

## JEE ADVANCED-2014

### MATHEMATICS

Q41. Sol.

$$M = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$$

$$|M| = ab - c^2$$

$$\begin{bmatrix} a \\ c \end{bmatrix} = [cb]^T = \begin{bmatrix} c \\ b \end{bmatrix}$$

$$\Rightarrow a = b = c$$

$$|M| = 0 \Rightarrow M \text{ is not invertible}$$

(b)

$$[cb] = \begin{bmatrix} a \\ c \end{bmatrix}^T = [ac]$$

$$\Rightarrow a = b = c \text{ } M \text{ is not invertible}$$

(c)

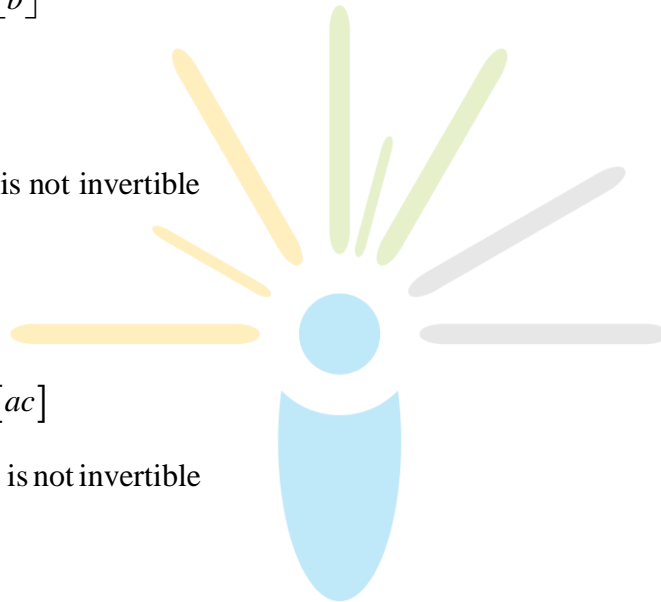
$$[M] = ab - 0 = ab \neq 0$$

$\therefore M$  is invertible

(d)

$$[M] = ab - c^2 \neq 0$$

$\therefore M$  is invertible



**Q42. Sol.** let equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

It passes through (0,1)

$$0 + 1 + 2g(0) + 2f + c = 0 \quad \dots\dots\dots(1)$$

$$(x-1)^2 + y^2 = 16 \Rightarrow x^2 + y^2 - 2x - 15 = 0 \quad g_1 = -1 \quad C_1 = -15$$

$$X^2 + y^2 = 1 \Rightarrow x^2 + y^2 = 1 \quad g_2 = 0 \quad C_2 = -1$$

Orthogonality condition  $2g_1g_2 + 2f_1f_2 = c_1 + c_2$   
 $2y(-1) + 2f(0) = -15 + c$

