JEE ADVANCED-2012

MATHEMATICS

[Time: 3 Hours] [Maximum Marks: 210]

A. General Instructions:

1. This booklet is your Question paper. Do not break the seats of his booklet before being instructed to do so by the invigilators.

2. Blank spaces and blank pages are provided in this booklet for your rough work. No additional sheets will be provided for rough work.

3. Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers, and electronic gadgets are NOT allowed inside the examination hall.

4. Answers to the questions and personal details are to be filled on a two-part carbon-less paper, which is provided separately. You should not separate these parts. The invigilator will separate them at the end of examination. The upper sheet is machine-gradable Objective Response Sheet (ORS) which will be taken back by the invigilator.

5. **Using a black ball point pen, darken the bubbles on the upper original sheet.** Apply sufficient pressure so that the impression is created on the bottom sheet.

6. DO NOT TAMPER WITH /MUTILATE THE ORS OR THE BOOKLET.

7. On breaking the seals of the booklet check that it contains 28 pages and all 60 questions and corresponding answer choices are legible. Read carefully the instructions printed at the beginning of each section.

B. Filling the Right Part of the ORS

8. The ORS also has a CODES printed on its left and right parts.

9. Check that the same CODE is printed on the ORS and on this booklet. IF IT IS NOT THEN **ASK FOR A CHANGE OF THE BOOKLET.** Sign at the place provided on the ORS affirming that you have verified that all the code are same.

10. Write your Name, Registration Number and the name of examination centre and sign with pen in the boxes provided on the right part of the ORS. **Do not write any of this information anywhere else.** Darken the appropriate bubble UNDER each digit

of your Registration Number in such a way that the impression is created on the bottom sheet. Also darken the paper CODE given on the right side of **ORS(R4)**.

C. Question paper format and Marking scheme:

11. **Section I** contains **10 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

12. **Section II** contains **5 multiple choice questions.** Each question has four choice (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

13. **Section III** contains **5 questions.** The answer to each question is a **single digit integer,** ranging from 0 to 9 (both inclusive).

D. Marking Scheme

14. For each question in **Section I,** you will be awarded **3 marks** if you darken the bubble corresponding to the correct answer **ONLY** and **zero marks** if no bubbles are darkened. In all other cases, **minus one (–1)** mark will be awarded in this section.

15. For each question in **Section II,** you will be awarded **4 marks** if you darken **ALL** the bubble)s) corresponding to the correct answer(s) **ONLY.** In all other cases **zero (0) marks** will be awarded. **No negative marks** will be awarded for incorrect answer in this section.

16. For each question in **Section III,** you will be awarded **4 marks** if you darken the bubble corresponding to the correct answer **ONLY.** In all other cases **zero (0) marks** will be awarded. **No negative marks** will be awarded for incorrect answer in this section.

SECTION I

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

41. If $\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$, then

(A) $a=1,b=4$

(B) $a=1,b=-4$

(C) $a=2,b=-3$

(D) $a=2,b=3$

42. Let $P=\left[a_{ij}\right]$ be a 3×3 matrix and let $Q=\left[b_{ij}\right]$, where $b_{ij}=2^{i+j}a_{ij}$ for $1\le i,j\le3$. If the determinant of P is 2, then the determinant of the matrix Q is

(A) 2^{10}

(B) 2^{11}

(C) 2^{12}

(D) 2^{13}

43. The locus of the mid–point of the chord of contact of tangents drawn from points lying on the straight line $4x-5y=20$ to the circle $x^2+y^2=9$ is

(A) $20\left(x^2+y^2\right)-36x+45y=0$

(B) $20\left(x^2+y^2\right)+36x-45y=0$

(c) $36\left(x^2+y^2\right)-20x+45y=0$

(D) $36\left(x^2+y^2\right)+20x-45y=0$

44. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

(A) 75

(B) 150

(C) 210

(D) 243

45. The integral $\int \frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}\,dx$ equals (for some arbitrary constant K)

(A) $-\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(B) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(C) $-\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(D) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

