JEE ADVANCED-2017

MATHMATICS

37. Sol. (A,B)

$$\frac{P(X\cap Y)}{P(Y)}=\frac{1}{2}$$

$$\frac{P(Y\cap X)}{P(X)}=\frac{2}{5}$$

$$P(X\cap Y)=\frac{P(Y)}{2}=\frac{2}{5}P(X)=\frac{2}{5}.\frac{1}{3}=\frac{2}{25}\Rightarrow P(Y)=\frac{4}{15}$$

$$\frac{P(\overline{X}\cap Y)}{P(Y)}=\frac{P(Y)-P(X\cap Y)}{P(Y)}=\frac{\frac{4}{15}-\frac{2}{15}}{\frac{4}{15}}=\frac{2}{4}=\frac{1}{2}$$

$$P(X\cup Y)=P(X)+P(Y)-P(X\cap Y)=\frac{1}{3}+\frac{4}{15}-\frac{2}{15}=\frac{7}{15}$$

38. Sol.(C,D)

$e^x\in(1,e)$ for $x\in(0,1)$

and $0<\int_0^x f(t)\sin t\,dt<1\text{in}(0,1)\quad\Rightarrow\quad(A)\text{is wrong}$

$f(x)+\int_0^{\pi/2} f(t)\sin t\,dt>0\quad\Rightarrow\quad(B)\text{is wrong}$

Let $g(x)=x-\int_0^{\frac{\pi}{2}-x} f(t)\cos t dt\quad\Rightarrow\quad g(0)=-\int_0^{\frac{\pi}{2}} f(t)\cos t\,dt<0$

$$g(1) = 1 - \int_0^{\frac{\pi}{2}-1} f(t)\cos t\, dt > 0 \quad \Rightarrow \quad (C)\, is\, correct$$

$$Let\, h(x) = x^9 - f(x)$$

$$h(0) = -f(0) < 0 \qquad \Rightarrow \quad (D)\, is\, correct$$

$$h(1) = 1 - f(1) > 0$$

39. Sol.(B, D)

$$\frac{a(x+iy)+b}{x+iy+1} = \frac{ax+b+iay}{x+1+iy} \times \frac{(x+1)-iy}{(x+1)-iy} = \frac{(ax+b)(x+1)+ay^2}{(x+1)^2+y^2} + \frac{i(ay(x+1)-y(ax+b))}{(x+1)^2+y^2}$$

$$\Rightarrow \frac{ay(x+1)-y(ax+b)}{(x+1)^2+y^2} = y \qquad \Rightarrow \frac{ay-by}{(x+1)^2+y^2} = y \qquad (\because a-b=1, y \neq 0)$$

$$\Rightarrow (x+1)^2 + y^2 = 1 \qquad \Rightarrow x+1 = \pm\sqrt{1-y^2} \qquad x = -1 \pm \sqrt{1-y^2}$$

40. Sol. (A, C, D)

$y = 2x+1$ is tangent to $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$

$$c^2 = a^2m^2 - b^2$$

$$1 = 4a^2 - 16 \Rightarrow a^2 = \frac{17}{4}$$

[Check if $p^2 = q^2 + r^2$]

41. Sol. (A, B, D)

$$f(x) = x\cos\left(\pi\left(x+[x]\right)\right)$$

Check continuity at $x = n$

$$f(n) = n\cos 2n\pi = n$$

$$f\left(n^{+}\right) = n\cos 2n\pi = n$$

$$f\left(n^{-}\right) = n\cos(2n-1)\pi = -n$$

It is discontinuous at all integer points except 0

42. Sol. (A, C)

$$A = B^2 \Rightarrow |A| = |B|^2 = +ve$$

(A) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = 1(-1) = \text{negative}$

Matrix B can not be possible

(B) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = 1(1-0) = \text{positive}$

Matrix B can be possible

Ex. $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & -1 \end{vmatrix}\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 2 & -1 \end{vmatrix}$

(C) $\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = -1 = \text{negetive}$

Matrix B can not be possible

(D) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 = \text{positive}$

Matrix B can be I

43. **Sol. (B)**

$8x - ky + \left(k^2 - 8h\right) = 0$

$2x + y - p = 0$

Comparing the coefficient of x, y and constant term, we get

$4 = -k = \frac{k^2 - 8h}{-p}$

$k = -4$

$16 - 8h = -4p$

$4 - 2h = -p \Rightarrow p = 2h - 4$

44. Sol. $D=0$

$$\begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{vmatrix}=0$$

$$\begin{vmatrix} (1+\alpha+\alpha^2) & (2\alpha+1) & (\alpha^2+\alpha+1) \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{vmatrix}=0$$

