JEE ADVANCED-2014

MATHEMETICS

Q41. Sol.

$$M = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$$

$$|M| = ab - c^2$$

$$\begin{bmatrix} a \\ c \end{bmatrix} = [cb]^T = \begin{bmatrix} c \\ b \end{bmatrix}$$

$$\Rightarrow a = b = c$$

$|M| = 0 \Rightarrow M$ is not invertible

(b)

$$[cb] = \begin{bmatrix} a \\ c \end{bmatrix}^T = [ac]$$

$\Rightarrow a = b = c\, M$ is not invertible

(c)

$$[M] = ab - 0 = ab \neq 0$$

$\therefore M$ is invertible

(d)

$$[M] = ab - c^2 \neq 0$$

$\therefore M$ is invertible

Q42. Sol. let equation of circle be

$x^2+y^2+2yx+2fy+c=0$

It spasses though $(0,1)$

$0+1+2g(0)+2f+c=0$ ....(1)

$(x-1)^2+y^2=16 \Rightarrow x^2+y^2-2x-15=0 \quad g_1=-1 \quad C_1=-15$
$X^2+y^2=1 \quad \Rightarrow x^2+y^2=1 \quad g_2=0 \quad C_2=-1$

Orthogonaliy rendition $2g_1g_2+2f_1f_2=c_1+c_2$
$2y(-1)+2f(0)=-15+c$

$2g(0)+2f(0)=c-1$

$(c=1) \quad -(3)$

from $(1), 2$ and (3) $\quad g=7 \quad f=-1$

so, $x^2+y^2+14x-2y+1=0$

$(x+7)^2+(y-1)^2=49$

$\{\text{center}=(-7,1), \text{radius}=7\}$

Q43. Sol. (A) $b=((\bar{b}.\bar{z})(\bar{z}-\bar{x})$

Multiplying $by\bar{z}$

$(\bar{b}.\bar{z})=(\bar{b}.\bar{z})(\bar{z}.\bar{z})-(\bar{x}.\bar{z})$

$$1 = 2 - \sqrt{2}\sqrt{2} \times \frac{1}{2}$$

Option A is correct

(B) $\vec{a} = \left(\vec{a}\vec{y}\right)\left(\vec{y} - \vec{z}\right)$

Multiplying by $\vec{y}$

$$\left(\vec{a}\vec{y}\right) = \left(\vec{a}\vec{y}\right)\left(\vec{y}^{2} - \vec{y}\vec{z}\right)$$

$$1 = 2 - \sqrt{2}\sqrt{2.\frac{1}{2}}$$

So Option (B) is correct

Q44. Sol. (B,C)

$$L_1 : \frac{x}{1} = \frac{y}{1} = \frac{7-1}{0} Q\left(t_1, t_1, 1\right)$$

$$L_2 : \frac{x}{1} = \frac{y}{-1} = \frac{7+1}{0} R\left(t_2, -t_2, -1\right)$$

$$dv \text{ or } PQ = \left(\lambda - t_1, \lambda - t_1, \lambda - 1\right) = dv \text{ or } 1_2$$

$$dv \text{ or } PR = \left(\lambda - t_2, \lambda + t_2, \lambda + 1\right) = dv \text{ or } 1_1$$

$$\lambda - 1 = 0 \text{ or } \lambda + 1 = 0 \Rightarrow \lambda = 1 \text{ or } \lambda = -1$$

Q45 Sol. Sol. max $\{f\ x : x \in [0.1]\} = \max\{g(x) = x \in [0.1]\}$

For same $c \in [0.1]$

$F(c) = g(c)$

$\Rightarrow [F(c)]^2 = [g(c)]^2$

So, option (A) and (D) are correct

Q46 Sol. $MN = NM \Rightarrow M$ and N are Aymmethic

$|M^2 + MN^2| = |M||M + N^2|$

Q47. Sol.

$F(x) = x^5 - 5x + a$

$f'(x) = 5x^4 - 5. \Rightarrow f'(x) = 0 \text{ at } x = \pm 1 f(0) = a$

$f(-\infty) = -\infty f(1) = -4 + a$

$f(\infty) = \infty f(-1) = -4 + a$

For $a = 4$
$F(1) = -4 + a = 0$
$F(-1) = 4 + a = 8$
For $(0) = -4$
For $a > 4$ graph will shift upwords

For $a = 4$
$F(1) = -4 + a = -8$
$F(-1) = 0$
$F(0) = -4$
$\Rightarrow$ for $-4 < a < 4$
Graph will cut real $x -$ anisdt 3 point

Q48. Sol.

