JEE ADVANCED-2012

MATHEMATICS

[Time: 3 Hours] [Maximum Marks: 198]

A. General Instructions :

1. This booklet is your Question paper. Do not break the seats of this booklet before being instructed to do so by the invigilators.

2. The question paper CODE is printed on the right hand top corner of this page and on the back page of this booklet.

3. Blank spaces and blank pages are provided in this booklet for your rough work. No additional sheets will be provided for rough work.

4. Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers, and electronic gadgets are NOT allowed inside the examination hall.

5. Answers to the questions and personal details are to be filled on a two-part carbon-less paper, which is provided separately. You should not separate these parts. The invigilator will separate them at the end of examination. The upper sheet is machine-gradable Objective Response Sheet (ORS) which will be taken back by the invigilator. You will be allowed to take away the bottom sheet at the end of the examination.

6. **Using a black ball point pen, darken the bubbles on the upper original sheet.** Apply sufficient pressure so that the impression is created on the bottom sheet.

7. DO NOT TAMPER WITH /MUTILATE THE ORS OR THE BOOKLET.

8. On breaking the seals of the booklet check that it contains 36 pages and all 60 questions and corresponding answer choices are legible. Read carefully the instructions printed at the beginning of each section.

B. Filling the Right Part of the ORS:

9. The ORS has CODES printed on its left and right parts.

10. Check that the same CODE is printed on the ORS and on this booklet. **IF IT IS NOT THEN ASK FOR A CHANGE OF THE BOOKLET.** Sign at the place provided on the ORS affirming that yon have verified that all the codes are same.

11. Write your Name, Registration Number and the name of examination centre and sign with pen in the boxes provided on the right part of the ORS. **Do not write any of this information anywhere else.** Darken the appropriate bubble UNDER each digit of

your Registration Number in such a way that the impression is created on the bottom sheet. Also darken the paper CODE given on the right side of ORS(R_4).

C. Question Paper Format:

12. **Section I** contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

13. **Section II** contains **3 paragraphs** each describing theory, experiment, data etc. There are **6 multiple choice questions** relating to three paragraphs with **2 questions on each paragraph.** Each question of a particular paragraph has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

14. **Section III** contains **6 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE are correct.

D. Marking Scheme

15. For each question in **Section I and Section II,** you will be awarded **3 marks** if you darken the bubble corresponding to the correct answer **ONLY** and **zero (0) marks** if no bubbles are darkened. In all other cases, **minus one (–1) mark** will be awarded in these sections.

16. For each question in **Section III**, you will be awarded **4 marks** if you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY.** In all other cases **zero (0) marks** will be awarded. **No negative marks** will be awarded for incorrect answer(s) in this section.

SECTION I

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

41. Let $a_1, a_2, a_3, \ldots$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$

(A) 22

(B) 23

(C) 24

(D) 25

42. The equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3,1,-1)$ is

(A) $5x - 11y + z = 17$

(B) $\sqrt{2}x + y = 3\sqrt{2} - 1$

(C) $x + y + z = \sqrt{3}$

(D) $x - \sqrt{2}y = 1 - \sqrt{2}$

43. Let PQR be a triangle of area Δ with $a = 2, b = \frac{7}{2}$ and $c = \frac{5}{2}$, where a, b, and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals

(A) $\frac{3}{4\Delta}$

(B) $\frac{45}{4\Delta}$

(C) $\left(\frac{3}{4\Delta}\right)^2$

(D) $\left(\frac{45}{4\Delta}\right)^2$

44. If $\vec{a}$ and $\vec{b}$ are vectors such that $\left|\vec{a}+\vec{b}\right|=\sqrt{29}$ and $\vec{a}\times\left(2\hat{i}+3\hat{j}+4\hat{k}\right)=\left(2\hat{i}+3\hat{j}+4\hat{k}\right)\times\vec{b}$, then a possible value of $\left(\vec{a}+\vec{b}\right)\cdot\left(-7\hat{i}+2\hat{j}+3\hat{k}\right)$ is

(A) 0

(B) 3

(C) 4

(D) 8

45. If P is a 3×3 matrix such that $P^T=2P+1$, where P^T is the transpose of P and I is the 3×3 identity matrix then there exists a column matrix $X=\begin{bmatrix}x\\y\\z\end{bmatrix}\neq\begin{bmatrix}0\\0\\0\end{bmatrix}$ such that

(A) $PX=\begin{bmatrix}0\\0\\0\end{bmatrix}$

(B) $PX=X$

(C) $PX=2X$

(D) $PX=-X$

46. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation $\left(\sqrt[3]{1+a}-1\right)x^2+\left(\sqrt{1+a}-1\right)x+\left(\sqrt[6]{1+a}-1\right)=0$ where $a>-1$. Then $\lim\limits_{a\to o^+}\alpha(a)$ and $\lim\limits_{a\to o^+}\beta(a)$ are

(A) $-\frac{5}{2}$ and 1

(B) $-\frac{1}{2}$ and -1

(C) $-\frac{7}{2}$ and 2

(D) $-\frac{9}{2}$ and 3

47. Four fair dice D_1, D_2, D_3 and D_4, each having six faces numbered 1, 2, 3, 4, 5, and 6, are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is

(A) $\frac{91}{216}$

(B) $\frac{108}{216}$

(C) $\frac{125}{216}$

(D) $\frac{127}{216}$

48. The value of the integral $\int_{-\pi/2}^{\pi/2} \left(x^2 + \ln \frac{\pi + x}{\pi - x} \right) \cos x \, dx$ is

(A) 0

(B) $\frac{\pi^2}{2} - 4$

(C) $\frac{\pi^2}{2} + 4$

(D) $\frac{\pi^2}{2}$

SECTION II

This section contains **6 multiple choice questions** relating to three paragraphs with **two questions on each paragraph.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

Paragraph for Questions 49 and 50

A tangent PT is drawn to the circle $x^2+y^2=4$ at the point $P\left(\sqrt{3},1\right)$. A straight line L, perpendicular to PT is a tangent to the circle $(x-3)^2+y^2=1$.

