JEE Main - 2023

Mathematics

Section A

This Section A contains 20 multiple choice questions from 1 to 20. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which only one is correct.

1. **Statement-1:** Total number of positive integral solutions of the equation $x_1.x_2.x_3.x_4.x_5 = 420$

   Statement-2: the total number of divisors of 420 is 25

   (A) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

   (B) Statement-1 is true, Statement-2 is true, Statement-2 is NOT a correct explanation for Statement-1.

   (C) Statement-1 is True, Statement-2 is False

   (D) Statement-1 is False, Statement-2 is True

2. In an cylindrical vessel there are 12 green and 8 yellow balls that are arranged randomly. If 10 balls are randomly drawn one-by-one with replacement, then the variance of the number of green balls drawn is

   (A) $\frac{12}{5}$

   (B) $\frac{6}{25}$

   (C) 6

   (D) 4

3. If $|z| \le 1, |\omega| \le 1$ where z and ω be two complex numbers such that and $|z - i\omega| = |z - i\overline{\omega}| = 3$ then z equals

   (A) $1 \text{or} i$

   (B) $i \text{or} -i$

   (C) $1 \text{or} -1$

   (D) $i \text{or} -1$

4. $e^{\lim\limits_{n\to\infty} n\left[\left(\frac{n}{n+1}\right)^{\alpha}+\sin\left(\frac{1}{n}\right)-1\right]}$, where $\alpha \in Q$ is equal to

(A) $e^{-\alpha}$

(B) $-\alpha$

(C) $e^{1-\alpha}$

(D) $e^{1+\alpha}$

5. If $4f(\cos x)+f(\sin x)=10x$ then $\frac{d}{dx}f(\sin x)=$

(A) $\frac{10}{3}\cos x$

(B) $\frac{10}{3}\sin x$

(C) $\frac{10}{3}\sec x$

(D) $\frac{3}{10}\cos x$

6. If $\int\frac{x^3+x}{x^4-9}dx=\frac{1}{A}f(x)+\frac{1}{B}\log\left|\frac{x^2-3}{x^2+3}\right|+c$ then

(A) $A=4, b=12, f(x)=\log|x^4-9|$

(B) $A=12, B=4, f(x)=\log|x^4+9|$

(C) $A=4, B=12, f(x)=\log|x^4+9|$

(D) $A=12, B=4, f(x)=\log|x^4-9|$

7. A Pole standing on a plain road surface which is leaning towards East. The angles of elevation of the top with respect to two points situated at distance a and b exactly due west on it are respectively α and β. Then the height of the top of pole from the ground is

(A) $\frac{(b-a)\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}$

(B) $\frac{(a-b)\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}$

(C) $\frac{(b-a)\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$

(D) $\frac{(a-b)\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$

8. $\sum_{r=0}^{n} \tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right)$ is equal to

(A) $\tan^{-1}\left(2^n\right)$

(B) $\tan^{-1}\left(2^n\right)-\frac{\pi}{4}$

(C) $\tan^{-1}\left(2^{n+1}+2^n\right)$

(D) $\tan^{-1}\left(\frac{2^{n+1}-1}{2+2^n}\right)$

9. If α and β are the roots of the equation $2x^2+6x+a=0, a<0$ then what is the lowest prime integral value of $2a$, if $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}<2$

(A) 10
(B) 11
(C) 12
(D) 13

10. A vector perpendicular to the plane containing the vector $\hat{i}+2\hat{j}+\hat{k}$ and $-2\hat{i}+\hat{j}+3\hat{k}$ is inclined to the vector $\hat{i}+\hat{j}+\hat{k}$ at an angle

(A) $\tan^{-1}\left(\frac{1}{3}\right)$

(B) $\tan^{-1}\left(\frac{2}{3}\right)$

(C) $\tan^{-1}\left(\frac{5}{3}\right)$

(D) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

11. The normal to the curve $y=x^2-x+1$, drawn at the points with the abscissa $x_1=0, x_2=-1$ and $x_3=\frac{5}{2}$

(A) are parallel to each other
(B) are pair wise perpendicular
(C) are concurrent
(D) are not concurrent

12. If the tangent to the parabola $y^2 = 4ax$ meets the axis in T and tangent at the vertex Q in A and the rectangle $T\ OA\ G$ is completed, then the locus of G is

(A) $y^2 + 2ax = 0$

(B) $x^2 + ay = 0$

(C) $y^2 + ax = 0$

(D) $x^2 + 2ay = 0$

13. Let $P(4\sec\theta, \tan\theta)$ and $Q\ (4\sec\phi, 3\tan\phi)$, where $\theta + \phi = \frac{\pi}{2}$, be two points on the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$. If (h, k) is the point of intersection of the normals at P & Q then k is equal to.

