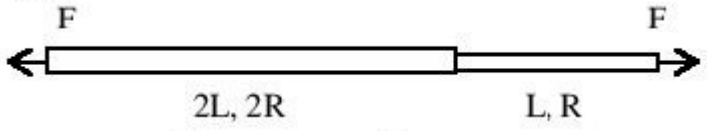
JEE(ADVANCED)-2013 PAPER 1

PHYSICS

1.Sol: (C)



$$k_1 = \frac{\pi 4R^2 x}{2L}, k_2 \quad \frac{\pi R^2 y}{L}$$

$$F = k_1 x = k_2 y \quad \Rightarrow \frac{y}{x} = \frac{k_1}{k_2} = 2$$

2.Sol: (D)

$$dw = \vec{F}.d\vec{r} = \vec{F}\left(dx\hat{i} + dy\hat{j}\right) = K\int \frac{xdx}{\left(x^2+y^2\right)^{3/2}} + \frac{ydy}{\left(x^2+y^2\right)^{3/2}}$$

$$x^2 + y^2 = a^2$$

$$w = \frac{K}{a^3}\int_a^0 xdx + \int_a^0 ydy = \frac{K}{a^3}\left(\frac{-a^2}{2} + \frac{a^2}{2}\right) = 0$$

3.Sol: (A)

$$R_1 = \frac{L}{kA} + \frac{L}{2kA} = \frac{3L}{2kA}$$

$$\frac{1}{R_2} = \frac{1}{\left(\frac{L}{kA}\right)} + \frac{1}{\left(\frac{L}{2kA}\right)} = \frac{3kA}{L}$$

$$R_2 = \frac{L}{3kA}$$

$$\Delta Q_1 = \Delta Q_2$$

$$\frac{\Delta T}{R_1}t_1 = \frac{\Delta T}{R_2}t_2$$

$$\Rightarrow t_2 = \frac{R_2}{R_1}t_1 = 2\sec.$$

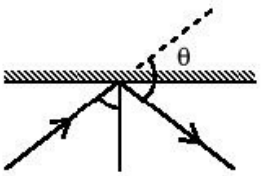
4.Sol: (A)

Let angle between the directions of incident ray and reflected ray be θ

$$\cos\theta = \frac{1}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right).\frac{1}{2}\left(\hat{i}-\sqrt{3}\hat{j}\right)$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = 120°$$



5.Sol: (B)

Main scale division $(s) = .05\text{cm}$

Vernier scale division $(v) = \frac{49}{100} = .049$

Least count $= .05 - .049 = .001\text{cm}$

Diameter: $5.10 + 24 \times .001$

$= 5.124\text{cm}$

6.Sol: (D)

$$PV = nRT = \frac{m}{M}RT$$

$$\Rightarrow PM = \rho RT$$

$$\frac{\rho_1}{\rho_2} = \frac{P_1M_1}{P_2M_2} = \left(\frac{P_1}{P_2}\right)\times\left(\frac{M_1}{M_2}\right) = \frac{4}{3}\times\frac{2}{3} = \frac{8}{9}$$

Here ρ_1 and ρ_2 are the densities of gases in the vessel containing the mixture.

7.Sol: (B)

$$\frac{I_{max}}{2} = I_m\cos^2\left(\frac{\phi}{2}\right)$$

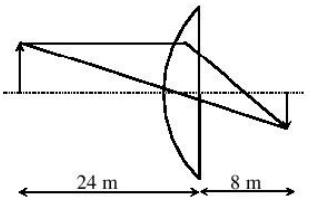
$$\Rightarrow \cos^2\left(\frac{\phi}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\phi}{2} = \frac{\pi}{4}$$

$$\Rightarrow \phi = \frac{\pi}{4}(2n+1)$$

$$\Rightarrow \Delta x = \frac{\lambda}{2\pi}\phi = \frac{\lambda}{2\pi}\times\frac{\pi}{2}(2n+1) = \frac{\lambda}{4}(2n+1)$$

8.Sol: (C)



$$\mu = \frac{\lambda_a}{\lambda_m} = \frac{3}{2}$$

$$\Rightarrow \frac{1}{f} = \frac{\mu-1}{R} = \frac{1}{2R}$$

$$\Rightarrow \frac{1}{f} - \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \frac{1}{8} - \frac{1}{-24} = \frac{1}{2R}$$

$$\Rightarrow \frac{3+1}{24} = \frac{1}{2R}$$

$$\Rightarrow R = 3\,\text{m}$$

9.Sol: (A)

Velocity of particle performing projectile motion at highest point

$= v_1 = v_0\cos\alpha$

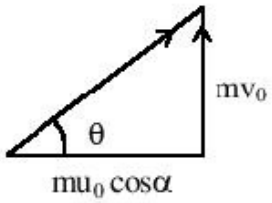
Velocity of particle thrown vertically upwards at the position of collision

$$= v_2^2 = u_0^2 - 2g\frac{u^2\sin^2\alpha}{2g} = v_0\cos\alpha$$

So, from conservation of momentum

$$\tan\theta = \frac{mv_0\cos\alpha}{mu_0\cos\alpha} = 1$$

$$\Rightarrow \theta = \pi/4$$



10.Sol: (B)

$t = 100 \times 10^{-9}$ sec, $P = 30 \times 10^{-3} Watt, C = C \times 10^{8}\ m/s$

$$Momentum = \frac{Pt}{C} = \frac{30 \times 10^{-3} \times 100 \times 10^{-9}}{3 \times 10^{8}} = 1.0 \times 10^{-17}\ kg\ ms^{-1}$$

11.Sol: (B, D)

After switch S_1 is closed, C_1 is charged by $2CV_0$, when switch S_2 is closed, C_1 and C_2 both have upper plate charge CV_0.