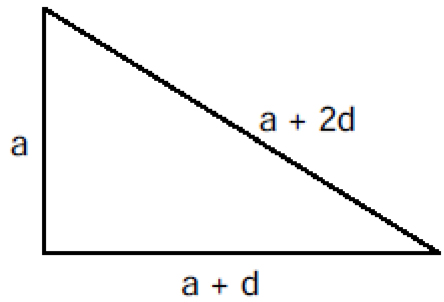
$$\begin{vmatrix} (1+\alpha+\alpha^2) & (2\alpha+1) & 0 \\ \alpha & 1 & 0 \\ \alpha^2 & \alpha & 1-\alpha^2 \end{vmatrix}=0 \Rightarrow (1-\alpha^2)(1+\alpha+\alpha^2-2\alpha^2-\alpha)=0 \Rightarrow (1-\alpha^2)=0$$

$\alpha=-1$ or 1

for $\alpha=1$, system of linear equation has no solution

$\therefore \alpha=-1$ so $1+\alpha+\alpha^2=1$

45. Sol. (6)



$$\frac{1}{2}a(a+d)=24 \quad \Rightarrow \quad a(a+d)=48 \cdots\cdots(1)$$

$$a^2+(a+d)^2=(a+2d)^2 \quad \Rightarrow \quad 3d^2+2ad-a^2=0$$

$$(3d-a)(a+d)=0$$

$$\Rightarrow \quad 3d=a \quad (\because a+d\neq 0)$$

$$\Rightarrow \quad d=2$$

$$\Rightarrow \quad a=6$$

$$\Rightarrow \quad a=6$$

so smallest side $=6$

46. Sol. (2)

$$g(x)=\int_x^{\pi/2}\frac{d}{dx}\left(f(t)\operatorname{cosec} t\right)dt$$

$$g(x)=f\left(\frac{\pi}{2}\right)\operatorname{cosec}\left(\frac{\pi}{2}\right)-f(x)\operatorname{cosec}x$$

$$g(x)=3-f(x)\operatorname{cosecc}x$$

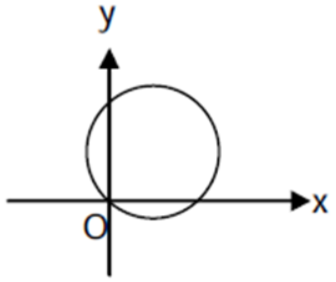
$$g(x)=3-\frac{f(x)}{\sin x}$$

$$\lim_{x\leftrightarrow 0} g(x) \quad =3-\lim_{x\leftrightarrow 0}\frac{f(x)}{\sin x}$$

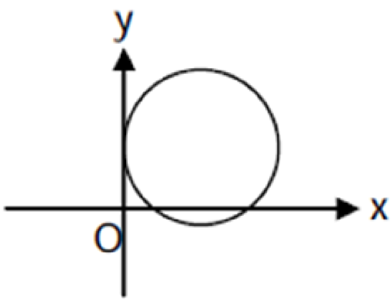
$$=3-\lim_{x\leftrightarrow 0}\frac{f'(x)}{\cos x}=3-\frac{1}{1}=2$$

47. Sol. (2)

Case-1 *passing through origin* $\Rightarrow p=0$



Case-II Touches y-axis and x-axis



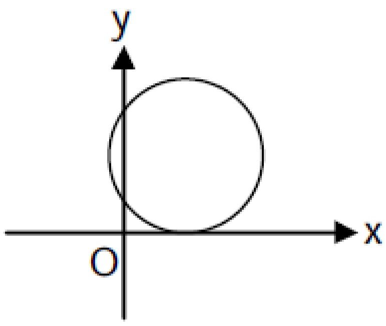
$f^2 - c = 0$ & $g^2 - c > 0$

$4 + p = 0 \quad 1 + p > 0$

$p = -4$

Not possible

Case-III Touches x-axis and cuts y-axis



$f^2 - c > 0$ & $g^2 - c = 0$

$4 + p > 0 \quad 1 + p = 0$

So two value of p are possible

48. Sol. (5)

$A, B, C, D, E, F, G, H, I, J$

$x = 10!$

$y = {}^{10}C_1 . {}^{10}C_2 . 8! \, {}^{9}C_8$

$$\frac{y}{9x} = \frac{{}^{10}C_1 . {}^{10}C_2 . 8! \times 9}{9 \times 10!} = \frac{10! \times 45}{9 \times 10!} = 5$$

