JEE ADVANCED-2017

MATHEMATICS

General Instructions :

Read the Instructions carefully: General:

1. This sealed booklet is your Question Paper. Do not break the seal till you are instructed to do so.

2. The question paper CODE is printed on the left hand top corner of this sheet and the right hand top corner of the back cover of this booklet

3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.

4. The paper CODE is printed on its left part as well as the right part of the ORS. Ensure that both these codes are identical and same as that on the question paper booklet If not contact the invigilator.

5. Blank spaces are provided within this booklet for rough work.

6. Write your name and roll number in the space provided on the back cover of this booklet

7. After breaking the seal of the booklet at 2:00 pm, verify that the booklet contains 36 pages and that all the 54 questions along with the options are legible. If not contact the invigilator for replacement of the booklet

8. You are allowed to take away the Question Paper at the end of the examination.

Optical Response Sheet

9. The ORS (top sheet) will be provided with an attached Candidate's Sheet (bottom sheet). The Candidate's Sheet is a carbon - less copy of the ORS.

10. Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's Sheet.

11. The ORS will be collected by the invigilator at the end of the examination.

12. You will be allowed to take away the Candidate's Sheet at the end of the examination,

13. Do not tamper with of mutilate the ORS. Do not use the ORS for rough work.

14. Write your name, roll number and code of the examination center, and sign with pen in the space provided for this purpose on the ORS. Do not write any of these details anywhere else on the ORS. Darken the appropriate bubble under each digit of your roll number.

Darken the Bubbles on the ORS

15. Use a Black Ball Point Pen to darken the bubbles on the ORS.

16. Darken the bubble completely.

17. The correct way of darkening a bubble is as:

18. The ORS is machine - gradable. Ensure that the bubbles are darkened in the correct way.

19. Darken the bubbles only if you are sure of the answer. There is no way to erase or "un-darken" a darkened bubble.

SECTION – 1

(Maximum Marks : 21)

- This section contains **SEVEN** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –1 In all other cases.

37. if $f : R \to R$ is a twice differentiable function such that $f''(x) > 0$ for all $x \in R$, and $f\left(\frac{1}{2}\right) = \frac{1}{2}, f(1) = 1$

(A) $f'(1) \le 0$

(B) $f'(1) > 1$

(C) $0 < f'(1) \le \frac{1}{2}$

(D) $\frac{1}{2} < f'(1) \le 1$

38. If $y = y(x)$ satisfies the differential equation

$8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1} dx, x > 0$ and $y(0) = \sqrt{7}$, then $y(256) =$

(A) 16

(B) 3

(C) 9

(D) 80

39. How many 3×3 matrices M with entries from $\{0,1,2\}$ are there, for which the sum of the diagonal entries of $M^T M$ is 5 ?

(A) 198

(B) 162

(C) 126

(D) 135

40. three randomly chosen no integers x, y and z are found to satisfy the equitation $x+y+z=10$.

Then the probability that z is even, is

(A) $\frac{1}{2}$

(B) $\frac{36}{55}$

(C) $\frac{6}{11}$

(D) $\frac{5}{11}$

41. Let $S=\{1,2,3,\ldots\ldots,9\}$. For $k=1,2,\ldots\ldots,5$, let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1+N_2+N_3+N_4+N_5=$

(A) 210

(B) 252

(C) 126

(D) 125

42. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

$\overrightarrow{OP}.\overrightarrow{OQ}+\overrightarrow{OR}.\overrightarrow{OS}=\overrightarrow{OR}.\overrightarrow{OP}+\overrightarrow{OQ}.\overrightarrow{OS}=\overrightarrow{OQ}.\overrightarrow{OR}+\overrightarrow{OP}.\overrightarrow{OS}$ Then the triangle PQR has S as its

(A) centroid

(B) orthocenter

(C) incentre

(D) circumcenter

43. The equation of the plane passing through the point $(1,1,1)$ and perpendicular to the planes $2x+y-2z=5$ and $3x-6y-2z=7$ is

(A) $14x+2y-15z=1$

(B) $-14x+2y+15z=3$

(C) $14x-2y+15z=27$

(D) $14x+2y+15z=31$

SECTION – 2

(Maximum Marks : 28)

- This section contains **SEVEN** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is correct.
- For each question, darken the bubble(s) corresponding to the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +4 If only the bubble corresponding to the correct option is darkened.

Partial Marks : +1 For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.

Zero Marks : +1 If none of the bubbles is darkened.

Negative Marks : –2 In all other cases.

