
- (D) 3

5. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$ then D is

- (A) divisible by x but not y
- (B) divisible by y but not x
- (C) divisible by neither x nor y
- (D) divisible by both x and y

6. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is

- (A) $(3, \infty)$
- (B) $(-\infty, -3)$
- (C) $(-3, 3)$
- (D) $(-3, \infty)$

7. **Direction :** Question is Assertion -Reason type. This question contains two statements: statements-1 (Assertion) and Statements -2 (Reason). Each of these question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

Statement -1: $\sum_{r=0}^n (r+1)^n C_r = (n+2)2^{n-1}$

Statement-2: $\sum_{r=0}^n (r+1)^n C_r x^r = (1+x)^n + nx(1+x)^{n-1}$

- (A) Statement -1 is true, statement -2 is false
- (B) Statement -1 is false, statement -2 is true
- (C) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for Statement -1
- (D) Statement -1 is true, Statement -2 is true ;Statement -2 is not a correct explanation for Statement -1

8. The solution for x of the equation $\int_{\sqrt{x}}^x \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2}$ is
- (A) $\frac{\sqrt{3}}{2}$
 (B) $2\sqrt{2}$
 (C) 2
 (D) None
9. Let ABC be a triangle with $\angle A = 45^\circ$. Let P be a point on side BC with $PB = 3$ and $PC = 5$. If O is circumcenter of triangle ABC then length OP is
- (A) $\sqrt{18}$
 (B) $\sqrt{17}$
 (C) $\sqrt{19}$
 (D) $\sqrt{15}$
10. The maximum sum of the series $20 + 19\frac{1}{3} + 18\frac{2}{3} + \dots$ is
- (A) 310
 (B) 300
 (C) 320
 (D) None of these
11. Two distinct numbers a and b are chosen randomly from the set $\{2, 2^2, 2^3, \dots, 2^{25}\}$. Then the probability that $\log_a b$ is an integer is _____
- (A) $\frac{21}{300}$
 (B) $\frac{31}{100}$
 (C) $\frac{31}{300}$
 (D) $\frac{21}{100}$

12. Find the value of $\lim_{n \rightarrow \infty} n \left[\frac{1}{(n+1)(n+2)} + \frac{1}{(n+2)(n+4)} + \dots + \frac{1}{6n^2} \right]$

- (A) $\log\left(\frac{5}{2}\right)$
- (B) $\log\left(\frac{3}{2}\right)$
- (C) 0
- (D) 1

13. For all real values of x , find the minimum value of $\frac{1-x+x^2}{1+x+x^2}$

- (A) $\frac{1}{2}$
- (B) $\frac{1}{3}$
- (C) $\frac{2}{5}$
- (D) $\frac{5}{6}$

14. If $\cos^{-1} x - \cos^{-1} \frac{y}{2} = \alpha$, then $4x^2 - 4xy \cos \alpha + y^2$ is equal to

- (A) $2 \sin 2\alpha$
- (B) 4
- (C) $4 \sin^2 \alpha$
- (D) $-4 \sin^2 \alpha$

15. If the sum of all solutions of the equation

$$8 \cos x \cdot \left[\cos\left(\frac{\pi}{6} + x\right) \cdot \cos\left(\frac{\pi}{6} - x\right) - \frac{1}{2} \right] = 1 \text{ in } [0, 2\pi] \text{ is } k\pi, \text{ then } k \text{ is equal to}$$

- (A) $\frac{8}{9}$
- (B) $\frac{13}{9}$
- (C) 5
- (D) 6

16. if $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \cdot \sin^{-1} \left(\frac{\sin x + \cos x}{2b} \right) + c$, where c is constant of Integration, then ordered pair (a, b) is

- (A) $\left(\frac{-1}{3} \right)$
- (B) $\left(1, \frac{3}{2} \right)$
- (C) $\left(1, \frac{2}{3} \right)$
- (D) $(1, 3)$

17. If the circle $x^2 + y^2 - 2gx + 6y - 19c = 0$, where $g, c \in R$ passes through the point $(6, 1)$ and its centre lies on the line $x - 2cy = 8$, then the length of intercept made by the circle on Y-axis is _____

- (A) $\sqrt{28}$
- (B) $2\sqrt{23}$
- (C) $2\sqrt{28}$
- (D) $\sqrt{23}$

18. The function $f(x) = x \cdot e^{x(1-x)}$, $x \in R$ is

- (A) Increasing in $\left(\frac{1}{2}, 1 \right)$
- (B) Decreasing in $(1, 3)$
- (C) Increasing in $\left(-\frac{1}{2}, 1 \right)$ and decreasing in $\left(-\infty, \frac{-1}{2} \right) \cup (1, \infty)$
- (D) Increasing in $\left(-\infty, \frac{-1}{2} \right) \cup (1, \infty)$ and decreasing in $\left(\frac{-1}{2}, 1 \right)$

