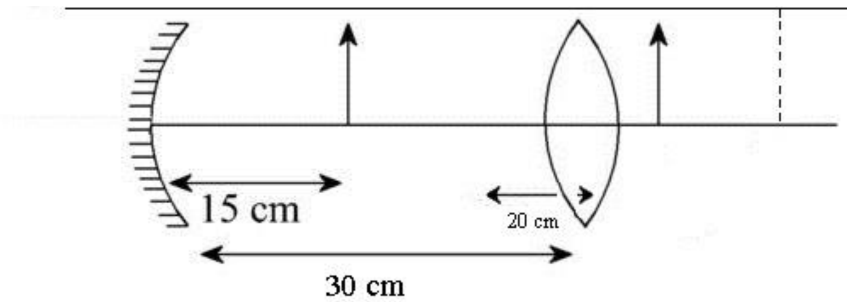
JEE ADVANCED-2015

PHYSICS

Q.1 Sol. (7)



Dotted line shows the image formed when two system is immersed in medium of $R.I = 7/6$. Image formed by mirror,

$v = -15\,\text{cm},\ f = -10\,\text{cm}$

So, $v = -30\,\text{cm}$.

$m = -2$

As, it is at the radius of curvature of the lens images forms at $20\ \text{cm}$ on the other side of lens and magnification now is 1. So, [Net magnification abs $(M_1) = 2$]

When kept in medium of $R.I = 7/6$.

$$\frac{1}{f\text{ lens}} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{2}{35}$$

$$f\text{ lens} = \frac{35}{2}\text{cm}$$

Focal length of mirror remains unchanged. So, new position of image,

$\mu = -20cm, \quad f = \frac{35}{2}$ cm

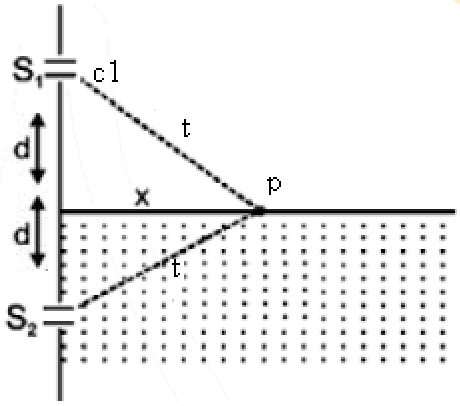
so, $v = 140$ cm .

$m_2 = -7$

Net magnification abs(M_2) $= (7) \times (2)$
$= 14 = M_2$

$$\left|\frac{M_2}{M_1}\right| = 7$$

Q.2 Sol. (3)



At point p path difference b/ω two waves from S_1 and S_2 will be

$\Delta p = (\mu - 1)t \quad$ where $\mu = \frac{4}{3}$

Now to get maxima at p (on the surface of water)

Path difference $\Delta p = \mathrm{m}\lambda$

$$(\mu-1)t = m\lambda \quad \Rightarrow \quad -(1)$$

Now $x^2 + d^2 = t^2 \quad -(2)$

From $(1)\&(2)$

$$(\mu-1)^2\left(x^2+d^2\right) = m^2\lambda^2$$

$$x^2 + d^2 = \frac{1}{(\mu-1)^2} m^2\lambda^2$$

$$x^2 = \frac{1}{(\mu-1)^2} m^2\lambda^2 - d^2$$

Now we have

$$x^2 = p^2 m^2 \lambda^2 - d^2$$

$$\Rightarrow p2 = \frac{1}{(\mu-1)^2} \Rightarrow \quad p = \frac{1}{\mu-1}$$

$$p = 3 \quad \text{where } \mu = \frac{4}{3}$$

Q.3 Sol. (7)

$$\frac{1}{2}mv_1^2 + \frac{1}{2}Iw_1^2 + g(30) = \frac{1}{2}mv^2 + \frac{1}{2}Iw^2$$

$$\frac{1}{2}mv_2^2 + \frac{1}{2}Iw_2^2 + g(27) = \frac{1}{2}mv^2 + \frac{1}{2}Iw^2$$

