JEE ADVANCED-2017

MATHMATICS

A. General Instructions :

1. This sealed booklet is your Question Paper. Do not break the seal till you are instructed to do so.

2. The question paper CODE is printed on the left hand top corner of this sheet and the right hand top comer of the back cover of this booklet

3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.

4. The paper CODE is printed on its left part as well as the right part of the ORS. Ensure that both these codes are identical and same as that on the question paper booklet If not contact the invigilator.

5. Blank spaces are provided within this booklet for rough work.

6. Write your name and roll number in the space provided on the back cover of this booklet

7. After breaking the seal of the booklet at 9:00 am, verify that the booklet contains 36 pages and that all the 54 questions along with the options are legible. If not contact the invigilator for replacement of the booklet

8. You are allowed to take away the Question Paper at the end of the examination.

Optical Response Sheet

9. The ORS (top sheet) will be provided with an attached Candidate’s Sheet (bottom sheet), The Candidate's Sheet is a carbon - less copy of the ORS.

10. Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's Sheet

11. The ORS will be collected by the invigilator at the end of the examination.

12. You will be allowed to take away the Candidate's Sheet at the end of the examination.

13. Do not tamper with of mutilate the ORS. Do not use the ORS for rough work.

14. Write your name, roll number and code of the examination center, and sign with pen in the space provided for this purpose on the ORS. Do not write any of these details anywhere else on the ORS. Darken the appropriate bubble under each digit of your roll number.

Darken the Bubbles on the ORS

15. Use a Black Ball Point Pen to darken the bubbles on the ORS.

16. Darken the bubble  completely.

17. The correct way of darkening a bubble is as: 

18. The ORS is machine - gradable. Ensure that the bubbles are darkened in the correct way.

19. Darken the bubbles only if you are sure of the answer. There is no way to erase or "un-darken" a darkened bubble.

SECTION -1 : (Maximum Marks : 28)

- This section contains **SEVEN** questions.
- Each question has FOUR options (A), (B), (C) and (D), **ONE OR MORE THAN ONE** of these four option(s) is(arc) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in <u>one of the following categories</u> :

Full Marks	+4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
Partial Marks	+1 For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.
Zero Marks	0 If none of the bubbles is darkened.
Negative Marks	-2 In all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks and darkening (A) and (B) will get -2 marks, as a wrong option is also darkened

37. Let X and Y be two events such that $P(X)=\frac{1}{3}, P(X\mid Y)=\frac{1}{2}$ and $P(Y\mid X)=\frac{2}{5}$ Then

(A) $P(Y)=\frac{4}{15}$

(B) $P(X'\mid Y)=\frac{1}{2}$

(C) $P(X\cup Y)=\frac{2}{5}$

(D) $P(X\cap Y)=\frac{1}{5}$

38. Let $f:R\to(0,1)$ be a continuous function. Then, which of the following function(s) has (have) the value zero at some point in the interval $(0,1)$?

(A) $e^x-\int_0^x f(t)\sin t\,dt$

(B) $f(x)+\int_0^{\frac{\pi}{2}} f(t)\sin t\,dt$

(C) $x-\int_0^{\frac{\pi}{2}-x} f(t)\cos t\,dt$

(D) $x^9-f(x)$

39. Let a, b, x and y be real number such that $a-b=1$ and $y \neq 0$. If the complex number $z=x+iy$ satisfies $\text{lm}\left(\frac{az+b}{z+1}\right)=y$, then which of the following is(are possible value(s) of x ?

(A) $1-\sqrt{1+y^2}$

(B) $-1-\sqrt{1-y^2}$

(C) $1+\sqrt{1+y^2}$

(D) $-1+\sqrt{1-y^2}$

40. If $2x-y+1=0$ is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{16}=1$, then then which of the following CANNOT be sides of a right-angled triangle?

(A) $a, 4, 1$

(B) $2a, 4, 1$

(c) $a, 4, 2$

(D) $2a, 8, 1$

41. Let $[x]$ be the greatest integer less than or equals to x. Then, at which of the following point(s) the function $f(x)=x\cos\left(\pi\left(x+[x]\right)\right)$ is discontinuous?

(A) $x=-1$

(B) $x=1$

(C) $x=0$

(D) $x=2$

42. Which of the following is(are) NOT the square of a 3×3 matrix with real entries?

(A) $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & -1\end{bmatrix}$

(B) $\begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & -1\end{bmatrix}$

(C) $\begin{bmatrix}-1 & 0 & 0\\0 & -1 & 0\\0 & 0 & -1\end{bmatrix}$

(D) $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

43. If a cord, which is not a tangent, of the parabola $y^2=16x$ has the question $2x+y=p$, and mid point (h,k), then which of the following is (are) possible values(s)of p,h and k ?