$$2g(0) + 2f(0) = c - 1$$

$$(c = 1) \quad \dots\dots(3)$$

from (1), 2 and (3)  $g = 7 \quad f = -1$

so,  $x^2 + y^2 + 14x - 2y + 1 = 0$

$$(x+7)^2 + (y-1)^2 = 49$$

$$\{\text{center} = (-7, 1), \text{radius} = 7\}$$

**Q43. Sol.** (A)  $b = ((\bar{b}\bar{z})(\bar{z}-\bar{x}))$

Multiplying by  $\bar{z}$

$$(\bar{b}\bar{z}) = (\bar{b}\bar{z})(\bar{z}\bar{z}) - (\bar{x}\bar{z})$$

$$1 = 2 - \sqrt{2}\sqrt{2} \times \frac{1}{2}$$

Option A is correct

$$(B) \vec{a} = (\vec{a}\vec{y})(\vec{y} - \vec{z})$$

Multiplying by  $\vec{y}$

$$(\vec{a}\vec{y}) = (\vec{a}\vec{y})(\vec{y}^2 - \vec{y}\vec{z})$$

$$1 = 2 - \sqrt{2}\sqrt{2} \cdot \frac{1}{2}$$

So Option (B) is correct

**Q44. Sol.** (B,C)

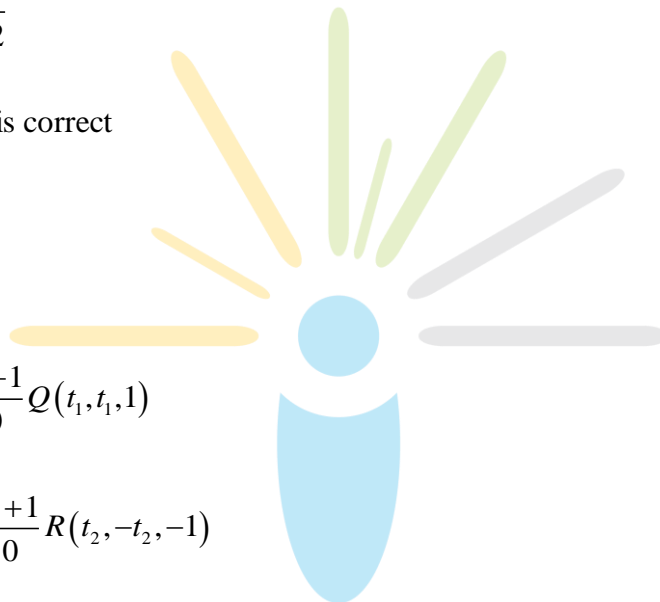
$$L_1 : \frac{x}{1} = \frac{y}{1} = \frac{z-1}{0} = Q(t_1, t_1, 1)$$

$$L_2 : \frac{x}{1} = \frac{y}{-1} = \frac{z+1}{0} = R(t_2, -t_2, -1)$$

$$dv \text{ or } PQ = (\lambda - t_1, \lambda - t_1, \lambda - 1) = dv \text{ or } l_2$$

$$dv \text{ or } PR = (\lambda - t_2, \lambda + t_2, \lambda + 1) = dv \text{ or } l_1$$

$$\lambda - 1 = 0 \text{ or } \lambda + 1 = 0 \Rightarrow \lambda = 1 \text{ or } \lambda = -1$$



**Q45 Sol.** Sol.  $\max \{f(x) : x \in [0,1]\} = \max \{g(x) : x \in [0,1]\}$

For same  $c \in [0,1]$

$$F(c) = g(c)$$

$$\Rightarrow [F(c)]^2 = [g(c)]^2$$

So, option (A) and (D) are correct

**Q46 Sol.**  $MN = NM \Rightarrow M$  and  $N$  are Aymmetric

$$|M^2 + MN^2| = |M||M + N^2|$$

**Q47. Sol.**

$$F(x) = x^5 - 5x + a$$

$$f'(x) = 5x^4 - 5. \Rightarrow f'(x) = 0 \text{ at } x = \pm 1, f(0) = a$$

$$f(-\infty) = -\infty, f(1) = -4 + a$$

$$f(\infty) = \infty, f(-1) = -4 + a$$

For  $a = 4$

$$F(1) = -4 + a = 0$$

$$F(-1) = 4 + a = 8$$

$$F(0) = -4$$

For  $a > 4$  graph will shift upwards

For  $a = 4$

$$F(1) = -4 + a = -8$$

$$F(-1) = 0$$

$$F(0) = -4$$

$\Rightarrow$  for  $-4 < a < 4$

Graph will cut real  $x$ -axis 3 points

**Q48. Sol.**

$$f(x) \int_{1/x}^x \frac{e^{t+\frac{1}{t}}}{t} dt$$

$$f(x) \frac{e^{-\left(\frac{1}{x} + \frac{1}{x}\right)} t(x)}{x} - \frac{e^{-\left(\frac{1}{x} + x\right)}}{\left(\frac{1}{x}\right)} \left\{ \frac{d}{d} \left( \frac{1}{x} \right) \right\}$$

$$= e^{-\left(\frac{1}{x} + \frac{1}{x}\right)} \left[ \frac{1}{x} + \frac{1}{x} \right]$$

$$= \frac{2}{x} e$$

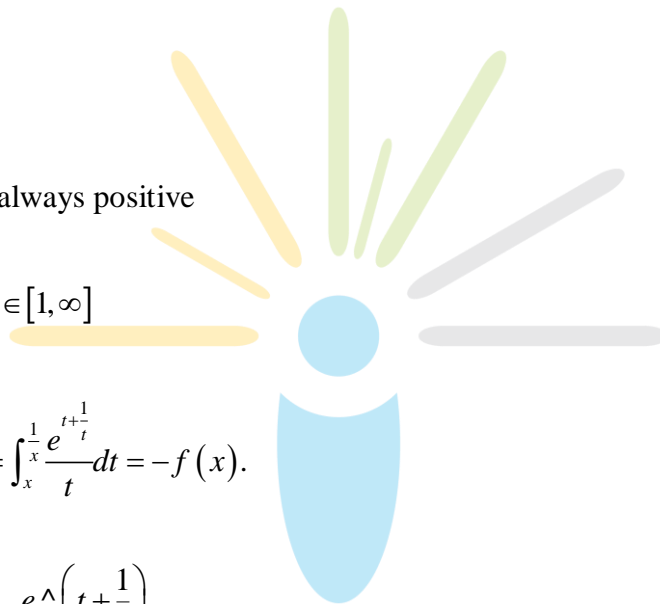
Now,  $e^{-\left(\frac{1}{x} + \frac{1}{x}\right)}$  is always positive