$$f\left(x\right)\int_{1/x}^{x}\frac{e^{t+\frac{1}{t}}}{t}dt$$

$$f\left(x\right)\frac{e^{-\left(x+\frac{1}{x}\right)}}{x}\frac{t\left(x\right)}{tx}-\frac{e^{-\left(\frac{1}{x}+x\right)}}{\left(\frac{1}{x}\right)}\left\{\frac{d}{d}\left(\frac{1}{x}\right)\right\}$$

$$=e^{-\left(x+\frac{1}{x}\right)}\left[\frac{1}{x}+\frac{1}{x}\right]$$

$$=\frac{2}{x}e$$

Now, $e^{-\left(x\frac{1}{2}\right)}$ is always positive

and $\frac{2}{x}>0$ for $x\in[1,\infty]$

Now $+f\left(\frac{1}{x}\right)=\int_{x}^{\frac{1}{x}}\frac{e^{t+\frac{1}{t}}}{t}dt=-f\left(x\right).$

and $f\left(2^{x}\right)=\int_{\frac{1}{2^{x}}}^{2x}\frac{e\wedge\left(t+\frac{1}{t}\right)}{t}dt=-f\left(2^{x}\right)$

$f\left(2^{-x}\right)=-f\left(2^{x}\right)\Rightarrow$ odd function

Q49 Sol. $F(x) = (\log(\sec x + \tan x))^3$

Now, $\sec^2 x - \tan^2 x = 1$

$(\sec x - \tan x) = 1/\sec x + \tan x = (\sec x + \tan x)^{-1}$

$\therefore f(-x) = \log(\sec x - \tan x))^3$

$= [-\log(\sec x - \tan x)]^3$

$= -(\log(\sec x - \tan x))^3 = f(x)$

$\therefore f(-x) = -f(x) \Rightarrow$ off function.

$F'(x) = 3(\log(\sec x - \tan x))^2 \sec^2 x \sec^2 x + \sec x \tan x/(\sec x + \tan x)$

$F'(x) = 3\underbrace{(\log(\sec x - \tan x))^3}_{positive} \quad \underbrace{\sec x}_{positive\ for\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)}$

$\therefore f'(n) > 0 \Rightarrow f^n$ is increasing $\Rightarrow$ one to one f^n

Ans. A, B, C

Q50. Sol. at $x = q \quad g(q^-) = 0$

$g(a^+) = \int_a^a +(t)\,dt \to 0$

at $(x = b) \quad g(b^-) = \int_a^b f(t)\,dt$

$g(b^+) = \int_a^b f(t)\,dt$

so $g(at) = g(a^{-})$ and $g(b^{-}) = g(b^{-})$

so, $g(x)$ is wntinuous at $x =$ 'a' and 'b'

$$g'(x) = \begin{cases} 0 \text{if } x < q \\ f(x) it\, a \le x \le b \\ 0 if\ x > b \end{cases}$$

$\therefore g'(x)$ is not differentiable at $x = a$ and 'b'

Q51. Sol. $\vec{a}\, x\, \vec{b} + \vec{b}\, x\, \vec{c} = p\,\vec{a} + q\vec{b} + r\vec{c}$

Taking dot product with B

$$\vec{b}.\left(\vec{a}\, x\, \vec{b}\right) + \vec{b}\left(\vec{b}\, x\, \vec{c}\right) = p\,\vec{a}.\vec{b} + q\vec{a}.\vec{b}\, q\,\vec{b}.\vec{b} + r\,\vec{c}.\vec{b}$$

$$0 = \frac{p}{2} + q + \frac{r}{2}$$

Taking dot with $\vec{a}$

$$\vec{a}.\left(\vec{a}\, x\, \vec{b}\right) + \vec{a}.\left(\vec{b}\, x\, \vec{c}\right) = p\,\vec{a}.\vec{a} + q\vec{a}.\vec{a} + q\,\vec{b}.\vec{a} + r\,\vec{c}.\vec{a}$$

$$0 + a.\left(\vec{b}x\vec{c}\right) = p + \frac{q}{2} + \frac{r}{2}(2)$$

Similarity with $\vec{C}$

$\vec{c}\left(\vec{a}x\vec{b}\right)=\frac{p}{2}+\frac{q}{2}+r\ (3)$

LHS of $(2)\,\varepsilon\,(3)$ are same

So :

$P+q\frac{1}{2}=\frac{r}{2}=\frac{p}{2}+½+r$

$p/2=r/2$

$p=2=r/2$

$p=r\ (4)$

from (1) (2) (4)

$p+q=0 \quad p=-q$

$r=-q$

putting value in q :

$=\frac{q^2+2q^2+q^2}{q^2}=4$

Q54. Sol.

$$\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}$$

$$\lim_{x\to 1}\left\{\frac{-ax+\left(\sin(x-1)+a+x-1\right)-x+1}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}$$

$$\lim_{x\to1}\left\{1+\frac{a-ax+1(x-1)}{x+\sin(x-1)-1}\right\}^{\frac{(1-\sqrt{x})^2}{1-\sqrt{x}}}$$

$$\lim_{x\to1}\left\{1+\frac{a(1-x)+1(1-x)}{(x-1)+\sin(x-1)}\right\}^{\frac{(1+\sqrt{x})(1-\sqrt{x})}{(1-\sqrt{2})}}\begin{pmatrix}\sin ce\\ x\neq1\end{pmatrix}$$

$$\lim_{x\to1}\left\{1+\frac{(a+1)(1-x)}{(x-1)+\sin(x-1)}\right\}^{1+\sqrt{x}}$$

$$\lim_{x\to1}\left\{1+\frac{\frac{5(a+1)(1-x)}{(x-1)}}{\frac{1+\sin x-1}{\lambda-1}}\right\}^{1+1}$$

$$\left\{1+\frac{-^{1}(a+1)}{\lim\limits_{\lambda-1}\frac{\sin\lambda-1}{\lambda-1}+1}\right\}^{2}$$