When S_3 is closed, then upper plate of C_2 becomes charged by $-CV_0$ and lower plate by $+CV_0$.

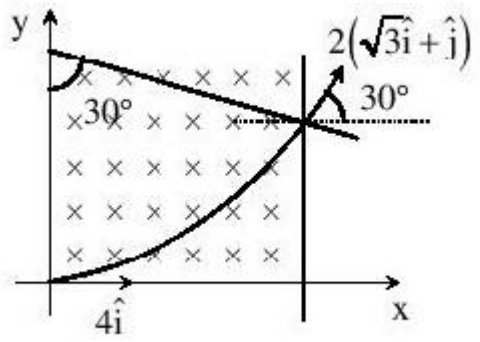
12.Sol: (A, C)

So magnetic field is along $-ve$, z-direction.

Time taken in the magnetic field $= 10 \times 10^{-3} = \frac{\pi M}{6QB}$

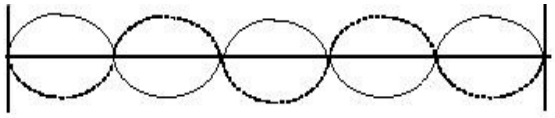
$$B = \frac{\pi M}{60 \times 10^{-3} Q} = \frac{1000 \pi M}{60Q}$$

$$\frac{50\pi M}{3Q}$$



13.Sol: (B, C)

$y = 0.01\text{m} \sin(20\pi x)\cos 200\pi t$



no. of nodes is 6

$$20\pi = \frac{2\pi}{\lambda}$$

$$\therefore \lambda = \frac{1}{10}\text{m} = 0.1\text{m}$$

length of the spring $= 0.5 \times \frac{1}{2} = 0.25$

Midpoint is the antinode

Frequency at this mode is $f = \frac{200\pi}{2\pi} = 100Hz$ f

$\therefore$ Fundamental frequency $= \frac{100}{5} = 20\text{Hz}$

14.Sol: (A, D)

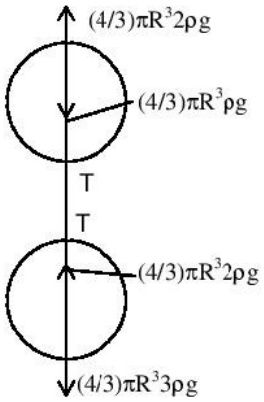
At equilibrium,

$\frac{4}{3}\pi R^3 2\rho g = \frac{4}{3}\pi R^3 \rho g + T$

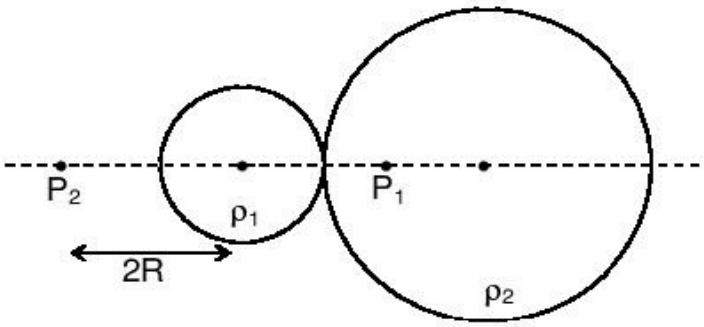
$T = \frac{4}{3}\pi R^3 \rho g$

$\therefore \Delta\ell = \frac{4}{3}\pi R^3 2\rho g$

For equilibrium of the complete system, net force of buoyancy must be equal to the total weight of the sphere which holds true in the given problem. So both the spheres are completely submerged.



15.Sol: (B, D)



At point $P_1, \frac{1}{4\pi\varepsilon_0}\frac{\rho_1(4/3)\pi R^3}{4r^2} = \frac{\rho_2 R}{3\varepsilon_0}$

$$\frac{\rho_1 R}{12} = \frac{\rho_2 R}{3}$$

$$\frac{\rho_1}{\rho_2} = 4$$

At point P_2,

$$\frac{\rho_1 (4/3)\pi R^3}{(2R)^2} + \frac{\rho_2 (4/3)\pi 8R^3}{(5R)^2} = 0$$

$$\therefore \frac{\rho_1}{\rho_2} = -\frac{32}{25}$$

16.Sol: (5)

The initial speed of 1^{st} bob (suspended by a string of length l_1)is $\sqrt{5gl_1}$.

The speed of this bob at highest point will be $\sqrt{gl_1}$.

When this bob collides with the other bob there speeds will be interchanged.

$$\sqrt{gl_1} = \sqrt{5gl_2} \Rightarrow \frac{l_1}{l_2} = 5$$

17.Sol: (5)

$$\text{Power} = \frac{dw}{dt} \Rightarrow w = 0.5 \times 5 = 2.5 = KF_f - KE_i$$

$$2.5 = \frac{M}{2}\left(v_f^2 - v_i^2\right)$$

$$\Rightarrow v_f = 5$$

18.Sol: (1)

Slope of graph is $h/e =$ *constant*

$\Rightarrow 1$

19.Sol: (4)

$$f = \left(1 - e^{-\lambda t}\right) = 1 - e^{-\lambda t} \approx 1 - (1 - \lambda t) = \lambda t$$

$$f = 0.04$$

Hence $\%\,decay \approx 4\%$

20.Sol: (8)

Conservation of angular momentum about vertical axis of disc

$$\frac{50(0.4)^2}{2}\times 10-\left[\frac{50(0.4)^2}{2}+4(6.25)(0.2)^2\right]\omega$$

$\omega = 8\,rad/\sec$