and  $\frac{2}{x} > 0$  for  $x \in [1, \infty]$

$$\text{Now } +f\left(\frac{1}{x}\right) = \int_x^{\frac{1}{x}} \frac{e^{t+\frac{1}{t}}}{t} dt = -f(x).$$

$$\text{and } f(2^x) = \int_{\frac{1}{2^x}}^{2^x} \frac{e^{t+\frac{1}{t}}}{t} dt = -f(2^x)$$

$$f(2^{-x}) = -f(2^x) \Rightarrow \text{odd function}$$



**Q49 Sol.**  $F(x) = (\log(\sec x + \tan x))^3$

Now,  $\sec^2 x - \tan^2 x = 1$

$(\sec x - \tan x) = 1/\sec x + \tan x = (\sec x + \tan x)^{-1}$

$\therefore f(-x) = \log(\sec x - \tan x)^3$

$= [-\log(\sec x + \tan x)]^3$

$= -(\log(\sec x + \tan x))^3 = -f(x)$

$\therefore f(-x) = -f(x) \Rightarrow$  odd function.

$F'(x) = 3(\log(\sec x + \tan x))^2 \sec^2 x \sec^2 x + \sec x \tan x / (\sec x + \tan x)$

$F'(x) = 3(\log(\sec x + \tan x))^3 \sec x$   
*positive* *positive for  $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$*

$\therefore f'(x) > 0 \Rightarrow f^n$  is increasing  $\Rightarrow$  one to one  $f^n$

Ans. A, B, C

**Q50. Sol.** at  $x = q$   $g(q^-) = 0$

$g(a^+) = \int_a^a f(t) dt \rightarrow 0$

at  $(x = b)$   $g(b^-) = \int_a^b f(t) dt$

$g(b^+) = \int_a^b f(t) dt$

so  $g(a^+) = g(a^-)$  and  $g(b^+) = g(b^-)$

so,  $g(x)$  is continuous at  $x = 'a'$  and  $'b'$

$$g'(x) = \begin{cases} 0 & \text{if } x < a \\ f'(x) & \text{if } a \leq x \leq b \\ 0 & \text{if } x > b \end{cases}$$

$\therefore g'(x)$  is not differentiable at  $x = a$  and  $'b'$

**Q51. Sol.**  $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$

Taking dot product with  $\vec{b}$

$$\vec{b} \cdot (\vec{a} \times \vec{b}) + \vec{b} \cdot (\vec{b} \times \vec{c}) = p\vec{a} \cdot \vec{b} + q\vec{a} \cdot \vec{b} + r\vec{c} \cdot \vec{b}$$

$$0 = \frac{p}{2} + q + \frac{r}{2}$$

Taking dot with  $\vec{a}$

$$\vec{a} \cdot (\vec{a} \times \vec{b}) + \vec{a} \cdot (\vec{b} \times \vec{c}) = p\vec{a} \cdot \vec{a} + q\vec{a} \cdot \vec{a} + r\vec{c} \cdot \vec{a}$$

$$0 + a \cdot (\vec{b} \times \vec{c}) = p + \frac{q}{2} + \frac{r}{2} \quad (2)$$