(A) $\frac{25}{4}$

(B) $\frac{25}{3}$

(C) $-\frac{25}{4}$

(D) $-\frac{25}{3}$

14. p, q, r, s are distinct integers in $A.P$ such that $S = p^2 + q^2 + r^2$ then $p + q + r + s$ is

(A) 0

(B) 1

(C) 2

(D) 3

15. The angle between the lines $(x^2 + y^2)\sin^2\alpha = (x\cos\theta - y\sin\theta)^2$ is

(A) 2α

(B) α

(C) $\alpha + \beta$

(D) $2(\alpha - \beta)$

16. The equation of the circle describes on the common chord of the circles $x^2 + y^2 + 4x = 0$ and $x^2 + y^2 + 4y = 0$ as diameter is

(A) $x^2 + y^2 - x - y = 0$

(B) $x^2 + y^2 + x - y = 0$

(C) $x^2+y^2+x+y=0$

(D) $x^2+y^2+2x+2y=0$

17. If the angle θ between the line $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda z}+4=0$ is such that the $\sin\theta=\frac{1}{3}$ then the value of λ is

(A) $-\frac{3}{5}$

(B) $\frac{5}{3}$

(C) $-\frac{4}{3}$

(D) $\frac{3}{4}$

18. The median of a set of 11 distinct observations is 35.5. If each of the largest 5 observations of the set is increased by 3, then the median of new set:

(A) is increased by 2

(B) is decreased by 2

(C) is two times the original median.

(D) remains the same as that of the original set.

19. Let three matrices $A=\begin{bmatrix}2 & 1\\ 4 & 1\end{bmatrix}: B\begin{bmatrix}-3 & -4\\ -6 & -5\end{bmatrix}$ and $C=\begin{bmatrix}\frac{5}{9} & -\frac{4}{9}\\ -\frac{2}{3} & \frac{1}{3}\end{bmatrix}$ then

$$t_r(A)+t_r\left(\frac{ABC}{2}\right)+t_r\left(\frac{A(BC)^2}{4}\right)+t_r\left(\frac{A(BC)^3}{8}\right)+....+\infty$$

(A) 6

(B) 9

(C) 12

(D) 15

20. The proposition $(p\Rightarrow -p)\wedge(-p\Rightarrow p)\Leftrightarrow F$ is a

(A) Tautology and contradiction

(B) Tautology

(C) Neither tautology nor contradiction

(D) Contradiction

Section-B

This Section B contains 10 Integer Type Questions from 21 to 30. Attempt any 5 Questions.

21. In an certain International conference of 200 people there are 39 Indian women and 43 Indian men. Of these Indian people 6 are doctors and 45 are either men or doctors. There are no foreign doctors. If $2A$ foreigners and B women doctors are attending the conference then the value of $2A-9B$ is .....

22. In the expansion of $\left(x+x^{p}\right)^{5}$ where $p=\log_{10} x$, if its third term is 10^{6} then How many Values are possible for x _____

23. If $\int_{0}^{1}\frac{\log(1+x)}{(1+x^{2})}dx=\frac{A\pi}{B}\text{Log}\,C$ then the value of $A+B-C=$ _____

24. There are two separate Tanks of water namely A and B with Tap. The capacity of tank A is double that of tank B. Both the tanks are filled completely with water, their inlets are closed and then the water is released simultaneously from tap of both the tanks. The rate of flow of water out of each tank at any instant of time is proportional to the quantity of water in the tank at that time. One hour after the water is released, the quantity of water in tank A is $\frac{3}{2}$ times the quantity of water in tank B. If both the tanks have the same quantity of after T hours where $T=\frac{\log P}{\log\frac{Q}{R}}$ then the value of $P+Q+R=$.....

25. The integral value of the area bounded by the curves $y=|x-2|$ and $y=5-|x|$ is

26. Let $f(x)=5e^{\sin^{2}x}-e^{\cos^{2}x}+2$, then the value of $\sqrt{5f_{\min}+f_{\max}}$, is _____

27. The line, $lx+my+n=0$ cut the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$ in points whose eccentric angles differ by $\frac{\pi}{2}$ then $\frac{a^{2}1^{2}+b^{2}m^{2}}{n^{2}}=$ _____

28. If the sum of infinite terms of the series $3+5.\frac{1}{4}+7.\frac{1}{4^{2}}+9.\frac{1}{4^{3}}+.....$ is $\frac{P}{Q}$ then $P+Q=$ _____

29. The perpendicular distance of the point $P(1,2,3)$ from the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$ is _____.

30. If $D=\begin{vmatrix} x! & (x+1)! & (x+2)! \\ (x+1)! & (x+2)! & (x+3)! \\ (x+2)! & (x+3)! & (x+4)! \end{vmatrix}$ where x is a positive integer then the value of D at $x=1$ is _____