JEE ADVANCED-2018 (PAPER-1)

MATHEMATICS

[Maximum Marks: 180]

- This question paper has total of eighteen (18) questions divided into three (03) sections (Section-1, Section-2 and Section-3).
- Total number of questions in Paper-1: Fifty-four (54).
- Paper-1 Maximum Marks: One Hundred Eighty (180).

Instructions for Section-1: Questions and Marking Scheme

- This section contains **SIX (06)** questions.
- Each question has **FOUR options** for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking chosen.
  Full Marks : **+4** If only (all) the correct option(s) is (are) chosen.
  Partial Marks: **+3** If all the four options are correct but ONLY three options are chosen.
  Partial Marks: **+2** If three or more options are correct but ONLY two options are chosen, both of which are correct options.
  Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks: 0 If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks: **–2** In all other cases.
- **For Example:** If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

Answering Section-1 Questions:

- To select the option(s), **using the mouse click** on the corresponding button(s) of the option(s).
- To deselect chosen option(s), click on the button(s) of the chosen option(s) again or click on the **Clear Response** button to clear all the chosen options.
- To change the option(s) of a previously answered question, if required, first click on the **Clear Response** button to clear all the chosen options and then select the new option(s).

- To mark a question ONLY for review (i.e. without answering it), click on the **Mark for Review & Next button**.
- To mark a question for review (after answering it), click on **Mark for Review & Next button** – answered question which is also marked for review will be evaluated.
- To save the answer, click on the **Save & Next** button – the answered question will be evaluated.

Instructions for Section-2: Questions and Marking Scheme

SECTION-2 (Maximum Marks: 24)

- This section contains **EIGHT (08)** questions. The answer to each question is **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, –0.33, –0.30,30.27, –127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme :

  Full Marks : **+3** If ONLY the correct numerical value is entered as answer.
  Zero Marks : **0** In all other cases.

Answering Section-2 Questions:

- Using the attached computer mouse, click on numbers (and/or symbols) on the on-screen virtual numeric keypad to enter the numerical value as answer in the space provided for answer.
- To change the answer, if required, first click on the **Clear Response** button to clear the entered answer and then enter the new numerical value.
- To mark a question ONLY for review (i.e. answering it), click on **Mark for Review & Next button** – the answered question which is also marked for review will be evaluated.
- To mark a question for review (after answering it), click **Mark for Review & Next button** – the answered question which is also marked for review will be evaluated.
- To save the answer, click on the **Save & Next button** – the answered question will be evaluated.

Instructions for Section-3: Questions and Marking Scheme

SECTION-3 (Maximum Marks: 12)

- This section contains **TWO (02)** paragraphs. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options corresponds to the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :
  Full Marks : **+3** If ONLY the correct option is chosen.
  Zero Marks :**0** If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks :**–1** In all other cases.

Answering Section-3 Questions:

- To select an option, using the mouse click on the corresponding button of the option.
- To deselect the chosen answer, click on the button of the chosen option again or click on the **Clear Response button**.
- To change the chosen answer, click on the button of another option.
- To mark a question ONLY for review (i.e. without answering it), click on **Mark for Review & Next button**.
- To mark a question for review (after answering it), click on **Mark for Review & Next button** – the answered which is also marked for review will be evaluated.
- To save the answer, click on the **Save & Next button** – the answered question will be evaluated.

SECTION – 1: [Maximum Marks: 24]

- This section contains **SIX (06)** questions.
- Each question has **FOUR options** for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking chosen.
  Full Marks : **+4** If only (all) the correct option(s) is (are) chosen.
  Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.
  Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen, both of which are correct options.
  Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks: **–2** In all other cases.

For Example: If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

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1. For a non-zero complex number z, let $\arg(z)$ denote the principal argument with $-\pi < \arg(z) \le \pi$. Then, which of the following statement(s) is (are) FALSE?