49. A possible equation of L is

$x-\sqrt{3}y=1$

$x+\sqrt{3}y=1$

$x-\sqrt{3}y=-1$

$x+\sqrt{3}y=5$

50. A common tangent of the two circles is

$x=4$

$y=2$

$x+\sqrt{3}y=4$

$x+2\sqrt{2}y=6$

Paragraph for Questions 51 and 52

Let $f(x)=(1-x)^2\sin^2 x+x^2$ for all $x\in IR$, and let $g(x)=\int_1^x\left(\frac{2(t-1)}{t+1}-\ln t\right)f(t)\,dt$ for all $x\in(1,\infty)$.

51. Consider the statements:

    $\boldsymbol{P}$: There exists some $x\in R$ such that $f(x)+2x=2(1+x^2)$

    $\boldsymbol{Q}$: There exists some $x\in R$ such that $2f(x)+1=2x(1+x)$

    Then

    (A) both $\boldsymbol{P}$ and $\boldsymbol{Q}$ are true

    (B) $\boldsymbol{P}$ is true and $\boldsymbol{Q}$ is false

    (C) $\boldsymbol{P}$ is false and $\boldsymbol{Q}$ is true

    (D) both $\boldsymbol{P}$ and $\boldsymbol{Q}$ are false

52. Which of the following is true?

    (A) g is increasing on $(1,\infty)$

    (B) g is decreasing on $(1,\infty)$

    (C) g is increasing on $(1,2)$ and decreasing on $(2,\infty)$

    (D) g is decreasing on $(1,2)$ and increasing on $(2,\infty)$

Paragraph for Questions 53 and 54

Let a_n denote the number of all n-digit positive integers formed by the digits $0,1$ or both such that no consecutive or both such that no consecutive 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n = the number of such n-digit integers ending with digit 0.

53. The value of b_6 is

(A) 7

(B) 8

(C) 9

(D) 11

54. Which of the following is correct?

(A) $a_{17} = a_{16} + a_{15}$

(B) $c_{17} \neq c_{16} + c_{15}$

(C) $b_{17} \neq b_{16} + c_{16}$

(D) $a_{17} = c_{16} + b_{16}$

SECTION III

This section contains **6 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

55. For every integer n ,let a_n and b_n be real numbers. Let function $f : IR \to IR$ be given by $f(x) = \begin{cases} a_n + \sin \pi x, & \text{for } x \in [2n, 2n+1] \\ b_n + \cos \pi x, & \text{for } x \in [2n-1, 2n] \end{cases}$, for all integers n. If f is continuous, then which of the following hold(s) for all n?

(A) $a_{n-1} - b_{n-1} = 0$

(B) $a_n - b_n = 1$

(C) $a_n - b_{n+1} = 1$

(D) $a_{n-1} - b_n = -1$

56. If the straight lines $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is(are)

(A) $y + 2z = -1$

(B) $y + z = -1$

(C) $y - z = -1$

(D) $y - 2z = -1$

57. If the adjoint of a 3×3 matrix P is $\begin{vmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{vmatrix}$, then the possible value(s) of the determinant of P is (are)

(A) -2

(B) -1

(C) 1

(D) 2

58. Let $f:(-1,1)\to IR$ be such that $f(\cos 4\theta)=\dfrac{2}{2-\sec^2\theta}$ for $\theta\in\left(0,\dfrac{\pi}{4}\right)\cup\left(\dfrac{\pi}{4},\dfrac{\pi}{2}\right)$. Then the value(s) of $f\left(\dfrac{1}{3}\right)$ is (are)

(A) $1-\sqrt{\dfrac{3}{2}}$

(B) $1+\sqrt{\dfrac{3}{2}}$

(C) $1-\sqrt{\dfrac{2}{3}}$

(D) $1+\sqrt{\dfrac{2}{3}}$

59. Let X and Y be two events such that $P(X|Y)=\frac{1}{2}, P(Y|X)=\frac{1}{3}$ and $P(X\cap Y)=\frac{1}{6}$. Which of the following is (are) correct?

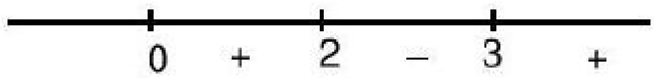
(A) $P(X\cup Y)=\frac{2}{3}$

(B) X and Y are independent

(C) X and Y are not independent

(D) $P(X^C\cap Y)=\frac{1}{3}$

60. If $f(x)=\int_0^x e^{t^2}(t-2)(t-3)\,dt$ for all $x\in(0,\infty)$, then



(A) f has a local maximum at $x=2$.

(B) f is decreasing on $(2,3)$

(C) there exists some $c\in(0,\infty)$ such that $f''(c)=0$

(D) f has a local minimum at $x=3$