JEE ADVANCED-2012

MATHEMETICS

SECTION – 1

Q41. Let M be a 2×2 symmetric matrix with integer entries. Then M is invertible if

(A) the first column of M is the transpose of the second row of M

(B) the second row of M is the transpose of the first column of M

(C) M is a diagonal matrix with nonzero entries in the main diagonal

(D) the product of entries in the main diagonal of M is not the square of an Integer

Q42. A circle S passes through the point $(0,1)$ and is orthogonal to the circles $(x-1)^2+y^2=16$ and $x^2+y^2=1$. Then

(A) radius of S is 8

(B) radius of S is 7

(C) centre of S is $(-7,1)$

(D) centre of S is $(-8,1)$

Q43. Let x, y and z be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\pi/3$. If a is a nonzero vector perpendicular to x and $y\ x\ z$ and b is nonzero vector perpendicular to y and $z^{x}x$, then

(A) $b=(b.z)(z-x)$

(B) $a=(a.y)(y-z)$

(C) $a.b=(a.y)(b.z)$

(D) $a=(a.y)(z-y)$

Q44. From a point $P(\lambda,\lambda,\lambda)$, perpendiculars PQ and PR are drawn respectively o the lines $y = x,\ z = 1$ and $y=-x, z=-1$. If P is such that $<QPR$ is a right angle, then the possible value(s) of λ is (are)

(A) $\sqrt{2}$

(B) 1

(C) -1

(D) $-\sqrt{2}$

Q45. For every pair of continuous functions $f,g:[0,1]\to\mathbb{R}$ such thatmax $\{f(x):x\in[0,1]\}=\max\{g(x):x\in[0,1]\}$, the correct statement(s) is(are) :

(A) $(f(c))^2+3f(c)=(g(c))^2+3g(c)$ for some $c\in[0,1]$

(B) $(f(c))^2+f(c)=(g(c))^2+3g(c)$ for some $c\in[0,1]$

(C) $(f(c))^2+3f(c)=(g(c))^2+g(c)$ for some $c\in[0,1]$

(D) $(f(c))^2=(g(c))^2$ for some $c\in[0,1]$

Q 46 Let M and N be two 3×3 matrices such that $MN = NM$. Further, if $M \neq N^2$ and $M^2 = N^4$, then

(a) Determinant of $\left(M^2 + MN^2\right)$ is 0

(b) There is a 3×3 non-zero matrix U such that $\left(M^2 + MN^2\right)U$ is the zero matrix

(c) Determinant of $\left(M^2 + MN^2\right) \geq 1$

(d) For a 3×3 matrix U, if $\left(M^2 + MN^2\right)U$ equals the zero matrix then U is the zero matrix

Q 47. Let a $\in \mathbb{R}$ and let $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = x^5 - 5x + a.$$

Then

(a) $f(x)$ has only one real root if $a > 4$

(b) $f(x)$ has only one real root if $a < 4$

(c) $f(x)$ has three real roots if $a < -4$

(d) $f(x)$ has three real roots if $-4 < a < 4$

Q48. Let $f:(0,\infty)\to\mathbb{R}$ be given by

$$f(x)=\int_{\frac{1}{x}}^{x} e^{-\left(t+\frac{1}{t}\right)}\frac{dt}{t}.$$

Then

(A) $f(x)$ is monotonically increasing on $[1,\infty)$

(B) $f(x)$ is monotonically decreasing on $\{0,1)$

(C) $f(x)+f(1/x)=0, \text{for all } x\in(0,\infty)$,

(D) $f(2^x)$ is an odd function of x on $\mathbb{R}$

Q49. Let $f:(-\pi/2,\pi/2)\to\mathbb{R}$ be given by

$$f(x)=(\log(\sec x+\tan x))^3$$

Then

(a) $f(x)$ is an odd function

(b) $f(x)$ is a one-one function

(c) $f(x)$ is an onto function

(d) $f(x)$ is an even function

Q50. Let $f:[a,b]\to[1,\infty)$ be a continuous function and let $g:\mathbb{R}$ be defined as

$$g(x)=\begin{cases}0 & \text{if } x<a,\\ \int_a^x f(t)\,dt & \text{if } a\le x\le b,\\ \int_a^b f(t)\,dt & \text{if } x>b.\end{cases}$$

Then

(a) $g(x)$ is continuous but no differentiable at a

(b) $g(x)$ is differentiable on $\mathbb{R}$

(c) $g(x)$ is continuous but not differentiable at b

(d) $g(x)$ is continuous and differentiable at either a or b but not both

SECTION – 2

Q51. Let a, b, and c be three non-coplanar unit vectors such that the angle between every pair of them is $\pi/3$. If $a \times b + b \times c = pa + qb + rc$, where p, q and r are scalars, then the value of $p^2 + 2q^2 + r^2/q^2$ is

Q52. Let $f:[0,\ 4\pi]\to[0,\pi]$ be defined by $f(x)=\cos^{-1}(\cos x)$. The number of points $x\in[0,4\pi]$ satisfying the equation

$F(x)=10-x/10$

Is

Q53. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines $x-y=0$ and $x+y=0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d^1(P)+d^2(P) \le 4$,is

Q54. The largest value of the non-negative integer a for which

$$\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}=\frac{1}{4}$$

Is

Q55. Let $n \ge 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is

Q56. Let a,b,c, be positive integers such that b/a is an integer. If a,b,c, are in geometric progression and the arithmetic mean of a,b,c is $b+2$, then the value of $a^2+a-14/a+1$ is

Q57. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1+n_2+n_3+n_4+n_5=20$. Then the number of such distinct arrangements (n_1,n_2,n_3,n_4,n_5) is

Q58. Let $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ be respectively given by $f(x)=x+1$ and $g(x)=x^2+1$. Define $h:\mathbb{R}\to\mathbb{R}$ by

$$h(x)=\begin{cases}\max\{f(x),g(x)\} & \text{if } x\le 0,\\ \min\{f(x),g(x)\} & \text{if } x>0.\end{cases}$$

The number of points at which $h(x)$ is not differentiable is

Q59. The slope of the tangent of the curve $\left(y-x^5\right)=x\left(1+x^5\right)^2$ at the point $(1,3)$ is

Q60. The value of

$$\int_0^1 4x^3\left\{\frac{d^2}{dx^2}\left(1-x^2\right)5\right\}dx$$

Is