

JEE ADVANCED-2012

MATHEMATICS

41. **Sol. (D)**

a_1, a_2, a_3 , are in *H.P.*

$\Rightarrow \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \dots$ are in *A.P.*

$\Rightarrow \frac{1}{a_n} = \frac{1}{a_1} + (n-1)d < 0$, where $\frac{\frac{1}{25} - \frac{5}{25}}{19} = d = \left(\frac{-4}{9 \times 25} \right)$

$\Rightarrow \frac{1}{5} + (n-1) \left(\frac{-4}{19 \times 25} \right) < 0$

$\frac{4(n-1)}{19 \times 5} > 1$

$n-1 > \frac{19 \times 5}{4}$

$n > \frac{19 \times 5}{4} + 1 \Rightarrow n \geq 25$.

42. **Sol. (A)**

Equation of required plane is

$$P \equiv (x+2y+3z-2) + \lambda(x-y+z-3) = 0$$

$$\Rightarrow (1+\lambda)x + (2-\lambda)y + (3+\lambda)z - (2+3\lambda) = 0$$

Its distance from $(3, 1, -1)$ is $\frac{2}{\sqrt{3}}$

$$\Rightarrow \frac{2}{\sqrt{3}} = \frac{|3(1+\lambda) + (2-\lambda) - (3+\lambda) - (2+3\lambda)|}{\sqrt{(\lambda+1)^2 + (2-\lambda)^2 + (3+\lambda)^2}}$$

$$= \frac{4}{3} = \frac{(-2\lambda)^2}{3\lambda^2 + 4\lambda + 14} \Rightarrow 3\lambda^2 + 4\lambda + 14 = 3\lambda^2$$

$$\Rightarrow \lambda = -\frac{7}{2} \Rightarrow -\frac{5}{2}x + \frac{11}{2}y - \frac{z}{2} + \frac{17}{2} = 0$$

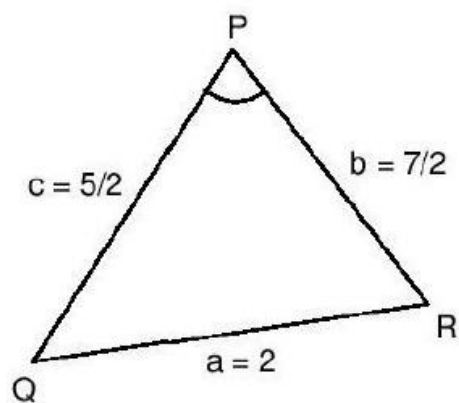
$$-5x + 11y - z + 17 = 0.$$

43. Sol. (C)

$$\frac{2\sin P - 2\sin P \cos P}{2\sin P + 2\sin P \cos P} = \frac{1 - \cos P}{1 + \cos P} = \frac{2\sin^2 \frac{P}{2}}{2\cos^2 \frac{P}{2}} = \tan^2 \frac{P}{2}$$

$$= \frac{(s-b)(s-c)}{s(s-a)}$$

$$= \frac{((s-b)(s-c))^2}{\Delta^2} = \frac{\left(\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\right)^2}{\Delta^2} = \left(\frac{3}{4\Delta}\right)^2$$



44. **Sol. (C)**

$$\vec{a} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{b}$$

$$(\vec{a} + \vec{b}) \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = \vec{0}$$

$$\Rightarrow \vec{a} + \vec{b} = \pm (2\hat{i} + 3\hat{j} + 4\hat{k}) \quad (\text{as } |\vec{a} + \vec{b}| = \sqrt{29})$$

$$\Rightarrow (\vec{a} + \vec{b}) \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= \pm (-14 + 6 + 12) = \pm 4.$$

45. **Sol. (D)**

$$\text{Give } P^T = 2P + I$$

$$\Rightarrow P = 2P^T + I = 2(2P + I) + I$$

$$\Rightarrow P + I = 0$$

$$\Rightarrow PX + X = 0$$

$$PX = -X.$$

46. **Sol. (B)**

$$\text{Let } 1 + a = y$$

$$\Rightarrow (y^{1/3} - 1)x^2 + (y^{1/2} - 1)x + y^{1/6} - 1 = 0$$

$$\Rightarrow \left(\frac{y^{1/3} - 1}{y - 1} \right) x^2 + \left(\frac{y^{1/2} - 1}{y - 1} \right) x + \frac{y^{1/6} - 1}{y - 1} = 0$$

Now taking $\lim_{y \rightarrow 1}$ on both the sides

$$\Rightarrow \frac{1}{3}x^2 + \frac{1}{2}x + \frac{1}{6} = 0$$

$$\Rightarrow 2x^2 + 3x + 1 = 0$$

$$x = -1, -\frac{1}{2}.$$

47. **Sol. (A)**

$$\text{Required probability} = 1 - \frac{6 \cdot 5^3}{6^4} = 1 - \frac{125}{216} = \frac{91}{216}.$$