$$\Rightarrow \frac{v_1^2}{2}+\frac{1}{2}\times\frac{R^2}{2}\times\frac{v_1^2}{R^2}+g\times 30=\frac{v_2^2}{2}+\frac{1}{2}\times\frac{R^2}{2}\times\frac{v_2^2}{R^2}+g\times 27$$

$$\Rightarrow \frac{3v_1^2}{4}+30g=\frac{3v_2^2}{4}+27g$$

$$\Rightarrow \frac{3}{4}\left(v_2^2-v_1^2\right)=3g$$

$$v_2^2-v_1^2=\frac{4}{3}\times 30$$

$$v_2^2=40+9$$

$$v_2=7$$

Q.4 Sol. (2)

It is given that

$$g=\frac{1}{4}g_0.$$

At height h , we have variation of g as

$$g=g_o\frac{1}{\left(1+\frac{h}{R}\right)^2}$$

$$\frac{1}{4}g_o=g_o\frac{1}{\left(1+\frac{h}{R}\right)^2}$$

$$\left(1+\frac{h}{R}\right)^2=4$$

$$1+\frac{h}{R}=2$$

$$\left[\frac{h}{R}=1\right] \Rightarrow [h=R] \quad \text{-(1)}$$

we have the formula for escape velocity an

$$v_{esc}=\sqrt{\frac{2GM}{R}} \quad \text{-(2)}$$

Now applying law of conversation of energy

$$\frac{1}{2}mv^2-\frac{GMm}{R}=\frac{1}{2}mv_f^2-\frac{GMm}{R+h}$$

$v_f=0$ (final velocity at max . height)

v from(1) $\quad h=R$.

$$\frac{1}{2}mv^2=\frac{GMm}{R}-\frac{GMm}{2R}.$$

$$\frac{1}{2}mv^2=\frac{GMm}{R}.$$

$$v_i^2=\frac{Gm}{R} \quad \Rightarrow \quad v_i=\sqrt{\frac{GM}{R}}$$

Here initial velocity is $v_i=\text{v}$

$$V=\sqrt{\frac{GM}{R}} \quad \text{-(3)}$$

$\Rightarrow$ from (2) & (3)

$$V_{ese}=\sqrt{2}\,v$$

$$N=2$$

Q.5 Sol. (2)

Let R_A and R_B are the radii of A and B respectively and P_A and P_B are the power emitted by A and B respectively.

According to question

$$R_A = 400R_B$$

& $P_A = 10^4 P_B$

Wien's displacement law

$$\frac{\lambda A}{\lambda B} = \frac{T_B}{T_A} \text{\_\_\_\_\_\_\_\_} (1)$$

Stefans – Boltzmann law

$$\frac{P_A}{P_B} = \frac{A_A T_A^4}{A_B T_B^4}$$

$$\frac{P_A}{P_B} = \frac{R_A^2 \lambda_B^4}{R_B^2 \lambda_A^4}$$

$$10^4 = 16 \times 10^4 \frac{\lambda_B^4}{\lambda_B^4}$$

$$\Rightarrow \frac{\lambda_A^4}{\lambda_B^4} = 16 \quad \Rightarrow \frac{\lambda_A}{\lambda_B} = 2$$

Q.6 Sol. (3)

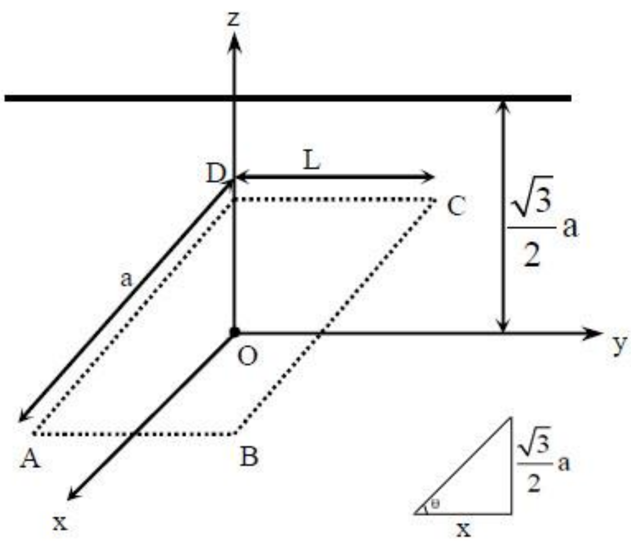
$$N = N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{\frac{1}{2}}}}$$