(A) $Arg(-1-i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$

(B) The function $f : R \to (-\pi, \pi]$, defined by $f(t) = \arg(-1+it)$ for all $t \in R$, is continuous at all points of R, where $i = \sqrt{-1}$

(C) For any two non-zero complex numbers z_1 and z_2, $\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$ is an integer multiple of 2π

(D) For any three given distinct complex numbers z_1, z_2 and z_3, the locus of the point z satisfying the condition $\arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi$, lies on a straight line.

2. In a triangle PQR, let $\angle PQR = 30^\circ$ and the sides PQ and QR have lengths $10\sqrt{3}$ and 10, respectively. Then, which of the following statement(s) is (are) TRUE?

(A) $\angle QPR = 45^\circ$

(B) The area of the triangle PQR is $25\sqrt{3}$ and $\angle QRP = 120^\circ$

(C) The radius of the incircle of the triangle PQR is $10\sqrt{3} - 15$

(D) The area of the circumcircle of the triangle PQR is 100π

3. Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes. Then, which of the following statement(s) is (are) TRUE?

(A) The line of intersection of P_1 and P_2 has direction ratios $1, 2, -1$

(B) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2

(C) The acute angle between P_1 and P_2 is 60°

(D) If P_3 is the plane passing through the point $(4,2,-2)$ and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point $(2,1,1)$ from the plane P_3 is $\frac{2}{\sqrt{3}}$

4. For every twice differentiable function $f : R \to [-2,2]$ with $(f(0))^2 + (f'(0))^2 = 85$, which of the following statement(s) is (are) TRUE?

(A) There exist $r, s \in R$, where $r < s$, such that f is one-one on the open interval (r,s)

(B) There exists $x_0 \in (-4,0)$ such that $|f'(x_0) \leq 1|$

(C) $\lim_{x \to \infty} f(x) = 1$

(D) There exists $\alpha \in (-4,4)$ such that $f(\alpha) + f''(\alpha) = 0$ and $f'(\alpha) \neq 0$

5. Let $f : R \to R$ and $g : R \to R$ be two non-constant differentiable functions. If $f'(x) = \left(e^{(f(x)-g(x))}\right) g'(x)$ for all $x \in R$, and $f(1) = g(2) = 1$, then which of the following statement(s) is (are) TRUE?

(A) $f(2) < 1 - \log_e 2$

(B) $f(2) > 1 - \log_e 2$

(C) $g(1) > 1 - \log_e 2$

(D) $g(1) < 1 - \log_e 2$

6. Let $f : [0,\infty) \to R$ be a continuous function such that $f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)\,dt$ for all $x \in [0,\infty)$. Then, which of the following statement(s) is (are)) TRUE?

(A) The curve $y = f(x)$ passes through the point $(1,2)$

(B) The curve $y = f(x)$ passes through the point $(2,-1)$

(C) The area of the region $\{(x,y) \in [0,1] \times R : f(x) \leq y \leq \sqrt{1-x^2}\}$ is $\frac{\pi - 2}{4}$

(D) The area of the region $\{(x,y) \in [0,1] \times R : f(x) \leq y \leq \sqrt{1-x^2}\}$ is $\frac{\pi - 1}{4}$

SECTION – 2: [Maximum Marks: 24]

- This section contains **EIGHT (08)** questions. The answer to each question is **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, –0.33, –0.30,,30.27, –127.30) using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.

- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : **+3** If ONLY the correct numerical value is entered as answer.
Zero Marks : **0** In all other cases.

7. The value of $\left(\left(\log_2 9\right)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times \left(\sqrt{7}\right)^{\frac{1}{\log_4 7}}$ is _____.

8. The number of 5 digit numbers which are divisible by 4, with digits from the set $\{1,2,3,4,5\}$ and the repetition of digits is allowed, is _____.

9. Let X be the set consisting of the first 2018 terms of the arithmetic progression $1,6,11,\cdots$, and Y be the set consisting of the first 2018 terms of the arithmetic progression $9,16,23,\cdots$ Then, the number of elements in the set $X \cup Y$ is _____.

