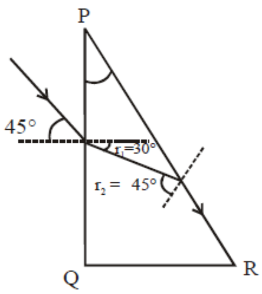
JEE ADVANCED -2016

PHYSICS

1. Sol. (A)



$$1\sin 45° = \sqrt{2}\sin r_1$$

$$r_2 - r_1 = \theta$$

$$\theta = 45° - 30°$$

$$\Rightarrow \theta = 15°$$

2. Sol. (B)

$$KE_{\max} = \frac{hC}{\lambda} - \phi$$

$$eV_s = \frac{hC}{\lambda} - \phi$$

$$1.6\times10^{-19}\times2 = \frac{h\times3\times10^{8}}{3000\times10^{-10}} - \phi \quad \ldots(\text{i})$$

$$1.6\times10^{-19}\times1 = \frac{h\times3\times10^{8}}{4000\times10^{-10}} - \phi \quad \ldots(\text{ii})$$

$$\text{From}\,(\text{ii})\,\phi = \frac{h\times 3\times 10^{8}}{4000\times 10^{-10}}1.6\times 10^{-19}$$

$$1.6\times 10^{-19}\times 2 = \frac{h\times 3\times 10^{8}}{3000\times 10^{-10}} - \frac{h\times 3\times 10^{8}}{4000\times 10^{-10}} + 1.6\times 10^{-19}$$

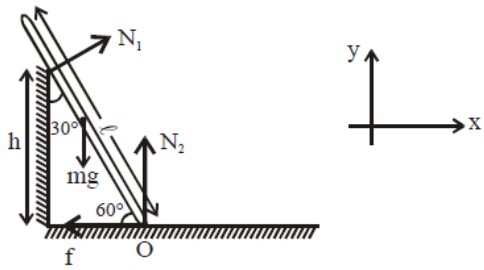
$$1.6\times 10^{-19} = \frac{h\times 3\times 10^{8}}{10^{-7}}\left(\frac{1}{3}-\frac{1}{4}\right) = \frac{h\times 3\times 10^{8}}{10^{-7}}\left[\frac{4-3}{12}\right]$$

$$1.6\times 10^{-19} = \frac{h\times 310^{8}}{10^{-7}}\times\frac{1}{12}$$

$$1.6\times 4\frac{10^{-19}\times 10^{-7}}{10^{8}} = h$$

$$6.4\times 10^{-34}\,Js = h$$

3. Sol. (D)



Force equation in x-direction ,

$$N_1\cos 30^\circ - f = 0 \quad \ldots(\text{i})$$

Force equation in y-direction ,

$$N_1\sin 30^\circ + N_2 - mg = 0 \quad \ldots(\text{ii})$$

Torque equation about O,

$$mg\frac{\ell}{2}\cos 60° - N_1\frac{h}{\cos 30°} = 0 \quad \ldots(\text{iii})$$

Also, given $N_1 = N_2 \quad \ldots(\text{iv})$

[Note taking reaction from floor as normal reaction only]

solving (i), (ii), (iii) & (iv) we have

$$\frac{h}{\ell} = \frac{3\sqrt{3}}{16} \;\&\; f = \frac{16\sqrt{3}}{3}$$

4. Sol. (B)

$$3000 - P = (120\times1)(4.2\times10^3)\frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{20}{60\times60\times3}$$

$$P = 2067\text{W}$$

5 Sol. (A)

This is the problem of RC circuit where the product RC is a constant.

