JEE ADVANCED-2018 (PAPER-2)

MATHEMATICS

[TOTAL MARK:180]

- This question paper has total of eighteen (18) questions divided into three (03) sections (Section-1, Section-2 and Section-3).
- Each part has total of eighteen (18) questions divided into three (03) sections (Sectlon-1, Section-2 and Sectlon-3).
- Total number of questions in Paper-1: Fifty-four (54).
- Paper-1 Maximum Marks: One Hundred Eighty (180).

Instructions for Section-1: Questions and Marking Scheme

SECTION-1 [Maximum Mark: 24]

- This section contains **SIX (06)** questions.
- Each question has **FOUR options** for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking chosen.

Full Marks : **+4** If only (all) the correct option(s) is (are) chosen.

Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.

Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen, both of which are correct options.

Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : **–2** In all other cases.

- **For Example:** If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

Answering Section-1 Questions:

- To select the option(s), **using the mouse click** on the corresponding button(s) of the option(s).
- To deselect chosen option(s), click on the button(s) of the chosen option(s) again or click on the **Clear Response** button to clear all the chosen options.

- To change the option(s) of a previously answered question, if required, first click on the **Clear Response** button to clear all the chosen options and then select the new option(s).
- To mark a question ONLY for review (i.e. without answering it), click on the **Mark for Review & Next button**.
- To mark a question for review (after answering it), click on **Mark for Review & Next button** – answered question which is also marked for review will be evaluated.
- To save the answer, click on the **Save & Next** button – the answered question will be evaluated.

Instructions for Section-2: Questions and Marking Scheme

SECTION-2 (Maximum Marks: 24)

- This section contains **EIGHT (08)** questions. The answer to each question is **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, –0.33, –0.30,30.27, –127.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct numerical value is entered as answer.

Zero Marks : **0** In all other cases.

Answering Section-2 Questions:

- Using the attached computer mouse, click on numbers (and/or symbols) on the on-screen virtual numeric keypad to enter the numerical value as answer in the space provided for answer.

- To change the answer, if required, first click on the **Clear Response** button to clear the entered answer and then enter the new numerical value.
- To mark a question ONLY for review (i.e. answering it), click on **Mark for Review & Next button** – the answered question which is also marked for review will be evaluated.
- To mark a question for review (after answering it), click **Mark for Review & Next button** – the answered question which is also marked for review will be evaluated.
- To save the answer, click on the **Save & Next button** – the answered question will be evaluated.

Instructions for Section-3: Questions and Marking Scheme

SECTION-3 (Maximum Marks: 12)

- This section contains **TWO (02)** paragraphs. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options corresponds to the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : –1 In all other cases.

Answering Section-3 Questions:

- To select an option, using the mouse click on the corresponding button of the option.
- To deselect the chosen answer, click on the button of the chosen option again or click on the **Clear Response button**.
- To change the chosen answer, click on the button of another option.
- To mark a question ONLY for review (i.e. without answering it), click on **Mark for Review & Next button**.
- To mark a question for review (after answering it), click on **Mark for Review & Next button** – the answered which is also marked for review will be evaluated.
- To save the answer, click on the **Save & Next button** – the answered question will be evaluated.

SECTION – 1: [Maximum Marks: 24]

- This section contains ELEVEN (11) questions.
- Each question has FOUR options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking chosen.

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.

Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.

Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct options.

Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks: **–2** In all other cases.

For Example: If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

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1. For any positive integer n, define $f_n : (0, \infty) \rightarrow R$ as

$$f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right) \text{ for all } x \in (0, \infty).$$

(Here, the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$)

Then, which of the following statement(s) is (are) TRUE?