Similarity with  $\vec{c}$

$$\vec{c}(\vec{a}\vec{b}) = \frac{p}{2} + \frac{q}{2} + r \quad (3)$$

LHS of (2) & (3) are same

So :

$$p + q \frac{1}{2} = \frac{r}{2} = \frac{p}{2} + \frac{1}{2} + r$$

$$p/2 = r/2$$

$$p = r$$

$$p = r \quad (4)$$

from (1) (2) (4)

$$p + q = 0 \quad p = -q$$

$$r = -q$$

putting value in  $q$  :

$$= \frac{q^2 + 2q^2 + q^2}{q^2} = 4$$



**Q54. Sol.**

$$\lim_{x \rightarrow 1} \left\{ \frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right\}^{\frac{1-x}{1-\sqrt{x}}}$$

$$\lim_{x \rightarrow 1} \left\{ \frac{-ax + (\sin(x-1) + a + x - 1) - x + 1}{x + \sin(x-1) - 1} \right\}^{\frac{1-x}{1-\sqrt{x}}}$$



$$\lim_{x \rightarrow 1} \left\{ 1 + \frac{a - ax + 1(x-1)}{x + \sin(x-1) - 1} \right\}^{\frac{(1-\sqrt{x})^2}{1-\sqrt{x}}}$$

$$\lim_{x \rightarrow 1} \left\{ 1 + \frac{a(1-x) + 1(1-x)}{(x-1) + \sin(x-1)} \right\}^{\frac{(1+\sqrt{x})(1-\sqrt{x})}{(1-\sqrt{2})}} \quad \left( \begin{array}{l} \text{since} \\ x \neq 1 \end{array} \right)$$

$$\lim_{x \rightarrow 1} \left\{ 1 + \frac{(a+1)(1-x)}{(x-1) + \sin(x-1)} \right\}^{1+\sqrt{x}}$$

$$\lim_{x \rightarrow 1} \left\{ 1 + \frac{\frac{5(a+1)(1-x)}{(x-1)}}{\frac{1 + \sin x - 1}{\lambda - 1}} \right\}^{1+1}$$

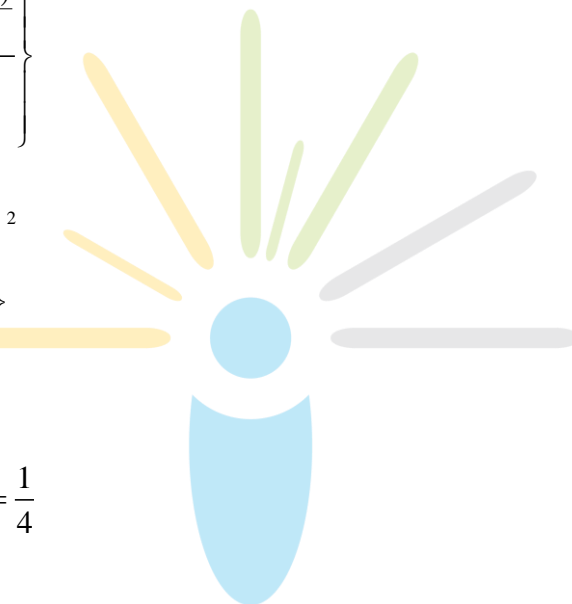
$$\left\{ 1 + \frac{-^1(a+1)}{\lim_{\lambda \rightarrow 1} \frac{\sin \lambda - 1}{\lambda - 1} + 1} \right\}^2$$