$$\frac{125}{1000} = \left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{8}\right) = \left(\frac{1}{2}\right)^n$$

$$n = 3$$

Q.7 Sol. (6)



Electric field at an element $(\perp^r$ comp.)

$$\frac{\lambda}{2\pi\epsilon_0\sqrt{\frac{3a^2}{4}+x^2}}\sin\theta$$ (x is the distance of element from origin)

$$\text{flux} = \phi = \int_{-a/2}^{a/2} E.ds$$

$$= \int_{-a/2}^{a/2} \frac{\lambda L}{2\pi\varepsilon_0 \sqrt{\frac{3a^2}{4} + x^2}} \frac{\sqrt{\frac{3a}{2}}}{\sqrt{\frac{3a}{2} + x^2}} dx$$

$$= \frac{\lambda L}{2\pi\varepsilon_0}\left(\frac{\pi}{6} + \frac{\pi}{6}\right)$$

$$= \frac{\lambda}{2\pi\varepsilon_0} \frac{2z}{6} \quad \Rightarrow \quad N = 6$$

Q.8 Sol. (2)

$$\frac{13.6}{n^2} + 10.4 = \frac{hc}{\lambda}$$

$$\frac{13.6}{n^2} + 10.4 = \frac{1242}{90}$$

$$n = 2$$

Q.9 Sol. (C)

Net electric field at the mid-point of both the linear charge system, $E = 0$

But, on small displacement given to $(+q)$, it will start oscillating, while $(-q)$ would be attracted toward positive plate

Hence, option, 'C' is correct.

Q.10 Sol. (B)

Now, for S_1

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$$\frac{1}{v}-\frac{-1.5}{(-50)}=\frac{1-1.5}{(-10)}$$

Solving, we get, $v=50\,\text{cm}$

For the S_2 tube, image should be at '∞'

Hence, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ $\quad[\text{as } u=-d+50]$

$$\frac{\mu_2}{v}-\frac{\mu_1}{(-d+50)}=\frac{\mu_2-\mu_1}{R}\quad[\text{as calculated below}]$$

$\mu_2=1.5, \mu_1=1$

as, $R=+10\,\text{cm}, v=\infty$

$\frac{\mu_2}{\infty}-\frac{\mu_1}{(-d+50)}=\frac{1.5-1}{10}$ Solving, we get $d=70\,\text{cm}$

Q.11 Sol. (A,B,C)

Force experience by conducting wire,

$F=\text{BIL}\sin\theta$

[θ: Angle between magnetic field and length]

If $\vec{B}$ is along z axis,

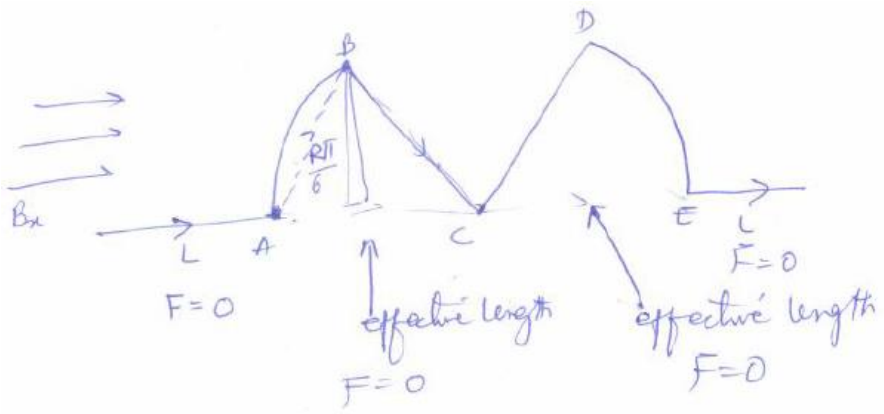
Then force $F = \text{BIL}\sin 90° + \text{BIR}\sin 90° + \text{BIR}\sin 90° + \text{BIL}\sin 90°$

$$\left|\vec{F}\right| \propto (L+R)$$

(Component of force may vary)