So due to leakage current, charge & current density will exponentially decay & will become zero at infinite time. So correct answer is (A) for any small element

Resistance $R = \dfrac{dr}{\sigma(2\pi r\ell)}$

Capacitance $C = \dfrac{\in 2\pi r\ell}{dr}$

Product $\text{R}\times C = \dfrac{\in}{\sigma} = \text{constant}$

$q = q_0 e^{-\left(\frac{t\sigma}{\in}\right)}$

$$I = \frac{dq}{dt} = \frac{q_0\sigma}{\in} e^{-\left(\frac{t\sigma}{\in}\right)}$$

$$\text{Current density} = \frac{I}{A} = \frac{q_0 \frac{\sigma}{\in} e^{-\frac{t\sigma}{\in}}}{2\pi r\ell}$$

$$j \propto e^{-\frac{t\sigma}{\in}}$$

6. Sol. (A, B, D)

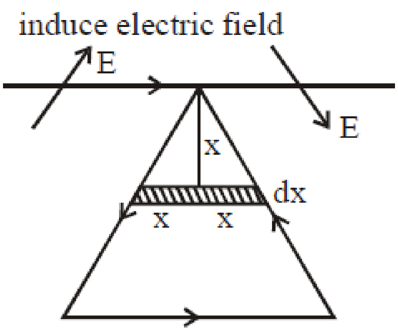
For parallel slab

$n_1 \sin\theta_i = n_2 \sin\theta_f$

And ℓ depends on refractive angle in slab

$\therefore \ell$ depends on refractive index of slab and independent of n_2

7. Sol. (B, D)



by direction of induced electric field, current in wire is in same direction of current along the hypotenuse.

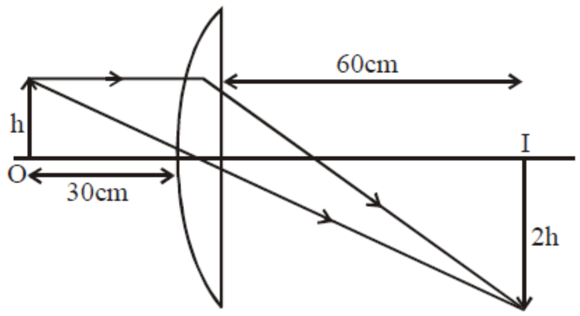
Flux through triangle if wire have current $i = \int_0^{0.1} \left(\frac{\mu_0 i}{2\pi x}\right)(2x dx) = \frac{\mu_0 i}{10\pi}$

$\Rightarrow$ Mutual inductance $= \frac{\mu_0}{10\pi}$

Induced emf in wire $= \frac{\mu_0}{10\pi}\frac{di}{dt} = \frac{\mu_0}{10\pi} \times 10 = \frac{\mu_0}{\pi}$

8. Sol. (A, D)

For lens

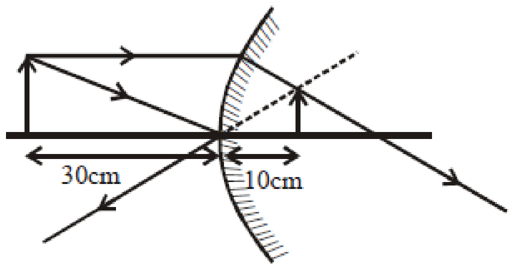


$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{60} - \frac{1}{(-30)} = \frac{1}{f} \Rightarrow f = 20\,\text{cm} \quad \ldots(\text{i})$

Also $\frac{1}{f} = (n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{(n-1)}{R} \quad (\ldots\text{ii})$

For reflection from convex mirror (curved surface)



$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{2}{R}$$

$$\frac{1}{+10}+\frac{1}{-30}=\frac{1}{f}=\frac{2}{R} \quad \ldots(\text{iii})$$

$R = 30\,\text{cm}$

from (i), (ii) & (iii)

$n = 2.5$.

faint image erect & virtual

9. Sol. (C,D)

Because of non-uniform evaporation at different section, area of cross-section would be different at different sections.

Region of highest evaporation rate would have rapidly reduced area and would become break up cross-section.

Resistance of the wire as whole increases with time.

Overall resistance increases hence power decreases. At break up junction temperature would be highest, thus light of highest band frequency would be emitted at those cross-section.