$$\text{Now, } \left\{ 1 + \frac{-(aH)}{1+1} \right\}^2 = \frac{1}{4}$$

$$\left\{ 1 + \frac{2-a-1}{2} \right\}^2 = \frac{1}{4}$$

$$\frac{1-a}{2} = \pm \frac{1}{2} (1-a = \pm 1) a = 0 \text{ or } a = 2$$

Largest value. 2



**Q55. Sol.** Draw figure for cases  $n = 3, 4, 5$  At 5 we will get the required condition

**Q56. Sol.**  $abc$  are in  $G.P$

$$B = ar \quad r \text{ is integer}$$

$$C = ar^2$$

Now

$$A = ar + ar^2 = 3(arb + 2) \text{ (given)}$$

$$A + ar + ar^2 = 3ar + 6$$

$$Ar^2 - 3ar + (a - 6)$$

$H$  is quad ratio in  $r$

$$R = \frac{2a \pm \sqrt{4a^2 - 4(a-6)(a)}}{2a}$$

$$R = 1 \pm \frac{2a \pm \sqrt{4a^2 - 4a^2 + 24a}}{2a}$$

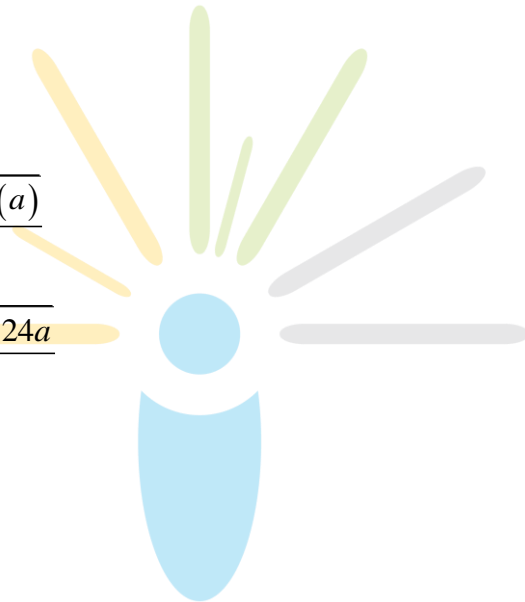
$$= 1 \pm \sqrt{\frac{24a}{4a^2}} = 1 \pm \sqrt{\frac{6}{a}}$$

For  $r$  to be integer

$$\sqrt{\frac{6}{a}} \text{ should be integer}$$

Which is possible when  $a=6$

$$\frac{a^2 + a - 14}{a + 1} = \frac{42 - 14}{7} = 4.$$



**Q57.Sol.** Possible values of  $n_1$  to fulfill above conditions is 1 & 2

For  $n_1 = 2$

$$n_2 = 3n_3 = 4n_4 = 5n_5 = 6+$$

Is the only possibility

For $n_1 = 1$	$n_1$	$n_2$	$n_3$	$n_4$	$n_5$	
Possible cases	1	2	3	5	9	
	1	2	3	6	8	
	1	2	3	4	10	
	1	2	4	5	8	
Hence total 7 distinct	1	2	4	6	7	arrangements
	1	3	4	5	7	

**Q58. Sol.**

$$f(n) = |x| + 1$$

$$g(x) = x^2 + 1$$

So, clearly from graph  $h(n)$  is not differentiable at  $x = -1, 0, 1$

→ Answer is 3

**Q59. Sol.**  $(y - x^5)^2 = x(1 + x^2)^2$

Diff w.r.t  $x$

$$2(y - x^5) \left[ \frac{dy}{dx} - 5x^4 \right] = x \left[ 2(1 + x^2)2x \right] + (1 + x^2)^2$$

At (1,3)

$$2(3-1)\left[\frac{dy}{dx}-5\right]=1[2*2*2(1)]+(1+1)^2 4\left[\frac{dy}{dx}-5\right]=8+4=12$$

$$\Rightarrow \frac{dy}{dx}=8$$

**Q60. Sol.**

$$\int_0^1 4x^3 \left\{ \frac{d^2}{dx^2} (1-x^2)^5 \right\} dx$$

$$\frac{\partial^2}{\partial x^2} (1-x^2)^5 = \frac{\partial}{\partial x} \frac{\partial}{\partial x} (1-x^2)^5$$

$$= \frac{\partial}{\partial x} (5(1-x^2)^4 (-2x))$$

$$= -10 \left[ x4(1-x^2)^3 (-2x) + (1-x^2)^4 \right]$$

$$= -10(1-x^2)^3 [-8x^2 + (1-x^2)]$$

$$= -10(1-x^2)^3 (1-9x^2)$$

$$\text{Now } \int_0^1 4x^3 (-10)(1-x^2)^3 [1-9x^2] dx$$

$$-40 \int_0^1 x^3 (1-x^2)^3 [1-9x^2] dx$$

$$\lambda \text{ let } x^2 = t \quad x \rightarrow 0 \quad t \rightarrow 0$$

$$2x dx = dt \quad x \rightarrow 1 \quad t \rightarrow 1$$

$$x dx = \frac{dt}{2}$$

Above becomes

$$-20 \int_0^1 t(1-t)^3(1-9t) \frac{dt}{2}$$

$$-20 \int_0^1 tt(1-t^3-3t+3t^2)(1-9t) dt$$

$$= -20 \int_0^1 (1-t^3-3t^2+3t^3)(1-9t) dt$$

$$= -20 \int_0^1 (t-t^4-3t^2+3t^3-9t^2+9t^5+27t^4-27t^4) dt$$

$$= -20 \int_0^1 (t-12t^2+30t-28t^4+9t^5) dt$$

$$= -20 \left( \frac{t^2}{2} - 4t^3 + \frac{15t^4}{2} - \frac{28t^5}{5} + \frac{3t^6}{2} \right)$$

$$= -20 \left[ \left( \frac{1}{2} - 4 + \frac{15}{2} - \frac{28}{3} + \frac{3}{2} \right) - (0) \right]$$

$$= -190 + 192$$

$$= 2$$