46. The point P is the intersection of the straight line joining the points $Q(2,3,5)$ and $R(1,-1,4)$ with the plane $5x-4y-z=1$. If S is the foot of the perpendicular drawn from the point $T(2,1,4)$ to QR, then the length of the line segment PS is

(A) $\frac{1}{\sqrt{2}}$

(B) $\sqrt{2}$

(C) 2

(D) $2\sqrt{2}$

47. Let $f(x)=\begin{cases} x^2\left|\cos\frac{\pi}{x}\right|, & x\neq 0 \\ 0, & x=0 \end{cases}, x\in$, then f is

(A) differentiable both at $x=0$ and at $x=2$

(B) differentiable at $x=0$ but not differentiable at $x=2$

(C) not differentiable at $x=0$ but differentiable at $x=2$

(D) differentiable neither at $x=0$ nor at $x=2$

48. Let z be a complex number such that the imaginary part of z is nonzero and $a=z^2+z+1$ is real. Then a cannot take the value

(A) -1

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{3}{4}$

49. The ellipse $E_1:\frac{x^2}{9}+\frac{y^2}{4}=1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point $(0,4)$ circumscribes the rectangle R. The eccentricity of the ellipse E_2 is

(A) $\frac{\sqrt{2}}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$

(D) $\frac{3}{4}$

50. The function $f:[0,3]\to[1,29]$, defined by $f(x)=2x^3-15x^2+36x+1$, is

(A) one–one and onto

(B) onto but not one–one

(C) one–one but not onto

(D) neither one–one nor onto

SECTION II

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

51. Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$, parallel to the straight line $2x-y=1$. The points of contact of the tangents on the hyperbola are

(A) $\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

(B) $\left(-\frac{9}{2\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$

(C) $\left(3\sqrt{3},-2\sqrt{2}\right)$

(D) $\left(-3\sqrt{3},2\sqrt{2}\right)$

52. Let $\theta, \varphi \in [0, 2\pi]$ be such that

$2\cos\theta(1-\sin\varphi) = \sin^2\theta\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\cos\varphi - 1, \tan(2\pi-\theta) > 0$ and

$-1 < \sin\theta < -\frac{\sqrt{3}}{2}$. Then φ **cannot** satisfy

(A) $0 < \varphi < \frac{\pi}{2}$

(B) $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$

(C) $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$

(D) $\frac{3\pi}{2} < \varphi < 2\pi$

53 If $y(x)$ satisfies the differential equation $y' - y\tan x = 2x\sec x$ and $y(0) = 0$, then

(A) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$

(B) $y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$

(C) $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}$

(D) $y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$

54. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is(are) true ?

(A) $P\left[X_1^c \middle| X\right] = \frac{3}{16}$

(B) $P\left[\text{Exactly two engines of the ship are functioning} \middle| X\right] = \frac{7}{8}$

(C) $P\left[X \middle| X_2\right] = \frac{5}{16}$

(D) $P\left[X \middle| X_1\right] = \frac{7}{16}$

55. Let S be the area of the region enclosed by $y = e^{-x^2}, y = 0, x = 0$ and $x = 1$. Then

(A) $S \geq \frac{1}{e}$

(B) $S \geq 1 - \frac{1}{e}$

(C) $S \leq \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$

(D) $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

SECTION III

This section contains **5 questions**. The answer to each question is single digit integer, ranging from 0 to 9 (*both inclusive*).

56. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $\left|\vec{a}-\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}-\vec{a}\right|^2=9$, then $\left|2\vec{a}+5\vec{b}+5\vec{c}\right|$ is

57. Let $f=IR\to IR$ be defined as $f(x)=|x|+\left|x^2-1\right|$. The total number of points at which f attains either a local maximum or a local minimum is

58. Let S be the focus of the parabola $y^2=8x$ and let PQ be the common chord of the circle $x^2+y^2-2x-4y=0$ and the given parabola. The area of the triangle PQS is

59. Let $p(x)$ be a real polynomial of least degree which has a local maximum at $x=1$ and a local minimum at $x=3$. If $p(1)=6$ and $p(3)=2$, then $p'(0)$ is

60. The value of $6+\log_{3/2}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots}}}\right)$ is