(A) $p=-1,h=1,k=-3$

(B) $p=2,h=3,k=-4$

(C) $p=-2,h=2,k=-4$

(D) $p=5,h=4,k=-3$

SECTION - 2 : (Maximum Marks : 15)

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive. For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories :

  Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.

  Zero Marks : 0 In all other cases.

44. For the real number of α, if the system

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Of linear equation, has infinitely many solution, then

$$1+\alpha+\alpha^2=\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

45. The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side?

46. Let $f:R\to R$ be a differentiable function such that $f(0)=0$, $f\left(\frac{\pi}{2}\right)=3$ and $f'(0)=1$. If

$$g(x)=\int_x^{\frac{\pi}{2}}\left[f'(t)\operatorname{cosec}t-\cot t\operatorname{cosec}t\, f(t)\right]dt \text{ for } x\in\left(0,\frac{\pi}{2}\right], \text{ then } \lim_{x\to 0} g(x)=$$

47. For how many value of p, the circle $x^2+y^2+2x+4y-p=0$ and the coordinate axes have exactly three common points ?

48. Words of length 10 are formed using A,B,C,D,E,F,G,H,I,J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no. other letter is repeated.

Then, $\frac{y}{9x}=$

SECTION - 3 : (Maximum Marks : 18)

- This section contains **SIX** questions of matching type.
- This section contains **TWO** tables (each having 3 columns and 4 rows).
- Based on each table, there are **THREE** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks +3 If only the bubble corresponding to the correct option is darkened.

Zero Marks 0 If none of the bubbles is darkened.

Answer Q.49, Q.50 and Q.51 by appropriately matching the information given in the three columns of the following table.

Column 1,2 and 3 contain conics, equation of tangents to the conics and points of contact, repectively.

Column-1	Column-2	Column-3
(I) $x^2+y^2=a^2$	(i) $my=m^2x+a$	(P) $\left(\frac{a}{m^2},\frac{2a}{m}\right)$
(II) $x^2+a^2y^2=a^2$	(ii) $y=mx+a\sqrt{m^2+1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}},\frac{a}{\sqrt{m^2+1}}\right)$

(III) $y^2 = 4ax$	(iii) $y = mx + \sqrt{a^2m^2 - 1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV) $x^2 - a^2y^2 = a^2$	(iv) $y = mx + \sqrt{a^2m^2 + 1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

49. For $a = \sqrt{2}$, if a tangent is drawn to a suitable conic (Column 1) and at the point of contact $(-1,1)$, then which of the following option is the only CORERECT combination for obtaining its equation?

50. The tangent to a suitable conic (column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3}x + 2y = 4$,then which of the following option is the only CORRECT combination?

(A) (IV) (iv) (S)

(B) (II) (iv) (R)

(C) (IV) (iii) (S)

(D) (II) (iii) (R)

51. if a tangent to a suitable conic (Column 1) is found to be $y = x + 8$ and its point of contact is $(8,16)$, then which of the following option is the only CORRECT combination?

(A) (III) (i) (P)

(B) (I) (ii) (Q)

(C) (II) (iv) (R)

(D) (III) (ii) (Q)

Answer Q.52, Q.53 and Q.54 by appropriately matching the information given in the three columns of the following table.

Let $f(x)=x+\log_e x-x\log_e x, x\in(0,\infty)$

- Column 1 contains information about zeros of $f(x), f'(x)$ and $f''(x)$ at infinitty.
- Column 2 contains information about the limiting behavior of $f(x), f'(x)$ and $f''(x)$ at infinity.
- Column 3 contains information about increasing/Decreasing nature of $f(x)$ *and* $f'(x)$.

Column1	Column 2	Column 3
(I) $f(x)=0$ for some $x\in(1,e^2)$	(i) (i) $\lim_{x\to\infty} f(x)=0$	(P) f is increasing $(0,1)$
(II) $f'(x)=0$ for some $x\in(1,e)$	(ii) $\lim_{x\to\infty} f(x)=-\infty$	(Q) f is decreasing (e,e^2)
(III) $f'(x)=0$ for some $x\in(0,1)$	(iii) $\lim_{x\to\infty} f'(x)=-\infty$	(R) f' is increasing $(0,1)$
(IV) $f''(x)=0$ for some $x\in(1,e)$	(iv) $\lim_{x\to\infty} f''(x)=0$	(S) f is decreasing (e,e^2)

52. Which of the following option is the only INCORRECT combination?

(A) (I) (iii) (P)

(B) (II) (iv) (Q)

(C) (II) (iii) (P)

(D) (III) (i) (R)

53. Which of the following option is the only CORRECT combination?

(A) (I) (ii) (R)

(B) (III) (iv) (P)

(C) (II) (iii) (S)

(D) (IV) (i) (S)

54. Which of the following option is the only CORRECT combination?

(A) (III) (iii) (R)

(B) (IV) (iv) (S)

(C) (II) (ii) (Q)

(D) (I) (i) (P)