(A) $\sum_{j=1}^{5} \tan^2\left(f_j(0)\right) = 55$

(B) $\sum_{j=1}^{10} \left(1 + f_j'(0)\right)\sec^2\left(f_j(0)\right) = 10$

(C) For any fixed positive integer n, $\lim_{x\to\infty} \tan\left(f_n(x)\right) = \frac{1}{n}$

(D) For any fixed positive integer n, $\lim_{x\to\infty} \sec^2\left(f_n(x)\right) = 1$

2. Let T be the line passing through the points $P(-2,7)$ and $Q(2,-5)$. Let F_1 be the set of all pairs of circles (S_1, S_2) such that T is tangent to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch each other at a point, say, M. Let E_1 be the set representing the locus of M as the pair (S_1, S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point $R(1,1)$ be F_1. Let E_2 be the set of the mid-points of the line segments in the set F_2. Then, which of the following statement(s) is (are) TRUE

(A) The point $(-2,7)$ lies in E_1

(B) The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does **NOT** lie in E_2

(C) The point $\left(\frac{1}{2}, 1\right)$ lies in E_2

(D) The point $\left(0, \frac{3}{2}\right)$, does **NOT** lie in E_1

3. Let S be the set of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1, b_2, b_2 \in R$ and the system of equation (in real variables)

$-x + 2y + 5z = b_1$

$2x - 4y + 3z = b_2$

$x - 2y + 2z = b_3$

has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one solution for each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$?

(A) $x + 2y + 3z = b_1, 4y + 5z = b_2$ and $x + 2y + 6z = b_3$

(B) $x + y + 3z = b_1, 5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$

(C) $-x + 2y - 5z = b_1, 2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$

(D) $x + 2y + 5z = b_1, 2x + 3z = b_2$ and $x + 4y - 5z = b_3$

4. Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q. Consider the ellipse whose center is at the origin $O(0,0)$ and whose semi-major axis is OQ. If the length of the minor axis of this ellipse is $\sqrt{2}$, then which of the following statement(s) is (are) TRUE?

(A) For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1

(B) For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is 1

(C) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{4\sqrt{2}}(\pi - 2)$

(D) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{16}(\pi - 2)$

5. Let s, t, r be non-zero complex numbers and L be the set of solutions $z = x + iy \left(x, y \in R, i = \sqrt{-1}\right)$ of the equation $sz + t\bar{z} + r = 0$, where $\bar{z} = x - iy$. Then, which of the following statement(s) is (are) TRUE?

(A) If L has exactly one element, then $|s| \neq |t|$

(B) If $|s| = |t|$, then L has infinitely many elements

(C) The number of elements in $L \cap \{z : |z - 1 + i| = 5\}$ is at most 2

(D) If L has more than one element, then L has infinitely many elements

6. Let $f : (0, \pi) \to R$ be a twice differentiable function such that $\lim_{t \to x} \frac{f(x)\sin t - f(t)\sin x}{t - x} = \sin^2 x$ for all $x \in (0, \pi)$. If $f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12}$, then which of the following statement(s) is (are) TRUE?

(A) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}}$

(B) $f(x) < \frac{x^4}{6} - x^2$ for all $x \in (0, \pi)$

(C) There exists $\alpha \in (0, \pi)$ such that $f'(\alpha) = 0$

(D) $f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$

SECTION – 2: [Maximum Marks: 24]

- This section contains **EIGHT (08)** questions. The answer to each question is **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 6.25, 7.00, –0.33, – 30,30.27, – 127.30) using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : **+3** If ONLY the correct numerical value is entered as answer.

Zero Marks : **0** In all other cases.

7. The value of the integral $\int_0^{\frac{1}{2}} \frac{1+\sqrt{3}}{\left((x+1)^2(1-x)^6\right)^{\frac{1}{4}}}dx$ is ________.

8. Let P be a matrix of order 3×3 such that all the entries in P are from the set $\{-1,0,1\}$. Then, the maximum possible value of the determinant of P is__________.

9. Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto function form Y to X, then the value of $\frac{1}{5!}(\beta-\alpha)$ is _______.

10. Let $f:R\to R$ be a differentiable function with $f(0)=0$. If $y=f(x)$ satisfies the differential equation $\frac{dy}{dx}=(2+5y)(5y-2)$, then the value of $\lim_{x\to\infty} f(x)$ is________.

11. Let $f:R\to R$ be a differentiable function with $f(0)=1$ and satisfying the equation

$f(x+y)=f(x)f'(y)+f'(x)f(y)$ for all $x,y\in R$.

Then, the value of $\log_e(f(4))$ is ________.