For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

44. If $f : R \to R$ is a differentiable function such that $f'(x) > 2f(x)$ for all $x \in R$, and $f(0) = 1$, then

(A) $f(x) > e^{2x}$ in $(0, \infty)$

(B) $f'(x) < e^{2x}$ in $(0, \infty)$

(C) $f(x)$ is increasing in $(0, \infty)$

(D) $f(x)$ is decreasing in $(0, \infty)$

45. If $I=\sum_{k=1}^{98}\int_{k}^{k+1}\frac{k+1}{x(x+1)}dx,$ then

(A) $I>\log_e 99$

(B) $I<\log_e 99$

(C) $I<\frac{49}{50}$

(D) $I>\frac{49}{50}$

46. If the line $x=\alpha$ divides the area of region $R=\{(x,y)\in R^2 : x^3\le y\le x, 0\le x\le 1\}$ in to two equal parts, then

(A) $2\alpha^4-4\alpha^2+1=0$

(B) $\alpha^4+4\alpha^2-1=0$

(C) $\frac{1}{2}<\alpha<1$

(D) $0<\alpha\le\frac{1}{2}$

47. let α and β be nonzero real number such that $2(\cos\beta-\cos\alpha)+\cos\alpha\cos\beta=1.$ Then which of the following is/are true?

(A) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)-\tan\left(\frac{\beta}{2}\right)=0$

(B) $\tan\left(\frac{\alpha}{2}\right)-\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$

(C) $\tan\left(\frac{\alpha}{2}\right)+\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$

(D) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)+\tan\left(\frac{\beta}{2}\right)=0$

48. let $f(x)=\frac{1-x\left(1+|1-x|\right)}{|1-x|}\cos\left(\frac{1}{1-x}\right)$ *for* $x\neq 1$. *Then*

(A) $\lim_{x\to 1^+} f(x)=0$

(B) $\lim_{x\to 1^-} f(x)$ does not exist

(C) $\lim_{x\to 1^-} f(x)=0$

(D) $\lim_{x\to 1^+} f(x)$ does not exist

49. If $g(x)=\int_{\sin x}^{\sin(2x)}\sin^{-1}(t)\,dt$, then

(A) $g'\left(-\frac{\pi}{2}\right)=2\pi$

(B) $g'\left(-\frac{\pi}{2}\right)=-2\pi$

(C) $g'\left(\frac{\pi}{2}\right)=2\pi$

(D) $g'\left(\frac{\pi}{2}\right)=-2\pi$

50. If $f(x)=\begin{vmatrix}\cos(2x) & \cos(2x) & \sin(2x)\\ -\cos x & \cos x & -\sin x\\ \sin x & \sin x & \cos x\end{vmatrix}$, then

(A) $f(x)$ attains its minimum at $x=0$

(B) $f(x)$ attains its maximum at $x=0$

(C) $f'(x)=0$ at more than three points in $(-\pi,\pi)$

(D) $f'(x)=0$ at exactly three points in $(-\pi,\pi)$

SECTION – 3

(Maximum Marks : 12)

- This section contains **TWO** questions
- Based on each paragraph, there are **TWO** questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.

Zero Marks : 0 In all other cases.

PARAGRAPH 1

Let O be the origin, and $\overrightarrow{OX}$, $\overrightarrow{OY}$, $\overrightarrow{OZ}$ be three unit vectors in the directions of the sides $\overrightarrow{QR}$, $\overrightarrow{RP}$, $\overrightarrow{PQ}$, respectively, of a triangle PQR.

51. If the triangle PQR varies, then the minimum value of $\cos(P+Q)+\cos(Q+R)+\cos(R+P)$ is

(A) $-\frac{3}{2}$

(B) $\frac{3}{2}$

(C) $\frac{5}{3}$

(D) $-\frac{5}{3}$

52. $\left|\overrightarrow{OX} \times \overrightarrow{OY}\right| =$

(A) $\sin(P+Q)$

(B) $\sin(P+R)$

(C) $\sin(Q+R)$

(D) $\sin 2R$

PARAGRAPH 2

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$ where $\alpha \neq \beta$. For $n = 0, 1, 2, \ldots.,$ let $a_n = p\alpha^n + q\beta^n$.

FACT: If a and b are rational numbers and $a + b\sqrt{5} = 0,$, then $a = 0 = b$.

53. $a_{12} =$

(A) $a_{11} + 2a_{10}$

(B) $2a_{11} + a_{10}$

(C) $a_{11} - a_{10}$

(D) $a_{11} + a_{10}$

54. If $a_4 = 28$, then $p + 2q =$

(A) 14

(B) 7

(C) 21

(D) 12