Similarly, when $\left(\vec{B}\right)$ is along y-axis

If $\left(\vec{B}\right)$ is along x-axis , net effect length is zero.



Hence $A, B,$ and C

Q.12 Sol. (A,B,D)

Average kinetic energy per molecule is $\frac{3}{2}nRT$ & Average Energy per mole of the gas mixture would be, $2RT$.

Ratio of Speed of the sound in the gas mixture is that in Helium gas would be

$$\frac{V_{mix}}{V_{He}} = \frac{\sqrt{\dfrac{Y_{mix}RT}{M_{mix}}}}{\dfrac{Y_{He}RT}{M_{He}}}$$

Calculating, $Y_{mix} = \frac{(C_p)_{mix}}{(C_v)_{mix}} = \frac{C_v + R}{C_v}$

Also, $(C_v)_{mix} = \frac{n_1 Cv_1 + n_2 Cv_2}{n_1 + n_2} = \frac{\left(1\times\frac{3}{2} + 1\times\frac{5}{2}\right)R}{1+1}$

$= (Cp)_{mix} = (Cv)_{mix} + R$

$(Cp)_{mix} = 3R,$

$Y = \frac{Cp}{Cv} = \frac{3R}{2R} = \frac{3}{2}$

Also, $M_{mix} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2} = \frac{1\times4 + 1\times2}{1+1} = 3$

$V_{mix}\sqrt{\frac{VRT}{M}} = \sqrt{\frac{\frac{3}{2}RT}{3}} = \sqrt{\frac{1}{2}RT} \quad \ldots\ldots(\text{i})$

$V_{He}\sqrt{\frac{YRT}{M_{He}}} = \sqrt{\frac{\frac{5}{3}RT}{4}} = \sqrt{\frac{5}{12}RT} \quad \ldots\ldots(\text{ii})$

$\frac{V_{mix}}{V_{He}} = \sqrt{\frac{6}{5}}$

Ratio of RMS speed of Helium atom to that of Hydrogen molecule,

$\frac{(V)_{He}}{(V)_{H2}} = \frac{\sqrt{\frac{3RT}{M_{He}}}}{\sqrt{\frac{3RT}{MH_2}}} = \frac{\sqrt{\frac{1}{4}}}{\sqrt{\frac{1}{2}}}\frac{1}{\sqrt{2}}$

Q.13 Sol. (B)

Using $R = \rho \frac{L}{A}$

Calculating Resistances of Aluminium and Iron, and as they are connected in parallel,

$$R_{parallel} = \frac{R_{AL} \times R_{Fe}}{R_{AL} + R_{Fe}} \quad \ldots\ldots(\text{i})$$

$$R_{AL} = \frac{2.7 \times 10^{-8} \times 50}{\left[7^2 - 2^2\right]} = 30\,\mu\Omega$$

$$R_{Fe} = \frac{1 \times 10^{-7} \times 50}{2^2} = 1250\,\mu\Omega$$

Substituting these values in eqn. (i) and getting

$$\frac{1875}{64}\,\mu\Omega$$

Hence, B

Q.14 Sol. (A,C)

Using , $hv - hv^* = eV$

$$V = \frac{hc}{e\lambda} = \frac{hc}{e\lambda^*}$$

Finding the graph, option A and C are correct.

Q.15 Sol. (B,C)

$1\ \text{cm}$ on MSR is divided into 8 equal divisions and a screw gauge with 100 divisions .