12. Let P be a point in the first octant, whose image Q in the plane $x+y=3$ (that is, the line segment PQ is perpendicular to the plane $x+y=3$ and the mid-point of PQ lies in the plane $x+y=3$) lies on the z-axis z. Let the distance of P from the x-axis be 5. If R is the image of P in the xy- plane, then the length of PR is_______

13. Consider the cube in the first octant with sides OP, OQ and OR of length 1, along the x-axis, y-axis and z-axis, respectively, where $O(0,0,0)$ is the origin. Let $S\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ be the centre of the cube and T be the vertex of the cube opposite to the origin O such that S lies on the diagonal OT. If $\vec{p}=\overline{SP}, \vec{q}=\overline{SQ}, \vec{r}=\overline{SR}$ and $\vec{t}=\overline{ST}$, then the value of $\left|(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t})\right|$ is____________.

14. Let $X = \left({}^{10}C_1\right)^2 + 2\left({}^{10}C_2\right)^2 + 3\left({}^{10}C_3\right)^2 + \cdots + 10\left({}^{10}C_{10}\right)^2$ where ${}^{10}C_r, r \in \{1, 2, \cdots, 10\}$ denote binomial coefficients. Then the value of $\frac{1}{1430}X$ is ________

SECTION – 3 : (Maximum Marks : 12)

- This section contains **TWO (02)** paragraphs. Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options. **ONLY ONE** of these four options corresponds to the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : **+3** If ONLY the correct option is chosen.

Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : **–1** In all other cases.

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15. Let $E_1 = \left\{ x \in R : x \neq 1 \text{ and } \frac{x}{x-1} > 0 \right\}$ and

$$E_2 = \left\{ x \in E_1 : \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right) \text{is a real number} \right\}$$

(Here, the inverse trigonometric function $\sin^{-1} x$ assumes values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.)

Let $f : E_1 \to R$ be the function defined by $f(x) = \log_e\left(\frac{x}{x-1}\right)$

and $g : E_2 \to R$ be the function defined by $g(x) = \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)$

LIST-I	LIST-II
(P) The range of f is	(1) $\left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$
(Q) The range of g contains	(2) $(0,1)$
(R) The domain of f contains	(3) $\left[-\frac{1}{2}, \frac{1}{2}\right]$
(S) The domain of g is	(4) $(-\infty, 0) \cup (0, \infty)$

	(5) $\left(-\infty, \frac{e}{e-1}\right]$
	(6) $(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1}\right]$

16. In a high school, a committee has to be formed from a group of 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.

(ii) Let α_2 be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total number of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_3 be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls and such that both M1 and G1 are **NOT** in the committee together.

LIST-I	**LIST-II**
(P) The value of α_1 is	(1) 136
(Q) The value of α_2 is	(2) 189
(R) The value of α_3 is	(3) 192
(S) The value of α_4 is	(4) 200
	(5) 381
	(6) 461

17. Let $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a > b > 0$, be a hyperbola in the $xy-$plane whose conjugate axis LM subtends an angle of 60° at one of its vertices N. Let the area of the triangle LMN be $4\sqrt{3}$.

(P) The length of the conjugate axis of H is	(1) 8
(Q) The eccentricity of H is	(2) $\frac{4}{\sqrt{3}}$
(R) The distance between the foci of H is	(3) $\frac{2}{\sqrt{3}}$
(S) The length of the latus rectum of H is	(4) 4

18. Let $f_1 : R \to R$, $f_2 : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R : \left(-1, e^{\frac{\pi}{2}} - 2\right) \to R$ and $f_4 : R \to R$ be functions defined by

(i) $f_1(x) = \sin\left(\sqrt{1 - e^{-x^2}}\right)$

(ii) $f_2(x) = \begin{cases} \dfrac{|\sin x|}{\tan^{-1} x} & if \quad x \neq 0 \\ 1 & if \quad x = 0 \end{cases}$, where the inverse trigonometric function $tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(iii) $f_3(x) = \left[sin\left(log_e(x+2)\right)\right]$, where for $t \in R, [t]$ denotes the greatest integer less than or equal to t ,

(iv) $f_4(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

(P) The function f_1 is	(1) **NOT** continuous at $x = 0$
(Q) The function f_2 is	(2) continuous at $x = 0$ and **NOT** differentiable at $x = 0$
(R) The function f_3 is	(3) differentiable at $x = 0$ and its derivative is **NOT** continuous at $x = 0$
(S) The function f_4 is	(4) differentiable at $x = 0$ and its derivative is continuous at $x = 0$