10. The number of real solutions of the equation

$$\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i\right) = \frac{\pi}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty}\left(-\frac{x}{2}\right)^i - \sum_{i=1}^{\infty}(-x)^i\right)$$

lying in the interval $\left(-\frac{1}{2},\frac{1}{2}\right)$ is _____.

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\cos^{-1}x$ assume values in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and $[0,\pi]$, respectively).

11. For each positive integer n, let $y_n = \frac{1}{n}\left((n+1)(n+2)\cdots(n+n)\right)^{1/n}$.

For $x \in R$, let $[x]$ be the greatest integer less than or equal to x. If $\lim_{n\to\infty} y_n = L$, then the value of $[L]$ is _____.

12. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a} \cdot \vec{b} = 0$. For some $x, y \in R$, let $\vec{c} = x\vec{a} + y\vec{b} + (\vec{a} \times \vec{b})$. If $|\vec{c}| = 2$ and the vector $\vec{c}$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2\alpha$ is _____.

13. Let a, b, c be three non-zero real numbers such that the equation $\sqrt{3}\, a \cos x + 2b \sin x = c, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, has two distinct real roots α and β with $\alpha + \beta = \frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is _____.

14. A farmer F_1 has a land in the shape of a triangle with vertices at $P(0,0), Q(1,1)$ and $R(2,0)$. From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y = x^n (n > 1)$. If the area of the region taken away by the farmer F_2 is exactly 30% of the area of ΔPQR, then the value of n is

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SECTION – 3: (Maximum Marks : 12)

- This section contains **TWO (02)** paragraphs. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options corresponds to the correct answer.
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  Negative Marks : **–1** In all other cases.

PARAGRAPH "X"

Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$.

(*There are two questions based on PARAGRAPH "X", the question given below is one of them*)

15. Let E_1E_2 and F_1F_2 be the chords of S passing through the point $P_0(1,1)$ and parallel to the x-axis and the y-axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slope -1. Let the tangents to S at E_1 and E_2 meet at E_3, the tangents to S at F_1 and F_2 meet at F_3, and the tangents to S at G_1 and G_2 meet at G_3. Then, then, the points E_3, F_3, and G_3 lie on the curve.

(A) $x + y = 4$

(B) $(x-4)^2 + (y-4)^2 = 16$

(C) $(x-4) + (y-4) = 4$

(D) $xy = 4$

PARAGRAPH "X"

Let S be the circle in the xy-plane defined by the equation $x^2 + y^2 = 4$.

(*There are two questions based on PARAGRAPH "X", the question given below is one of them*)

16. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve

(A) $(x+y)^2 = 3xy$

(B) $x^{2/3} + y^{2/3} = 2^{4/3}$

(C) $x^2 + y^2 = 2xy$

(D) $x^2 + y^2 = x^2y^2$

PARAGRAPH "A"

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student $S_i, i = 1,2,3,4,5$. But, on the examination day, the five students are randomly allotted the five seats.

(*There are two questions based on PARAGRAPH "A", the question given below is one of them*)

17. The probability that, on the examination day, the student S_1 gets the previously allotted seat R_1, and **NONE** of the remaining students gets the seat previously allotted to him/her, is

(A) $\frac{3}{40}$

(B) $\frac{1}{8}$

(C) $\frac{7}{40}$

(D) $\frac{1}{5}$

PARAGRAPH "A"

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student $S_i, i = 1, 2, 3, 4, 5$. But, on the examination day, the five students are randomly allotted the five seats.

(***There are two questions based on PARAGRAPH "A", the question given below is one of them***)

18. For $i = 1, 2, 3, 4$, let T_i denote the event that the students S_i and S_{i+1} do **NOT** sit adjacent to each other on the day of the examination. Then, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is

(A) $\frac{1}{15}$

(B) $\frac{1}{10}$

(C) $\frac{7}{60}$

(D) $\frac{1}{5}$