10. Sol. (B, D)

We know,

$$F=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \quad \Rightarrow \frac{q^2}{\varepsilon_0}=\left(Fr^2\right)4\pi$$

So, dimension

$$\frac{q^2}{\varepsilon_0} = \dim\left(Fr^2\right) = MLT^{-2} \times L^2 ML^3T^{-2}$$

Similarly; $E = \frac{3}{2}K_BT \Rightarrow \dim\left(K_BT\right) = \dim\left(Energy\right) = ML^2T^{-2}$

(A) $\sqrt{\frac{nq^2}{\varepsilon k_B T}} = \sqrt{\frac{L^{-3} \times ML^3T^{-2}}{ML^2T^{-2}}} = \frac{1}{L}$

(B) $\sqrt{\frac{(E) \times \text{vol}}{Fr^2}} = \sqrt{\frac{ML^2T^{-2} \times L^3}{MLT^{-2} \times L^2}} = L$

(C) $\sqrt{\frac{Fr^2(\text{vol})^{2/3}}{(K\varepsilon)}} = \sqrt{\frac{MLT^{-2} \times L^2 \times L^2}{ML^2T^{-2}}} = \sqrt{L^3} = L^{3/2}$

(D) $\sqrt{\frac{Fr^2(\text{vol})^{1/3}}{\text{Energy}}} = \sqrt{\frac{MLT^{-2}L^2 \times L}{ML^2T^{-2}}} = L$

$$\left[\therefore \text{dimension } n = \dim\left(\frac{1}{\text{vol}}\right) = L^{-3}\right]$$

11. Sol (A, B, D)

As radius $r \propto \frac{n^2}{z}$

$$\Rightarrow \frac{\Delta r}{r} = \frac{\left(\frac{n+1}{z}\right)^2 - \left(\frac{n}{z}\right)^2}{\left(\frac{n}{z}\right)^2} = \frac{2n+1}{n^2} \approx \frac{2}{n} \propto \frac{1}{n}$$

as energy $E \propto \frac{z^2}{n^2}$

$$\Rightarrow \frac{\Delta E}{E} = \frac{\frac{z^2}{n^2} - \frac{z^2}{(n+1)^2}}{\frac{z^2}{(n+1)^2}} = \frac{(n+1)^2 - n^2}{n^2.(n+1)^2}.(n+1)^2$$

$$\Rightarrow \frac{\Delta E}{E} = \frac{2n+1}{n^2} \simeq \frac{2n}{n^2} \propto \frac{1}{n}$$

as angular momentum $L = \frac{nh}{2\pi}$

$$\Rightarrow \frac{\Delta L}{L} = \frac{\frac{(n+1)h}{2\pi} - \frac{nh}{2\pi}}{\frac{nh}{2\pi}} = \frac{1}{n} \propto \frac{1}{n}$$

12. Sol (A, B, D)

$$\vec{r} = \alpha t^3 \hat{i} + \beta t^2 \hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 3\alpha t^2 \hat{i} + 2\beta t \hat{j}$$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = 6\alpha t \hat{i} + 2\beta \hat{j}$$

At $t = 1$

$$(\text{A})\, \vec{v} = \times 3 \frac{10}{3} \times 1\hat{i} + 2 \times J \times 1\hat{j}$$

$$= 10\hat{i} + 10\hat{j}$$

$(\text{B})\vec{L} = \vec{r} \times \vec{p}$

$$= \left(\frac{10}{3} \times 1\hat{i} + 5 \times 1\hat{j}\right) \times 0.1\left(10\hat{i} + 10\hat{j}\right)$$

$$= -\frac{5}{3}\hat{k}$$

$$(\text{C})\vec{F} = m \times \left(6 \times \frac{10}{3} \times 1\hat{i} + 2 \times 5\hat{j}\right) = 2\hat{i} + \hat{j}$$