49. Sol. (A)

For $a = \sqrt{2}$, the equitation of the circle is : $x^2 + y^2 = 2$

Equation of tangent at $(-1, 1)$ is : $-x + y = 2$

Point of contact : $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}} \right) \Rightarrow \left(\frac{-\sqrt{2}}{\sqrt{2}}, \frac{\sqrt{2}}{\sqrt{2}} \right) \Rightarrow (-1, 1)$

50. Sol. (B)

(A) $x^2 + y^2 = \frac{13}{4}$

Equation of tangent at $\left(\sqrt{3}, \frac{1}{2} \right)$ is : $x + \sqrt{3} + \frac{y}{2} = -\frac{13}{4}$

$\therefore$ option (A) is incorrect.

(B) Satisfying the point $\left(\sqrt{3}, \frac{1}{2}\right)$ in the curve $x^2 + a^2y^2 = a^2$, we get $3 + \frac{a^2}{4} = a^2$

$\Rightarrow \frac{3a^2}{4} = 3 \quad \Rightarrow \quad a^2 = 4 \qquad \therefore$ the conic is: $x^2 + 4y^2 = 4$

Equation of tangent at $\left(\sqrt{3}, \frac{1}{2}\right)$ is : $\sqrt{3}x + 2y = 4$

51. Sol. (A)

The equation of given tangent is : $y = x + 8$

Satisfying the point $(8,16)$ in the curve $y^2 = 4ax$ we get, $a = 8$,

Now comparing the given tangent with the general tangent to the parabola, $y = mx + \frac{a}{m}$,

We get $m = 1$.

Point of contact is $\left(\frac{a}{m^2}, \frac{2a}{m}\right) \Rightarrow (8,16)$

52. Sol. (D)

$$f(x) = x + \ell nx - x\ell nx$$
$$f'(x) = 1 + \frac{1}{x} - \ell nx - x\left(\frac{1}{x}\right) = \frac{1}{x} - \ell nx$$
$$f''(x) = -\frac{1}{x^2} - \frac{1}{x} < 0 \;\forall\; x \in (0, \infty)$$

$\therefore f'(x)$ is strictly decreasing function for $x \in (0, \infty)$

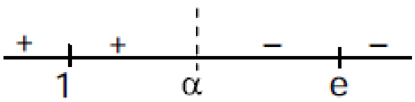
$$\left.\begin{array}{l}\lim\limits_{x\to\infty} f'(x) = -\infty \\ \lim\limits_{x\to 0+} f'(x) = \infty\end{array}\right\} f'(x)=0 \text{ has only one real root in } (0,\infty)$$

$$f'(1) = 1 > 0$$

$$f'(e) = \frac{1}{e} - 1 < 0$$

$\therefore f(x) = 0$ has one root in $(1, e)$

Let $f'(\alpha) = 0$, where $\alpha \in (1, e)$



$\therefore f(x)$ is increasing in $(0, \alpha)$ and decreasing in (α, ∞)

$f(1) = 1$ and $f(e^2) = e^2 + 2 - 2e^2 = 2 - e^2 < 0$

$f(x) = 0$ has one root in $(1, e^2)$

From column 1: I and II are correct.

From column 2: ii, iii and iv are correct.

From column 3: P,Q,S are correct.

53. Sol. (C)

$$f(x) = x + \ell nx - x\ell nx$$

$$f'(x) = 1 + \frac{1}{x} - \ell nx - x\left(\frac{1}{x}\right) = \frac{1}{x} - \ell nx$$

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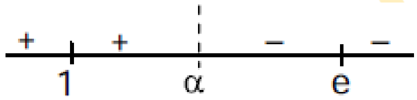

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54. Sol. (C)

$$f(x) = x + \ell nx - x\ell nx$$

$$f'(x) = 1 + \frac{1}{x} - \ell nx - x\left(\frac{1}{x}\right) = \frac{1}{x} - \ell nx$$

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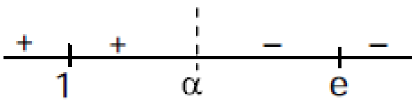

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