Now, $\left\{1+\frac{-(aH)}{1+1}\right\}^{2}=\frac{1}{4}$

$$\left\{1+\frac{2-a-1}{2}\right\}^{2}=\frac{1}{4}$$

$$\frac{1-a}{2}=\pm\frac{1}{2}(1-a=\pm1)\,a=0\,\text{or}\,a=2$$

Largest value. 2

Q55. Sol. Draw figure for cases $n = 3, 4, 5$ At 5 we will get the required condition

Q56. Sol. $a\,b\,c$ are in $G.P$

$B = ar$ r is integer

$C = ar^2$

Now

$A = ar + ar^2 = 3(ar\,b + 2)$(given)

$A + ar + ar^2 = 3ar + 6$

$Ar^2 - 3ar + (a - 6)$

H is quad ratio in r

$$R = \frac{2a \pm \sqrt{4a^2 - 4(a-6)(a)}}{2a}$$

$$R = 1 \pm \frac{2a \pm \sqrt{4a^2 - 4a^2 + 24a}}{2a}$$

$$= 1 \pm \sqrt{\frac{24a}{4a2}} = 1 \pm \sqrt{\frac{6}{a}}$$

For r to be integer

$\sqrt{\frac{6}{a}}$ should be integer

Which is possible when a=6

$$\frac{a^2 + a - 14}{a + 1} = \frac{42 - 14}{7} = 4.$$

Q57.Sol. Possible values of n_1 to fulfill above conditions is $1\&2$

For $n_1 = 2$

$n_2 = 3n_3 = 4n_4 = 5n_5 = 6+$

Is the only possibility

	n_1	n_2	n_3	n_4	n_5	
For $n_1 = 1$						
Possible cases	1	2	3	5	9	
	1	2	3	6	8	
	1	2	3	4	10	
	1	2	4	5	8	
Hence total 7distint	1	2	4	6	7	arrangements
	1	3	4	5	7	

Q58. Sol.

$f(n) = |x| + 1$

$g(x) = x^2 + 1$

So, clearly from graph $h(n)$ is not differentiable at $x = -1, 0, 1$

$\rightarrow$ Answer is 3

Q59. Sol. $(y - x^5)^2 = x(1 + x^2)^2$

Diff *w.r.t x*

$$2(y - x^5)\left[\frac{dy}{dx} - 5x^4\right] = x\left[2(1 + x^2)2x\right] + (1 + x^2)^2$$

At $(1,3)$

$$2(3-1)\left[\frac{dy}{dx}-5\right]=1\left[2*2*2(1)\right]+(1+1)^2 4\left[\frac{dy}{dx}-5\right]=8+4=12$$

$$=> \frac{dy}{dx}=8$$

Q60. Sol.

$$\int_0^1 4x^3\left\{\frac{d^2}{dx^2}(1-x^2)^5\right\}dx$$

$$\frac{\partial^2}{\partial x^2}(1-x^2)^5=\frac{\partial}{\partial x}\frac{\partial}{\partial x}(1-x^2)^5$$

$$=\frac{\partial}{\partial x}\left(5(1-x^2)^4(-2x)\right)$$

$$=-10\left[x4(1-x^2)^3(-2x)+(1-x^2)^4\right]$$

$$=-10(1-x^2)^3\left[-8x^2+(1-x^2)\right]$$

$$=-10(1-x^2)^3(1-9x^2)$$

$$\text{Now}\int_0^1 4x^3(-10)(1-x^2)^3\left[1-9x^2\right]dx$$

$$-40\int_0^1 x^3(1-x^2)^3\left[1-9x^2\right]dx$$

$$\lambda et\ x^2=t \quad x\to 0\,t\to 0$$
$$2x\,dx=dt \quad x\to 1\,t\to 1$$

$$x\,dx=\frac{dt}{2}$$

Above becomes

$$-20\int_0^1 t(1-t)^3(1-9t)\frac{dt}{2}$$

$$-20\int_0^1 tt\left(1-t^3-3t+3t^2\right)(1-9t)\,dt$$

$$=-20\int_0^1\left(1-t^3-3t^2+3t^3\right)(1-9t)\,dt$$

$$=-20\int_0^1\left(t-t^4-3t^2+3t^3-9t^2+9t^5+27t^4-27t^4\right)dt$$

$$=-20\int_0^1\left(t-12t^2+30t-28t^4+9t^5\right)dt$$

$$=-20\left(\frac{t^2}{2}-4t^3+\frac{15t^4}{2}-\frac{28t^5}{5}+\frac{3t^6}{2}\right)$$

$$=-20\left[(1/2-4+15/2-28/3+3/2)-(0)\right]$$

$$=-190+192$$
$$=2$$