19. Let $y = y(x)$ be the solution curve of differential equation

$$\frac{dy}{dx} + \left(\frac{2x^2 + 11x + 13}{x^3 + 6x^2 + 11x + 6} \right) y = \frac{(x+3)}{(x+1)}, x > -1$$

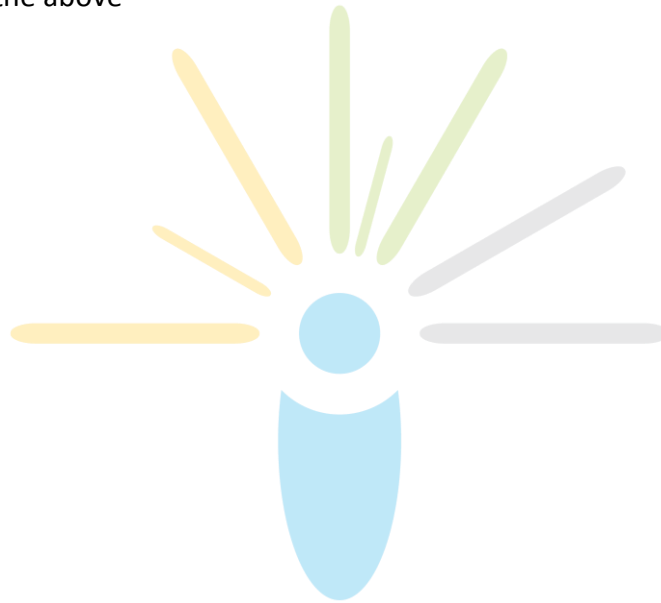
Which passes through point $(0, 1)$ then $y(2)$ is _____

- (A) $\frac{15}{29}$

- (B) $\frac{50}{27}$
- (C) $\frac{27}{50}$
- (D) $\frac{16}{3}$

20. If the length of latus rectum of a parabola, whose focus is (a, a) and the tangent at its vertex is $x + y = a$ is 24, then $|a|$ is equal to:

- (A) $4\sqrt{2}$
- (B) $\sqrt{2}$
- (C) $6\sqrt{2}$
- (D) None of the above

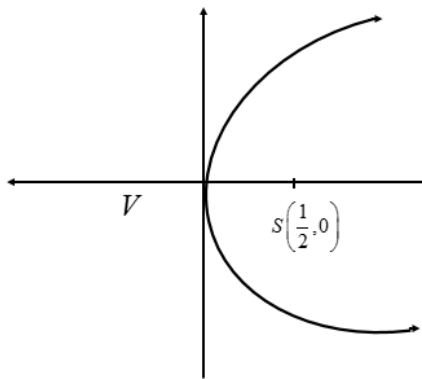


Section B

This Section B contains 10 Integer Type Questions from 21 to 30. Attempt any 5 Questions.

- 21.** At a certain conference of 100 people, there are 29 Indian women and 23 Indian men. Of these Indian people 5 are doctors and 24 are either men or doctors. There are no foreign doctors. How many foreigners and women doctors are attending the conference?
- 22.** Evaluate the following Limit:

$$\lim_{x \rightarrow 0} x^x = ?$$
- 23.** If $|z+4| \leq 3$, then the maximum value of $|z+7|$ is _____
- 24.** The mean and variance of 10 observations were calculated as 20 and 15 respectively by a student who took by mistake 25 instead of 15 for one observation. Then, the correct standard deviation is _____
- 25.** A man starts walking from point $P(-3,4)$, touches the x-axis at R , and then turns to reach at the point $Q(0,2)$. The man is walking at constant speed. If the man reaches point Q in minimum time, then $100(PR^2 + QR^2)$ is _____
- 26.** If the length of latus rectum of the ellipse $x^2 + 4y^2 + 2x + 8y - \lambda = 0$ is $\frac{7}{2}$ and l is the length of its major axis, then $\lambda + l$ is equal to _____
- 27.** A Circle of radius 2 unit passes through the vertex and the focus of parabola $y^2 = 2x$ and touches the parabola, $y = \left(x - \frac{1}{4}\right)^2 + \alpha$, where $\alpha > 0$. Then $(4\alpha - 8)^2 =$ _____



28. The positive value of Determinant of $|3A|$ if $Adj(adj(A)) = \begin{bmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{bmatrix}$ is

29. Let \vec{a} and \vec{b} be two vectors such that $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2$, $\vec{a} \cdot \vec{b} = 3$ and $|\vec{a} \times \vec{b}|^2 = 87$

Then $|\vec{a}|^2$ is equal to _____

30. let p and q be two real numbers such that $p + q = 3$ and $p^4 + q^4 = 369$. Then

$\left(\frac{1}{p} + \frac{1}{q}\right)^{-2}$ is equal to