48. **Sol. (B)**

$$\begin{aligned} & \int_{-\pi/2}^{\pi/2} \left\{ x^2 + \ln \frac{\pi+x}{\pi-x} \right\} \cos x \, dx \\ &= \int_{-\pi/2}^{\pi/2} x^2 \cos x \, dx + \int_{-\pi/2}^{\pi/2} \ln \left(\frac{\pi+x}{\pi-x} \right) \cos x \, dx \\ &= 2 \int_0^{\pi/2} x^2 \cos x \, dx \\ &= 2 \left[x^2 \sin x + 2x \cos x - 2 \sin x \right]_0^{\pi/2} \\ &= 2 \left[\frac{\pi^2}{4} - 2 \right] = \frac{\pi^2}{2} - 4. \end{aligned}$$

49. **Sol. (A)**

Equation of tangent at $P(\sqrt{3}, 1)$

$$\sqrt{3}x + y = 4$$

Slope of line perpendicular to above tangent is $\frac{1}{\sqrt{3}}$

So equation of tangents with slope $\frac{1}{\sqrt{3}}$ to $(x-3)^2 + y^2 = 1$ will be

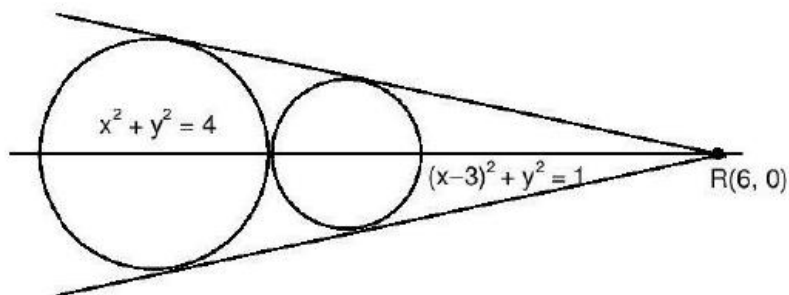
$$y = \frac{1}{\sqrt{3}}(x-3) \pm 1\sqrt{1 + \frac{1}{3}}$$

$$\sqrt{3}y = x - 3 \pm (2)$$

$$\sqrt{3}y = x - 1 \text{ or } \sqrt{3}y = x - 5.$$

50. **Sol. (D)**

Point of intersection of direct common tangents is $(6, 0)$



so let the equation of common tangent be

$$y - 0 = m(x - 6)$$

as it touches $x^2 + y^2 = 4$

$$\Rightarrow \left| \frac{0-0+6m}{\sqrt{1+m^2}} \right| = 2$$

$$9m^2 = 1+m^2$$

$$m = \pm \frac{1}{2\sqrt{2}}$$

So equation of common tangent

$$y = \frac{1}{2\sqrt{2}}(x-6), y = -\frac{1}{2\sqrt{2}}(x-6) \text{ and also } x = 2$$

51. **Sol. (C)**

$$f(x) = (1-x)^2 \sin^2 x + x^2 \quad \forall x \in \mathbb{R}$$

$$g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - \ln t \right) f(t) dt \quad \forall x \in (1, \infty)$$

For statement P :

$$f(x) + 2x = 2(1+x^2) \quad \dots(i)$$

$$(1-x)^2 \sin^2 x + x^2 + 2x = 2 + 2x^2$$

$$(1-x)^2 \sin^2 x = x^2 - 2x + 2 = (x-1)^2 + 1$$

$$(1-x)^2 (\sin^2 x - 1) = 1$$

$$-(1-x)^2 \cos^2 x = 1$$

$$(1-x)^2 \cdot \cos^2 x = -1$$

So equation (i) will not have real solution

So, P is wrong.

For statement Q :

$$2(1-x)^2 \sin^2 x + 2x^2 + 1 = 2x + 2x^2 \quad \dots(ii)$$

$$2(1-x)^2 \sin^2 x = 2x - 1$$

$$2 \sin^2 x = \frac{2x-1}{(1-x)^2} \quad \text{Let } h(x) = \frac{2x-1}{(1-x)^2} - \sin^2 x$$

Clearly $h(0) = -ve$, $\lim_{x \rightarrow 1^-} h(x) = +\infty$

So by IVT, equation (ii) will have solution.

So, Q is correct.

52. **Sol. (B)**

$$g'(x) = \left(\frac{2(x-1)}{x+1} - \ln x \right) f(x). \quad \text{For } x \in (1, \infty), f(x) > 0$$

$$\text{Let } h(x) = \left(\frac{2(x-1)}{x+1} - \ln x \right) \Rightarrow h'(x) = \left(\frac{4}{(x+1)^2} - \frac{1}{x} \right) = \frac{-(x-1)^2}{(x+1)^2 x} < 0$$

Also $h(1) = 0$ so, $h(x) < 0 \quad \forall x > 1$

$\Rightarrow g(x)$ is decreasing on $(1, \infty)$.