In Vernier callipers

Least Count $=1MSD-1VSD$

$$=1VSD=\frac{4}{5}MSD \quad [\because 5VSD=4MSD]$$

$$=\frac{1}{5}MSD$$

$$=\frac{1}{5}\times\frac{1}{8}\text{cm}$$

$$L.C_{vs}=\frac{1}{40}\text{cm}=0.025\,\text{cm}\ .$$

For screw gauge,

$$L.C_{sg}=\frac{Pitch}{No.of\ division\,on\,circular\ scale}$$

If $pitch=2\times L.C_{vs}=\frac{1}{20}\text{cm}$.

$$L.C_{SG}=\frac{1/20}{100}=0.005\,\text{cm}\ .$$

For screw gauge,

If Linear scale $L.C=2\times\frac{1}{40}=\frac{1}{20}\text{cm}$.

$$\text{Pitch}=2\times\frac{1}{20}\times\frac{1}{10}\text{cm}.$$

$$L.C._{SG}=\frac{140}{100}=0.01\text{mm}.$$

Q.16 Sol. (C,D)

$$L = K[h]^a[C]^b[G]^c$$

Writing dimensional formula of each terms

$$L = \left[ML^2T^{-1}\right]^a\left[LT^{-1}\right]^b\left[M^{-1}L^3T^{-2}\right]^c$$

$$a - c = 0$$

$$2a + b + 3c = 1$$

$$-a - b - 2c = 0$$

Solving the equation, we get

$$a = \frac{1}{2}, c = \frac{1}{2}, b = \frac{-3}{2}$$

Hence, $L \propto \sqrt{h}$
$\propto \sqrt{G}$

Similarly, for M

$$M \propto (h)\frac{1}{2}, (c)\frac{1}{2}, (G) - \frac{1}{2}$$

Hence, A, C, D are correct.

Q.17 Sol. (B,D)

$$E_1 = \frac{1}{2}mw_1^2a^2 = \frac{b^2}{2m} = \frac{a^2}{2mn^4} \quad \left[\because \frac{a}{b} = n^2\right]$$

$$\Rightarrow \quad m^2 w_1^2 = \frac{1}{n^4}$$

and,

$$E_2 = \frac{1}{2} m w_2^2 R^2 = \frac{R^2}{2m}$$

$$\Rightarrow \quad E_2 = \frac{1}{2} m w_2^2 \quad \left(\frac{a}{n}\right)^2 = \frac{\left(\frac{a}{n}\right) \wedge 2}{2m} \left[\because \frac{a}{R} = n\right]$$

$$m^2 w_2^2 = 1$$

so, $\dfrac{w^2}{w_1^2} = n^2 \quad -(a)$

Now,

$$\frac{E_1}{w_1} = \frac{b^2}{2mw_1} = \frac{a^2}{2mn^4 \cdot w^1} = \frac{a^2 \cdot n^2}{2mn^4 \cdot w^2} -$$

$$- \text{From -}(a)$$

$$= \frac{E_2}{w_2}$$

Q.18 Sol. (D)

Initial Angular Momentum $= MR^2 w$

As net external torque on the system is zero,

Angular momentum remains conserved.

Final $\vec{L} = \left[MR^2 + \frac{4}{8}\left(\frac{3R}{5}\right)^2 + \frac{M}{8} x^2\right] \frac{8w}{9}$

$$\Rightarrow \quad x^2 = \frac{16}{25}R^2$$

$$\Rightarrow x = \frac{4}{5}R$$

Q.19 Sol.(A→R,T)

In nuclear fusion Positron , neutrino and Gamma rays are released apart from the fusion products. It is responsible for the energy production in the core of the stars.

$(B \rightarrow P, S, T)$

Fission of ^{235}U happens by absorption of thermal neutrons. Heavy water is used as modulators in controlled fission. Neutrons, neutrinos, beta rays and Gamma rays are released.