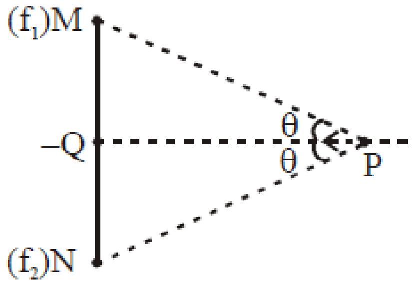
$(\text{D})\vec{\tau} = r \times \vec{F}$

$$= \left(\frac{10}{3}\hat{i} + 5\hat{j}\right) \times \left(2\hat{i} + \hat{j}\right)$$

$$= +\frac{10}{3}\hat{k} + 10\left(-\hat{k}\right)$$

$$= -\frac{20}{3}\hat{k}$$

13. Sol. (A, C, D)



$$f_M = \frac{C + V\cos\theta}{C} f_1$$

$$f_N = \frac{C + V\cos\theta}{C} f_2$$

$\Delta f = f_N - f_M$

$= \frac{C + V\cos\theta}{C}(f_2 - f_1)$

$\frac{d(\Delta f)}{dt} = -\frac{V}{C}(f_2 - f_1)\sin\theta\frac{d\theta}{dt}$

$\therefore \& \frac{d(\Delta f)}{dt}$ is maximum when $\theta = 90°$

$[\therefore C$ is correct$]$

$v_P = \left(1 + \frac{V}{C}\cos\theta\right)\Delta f$

$v_Q = \Delta f$

$v_R = \left(1 - \frac{V}{C}\cos\theta\right)\Delta f$

$\therefore v_P + v_r = 2v_Q$

14. Sol (3)

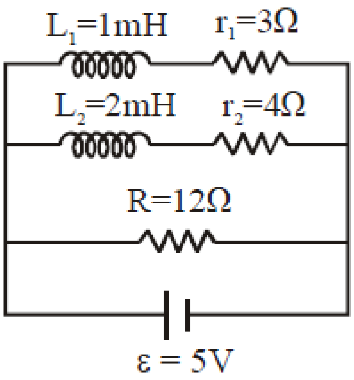
$V_T \propto \frac{r^2[d_m - d_L]}{n}$

$\frac{V_{TP}}{V_{TQ}} = \left(\frac{r_P}{r_Q}\right)^2 \times \frac{n_{L_2}}{n_{L_1}} \times \left[\frac{d_m - d_{L_1}}{d_m - d_{L_2}}\right]$

$\frac{V_{TP}}{V_{TQ}} = \left(\frac{2}{1}\right)^2 \times \frac{2}{3} \times \left[\frac{8 - 0.8}{8 - 1.6}\right]$

$\frac{V_{TP}}{V_{TQ}} = 3$

15. Sol (8)



$$I_{max} = \frac{\varepsilon}{R} = \frac{5}{12} A \left(\text{Initially at } t = 0\right)$$

$$I_{min} = \frac{\varepsilon}{R_{eq}} = \varepsilon\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{R}\right) \text{(finally in steady state)}$$

$$= 5\left(\frac{1}{3} + \frac{1}{4} + \frac{1}{12}\right)$$

$$= \frac{10}{3} A$$

$$\frac{I_{max}}{I_{min}} = 8$$

16. Sol. (9)

$${}^{12}_{5}B \rightarrow {}^{12}_{6}C + {}^{0}_{-1}e + \bar{v}$$

Mass defect $= (12.014 - 12)u$

$\therefore$ Released energy $= 13.041 MeV$

Energy used for excitation of ${}^{12}_{6}C = 4.041 Mev$

$\therefore$ Energy converted to KE of electron

=13.041-4.041=9MeV

17. Sol. (6)

$$\frac{hc}{\lambda}=\frac{12370}{970}$$

$$-13.6+12.7=-\frac{13.6}{n^2}$$

$$n^2=16$$

$$n=4$$

Number of lines $= {}^{n}C_2 = 6$

18. Sol. (9)

$P = eA\sigma T^4$ where T is in kelvin

$$\log_2 \frac{eA\sigma(487+273)^4}{P_0}=1 \quad \ldots(\text{i})$$

$$\log_2 \frac{eA\sigma(2767+273)^4}{P_0}=x \quad \ldots(\text{ii})$$

$(\text{ii})-(\text{i})$

$$\log_2\left(\frac{3040}{760}\right)^4 = x-1$$

$\therefore x=9$