53. **Sol. (B)**

$$a_n = b_n + c_n$$

$$b_n = a_{n-1}$$

$$c_n = a_{n-2} \Rightarrow a_n = a_{n-1} + a_{n-2}$$

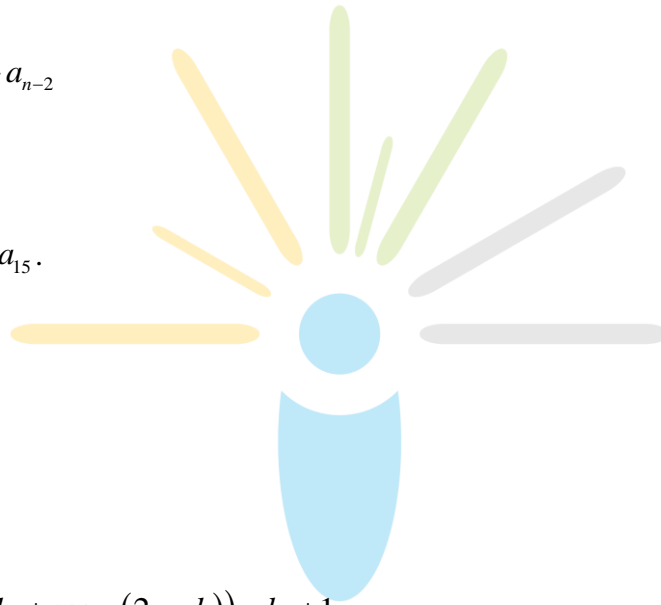
$$\text{As } a_1 = 1, a_2 = 2, a_3 = 3, a_4 = 5, a_5 = 8 \Rightarrow b_6 = 8.$$

54. **Sol. (A)**

$$\text{As } a_n = a_{n-1} + a_{n-2}$$

$$\text{for } n = 17$$

$$\Rightarrow a_{17} = a_{16} + a_{15}.$$



55. **Sol. (B, D)**

$$\text{At } x = 2n$$

$$L.H.L = \lim_{h \rightarrow 0} (b_n + \cos \pi(2n - h)) = b_n + 1$$

$$R.H.L = \lim_{h \rightarrow 0} (a_n + \sin \pi(2n + h)) = a_n$$

$$f(2n) = a_n$$

$$\text{For continuity } b_n + 1 = a_n$$

$$\text{At } x = 2n + 1$$

$$L.H.L = \lim_{h \rightarrow 0} (a_n + \sin \pi(2n + 1 - h)) = a_n$$

$$R.H.L = \lim_{h \rightarrow 0} (b_{n+1} + \cos(\pi(2n+1-h))) = b_{n+1} - 1$$

$$f(2n+1) = a_n$$

For continuity

$$a_n = b_{n+1} - 1$$

$$a_{n-1} - b_n = -1.$$

56. **Sol. (B, C)**

For given lines to be coplanar, we get

$$\begin{vmatrix} 2 & k & 2 \\ 5 & 2 & k \\ 2 & 0 & 0 \end{vmatrix} = 0 \Rightarrow k^2 = 4, k = \pm 2$$

For $k = 2$, obviously the plane $y + 1 = z$ is common in both lines

For $k = -2$ family of plane containing first line is $x + y + \lambda(x - z - 1) = 0$.

Point $(-1, -1, 0)$ must satisfy it

$$-2 + \lambda(-2) = 0 \Rightarrow \lambda = -1$$

$$\Rightarrow y + z + 1 = 0.$$

57. **Sol. (A, D)**

$$|\text{Adj } P| = |P|^2 \text{ as } (|\text{Adj}(P)| = |P|^{n-1})$$

$$\text{Since } |\text{Adj } P| = 1(3-7) - 4(6-7) + 4(2-1) = 4$$

$$|P| = 2 \text{ or } -2.$$

58. **Sol. (A, B)**

$$\text{For } \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right).$$

$$\text{Let } \cos 4\theta = 1/3$$

$$\Rightarrow \cos 2\theta = \pm \sqrt{\frac{1 + \cos 4\theta}{2}} = \pm \sqrt{\frac{2}{3}}$$

$$f\left(\frac{1}{3}\right) = \frac{2}{2 - \sec^2 \theta} = \frac{2 \cos^2 \theta}{2 \cos^2 \theta - 1} = 1 + \frac{1}{\cos 2\theta}$$

$$f\left(\frac{1}{3}\right) = 1 - \sqrt{\frac{3}{2}} \text{ or } 1 + \sqrt{\frac{3}{2}}.$$

59. **Sol. (A, B)**

$$P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)} = \frac{1}{2} \text{ and } \frac{P(X \cap Y)}{P(X)} = \frac{1}{3}$$

$$P(X \cap Y) = \frac{1}{6} \Rightarrow P(Y) = \frac{1}{3} \text{ and } P(X) = \frac{1}{2}$$

Clearly, X and Y are independent

$$\text{Also, } P(X \cup Y) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}.$$

60. Sol. (A, B, C, D)

$$f'(x) = e^{x^2}(x-2)(x-3)$$

Clearly, maxima at $x=2$, minima at $x=3$ and

decreasing in $x \in (2,3)$

$$f'(x) = 0 \text{ for } x=2 \text{ and } x=3 \text{ (Rolle's theorem)}$$

so there exist $c \in (2,3)$ for which

$$f''(c) = 0.$$