$(C \rightarrow P, Q, R)$

Beta decays happen in fission reaction, decays of $^{60}_{27}Co$ nucleus to $^{60}_{28}Ni$

(^{60}Co is a Synthetic radio isotope of cobalt which decays to ^{60}Ni)

$(D \rightarrow P, Q, R)$

β^- decay is followed by emission of antineutrino and not neutrino.

$\gamma -$ ray emission happens in fission, fusion, & decay of ^{60}Co .

Q.20 Sol.(A$\rightarrow$P,R,S,T)

$$U_1(x) = \frac{v_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$$

$$F_1(x) = -\frac{dU_1(x)}{dx}$$

$$= -\frac{v_0}{2}\times 2\left[1-\frac{x^2}{a^2}\right]\left[-\frac{2x^2}{a^2}\right]$$

$$= 2\frac{U_0}{G^2}.x.\left(1-\frac{x^2}{a^2}\right)$$

$$F_1(x) = 0 \quad \Rightarrow x = \pm a.$$

Force towards $x=0$ in the region $|x| < a;\ f(x) = -k(x-a)$

Potential Energy at $(x) = -a$

$$U_1(-a) = 0.$$

So, It may oscillate about $x = -a.$

$(B \rightarrow Q, S)$

$$U_2(x) = \frac{U_0}{2}\left(\frac{x}{2}\right)^2$$

$$F_2(x) = -\frac{dU_2}{dx}$$

$$= -\frac{U_0}{2a^2}.(2x) = -\frac{U_0}{a^2}x.$$

$F_2(x)=0 \Rightarrow x=0$

Hence $F_2(x)=\frac{-U_0}{a^2}\chi=-kx$

So, it experience attractive force towards $x=0$ in $|x|<a$.

$(C \to P,R,S,T)$

$U_3(x)=\frac{U_0}{2}\left[\left(\frac{x}{z}\right)^2\right]\exp.\left[-\left(\frac{x}{z}\right)^2\right]$

$F_3(x)=\frac{-dU_3}{dx}$

$=\frac{-U_0}{2}\left[\frac{2x}{a^2}.\exp\left[-\left(\frac{x}{a}\right)\right]+\left(\frac{x}{a}\right)^2.\left[\frac{-2x}{a^2}\right]\left[\exp-\left(\frac{x}{a}\right)^2\right]\right]$

$=-U_0.\exp.\left[-\left(\frac{x}{a}\right)\right]\left[\frac{x}{a^2}-\frac{x^3}{a^4}\right]=-U_0c^{-\left(\frac{x}{a}\right)}\left(\frac{x}{a}\right)\left[1-\frac{x^2}{a^2}\right]$

For $F_3(x)=0 \Rightarrow \frac{x}{a^2}=\frac{x^3}{a^4} \Rightarrow x^2=a^2$

$\Rightarrow x=\pm a$

Particle can oscillate about $x='-a'$ with energy $\frac{U_0}{4}$ if its energy at $'x=-a'$ is use than or equal to $\frac{U_0}{4}$.

$U_3(-a)=\frac{U_0}{2e}<\frac{U_0}{4.}$

It experiences attractive force towards $x=0$

$(D \to P, R)$

$$U_4(k) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$$

$$F(x) = -\frac{\alpha u_4}{\alpha_x}$$

$$= -\frac{U_0}{2}\left[\frac{1}{a} - \frac{1}{3a^3} \times 3x^2\right]$$

$$= -\frac{U_0}{2a}\left[1 - \frac{x^2}{a^2}\right]$$

$$F_4(x) = 0 \Rightarrow x = \pm a$$

Potential energy at $x = -a$

$$V(-a) = -\frac{V_0}{3}$$

$$|v(-a)|.\frac{V_0}{4}$$

So it can't oscillate about $x = -a$ with $\frac{V_0}{4}$.

Now,

$$F_4(x) = \frac{-V_0}{2a}\left[1 - \frac{x^2}{a^2}\right]$$

For $|x| < a4 \Rightarrow F_4(x) = \text{tve}$

So, force is acting along the same direction throughout $|x| < a$.