JEE ADVANCED-2018 (PAPER-1)

PHYSICS

1. **Ans: (B,C)**

Sol.

$$U=\frac{kr^2}{2}\Rightarrow \vec{F}=-kr$$
$$\frac{mv^2}{R}=+kR\Rightarrow v=\sqrt{\frac{k}{m}}R$$
$$\text{Angular momentum } L=mvr=\sqrt{mk}R^2$$

2. **Ans: (A,C)**

Sol.

$$m=1\text{kg}$$
$$m\vec{a}=\vec{F}=t\hat{i}+\hat{j}$$
$$(1)\vec{a}=t\hat{i}+\hat{j}$$
$$\frac{d\vec{v}}{dt}=t\hat{i}+\hat{j}$$

$$\vec{v}=\frac{t^2}{2}\hat{i}+t\hat{j}=\frac{1}{2}\hat{i}+\hat{j}$$
$$\vec{s}=\frac{t^3}{6}\hat{i}+\frac{t^2}{2}\hat{j}$$
$$\vec{s}(t=1)=\frac{1}{6}\hat{i}+\frac{1}{2}\hat{j}$$
$$\vec{\tau}=\vec{r}\times\vec{F}$$

$$=\left[\frac{1}{6}\hat{i}+\frac{1}{2}\hat{j}\right]\times\left[\hat{i}+\hat{j}\right]$$
$$=\frac{1}{6}\left(\hat{i}\times\hat{j}\right)+\frac{1}{2}\left(\hat{j}\times\hat{i}\right)$$

$$=\frac{1}{6}\hat{k}-\frac{1}{2}\hat{k}=\frac{-2}{6}\hat{k}=-\frac{1}{3}\hat{k}$$

$$|\tau|=\frac{1}{3}\text{ at } t=1\text{sec}$$

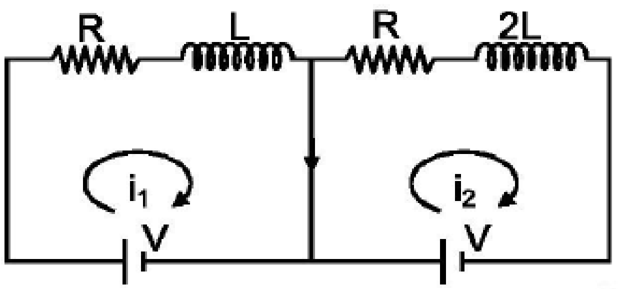
3. **Ans: (A,C)**

Sol. $h=\dfrac{2\sigma\cos\theta}{\rho g r}$

When lift is going up with constant acceleration $h=\dfrac{2\sigma\cos\theta}{\rho(g+a)r}$

4. **Ans: (B, D)**

Sol.



$$\Delta i\,(\text{current in middle wire})=i_1-i_2$$

$$=\frac{V}{R}\left(1-e^{\frac{Rt}{L}}\right)-\frac{V}{R}\left(1-e^{\frac{Rt}{2L}}\right)$$

$$=\frac{V}{R}\left(e^{\frac{Rt}{2L}}-e^{\frac{Rt}{L}}\right)$$

$$\text{For } i_{max}:\frac{d}{dt}(\Delta i)=0$$

$$\Rightarrow\quad \frac{V}{R}\left(-\frac{R}{L}e^{-\frac{Rt}{L}}+\frac{R}{2L}e^{\frac{Rt}{2L}}\right)=0\quad\Rightarrow\quad e^{\frac{Rt}{L}}=\frac{1}{2}e^{-\frac{Rt}{2L}}$$

$$\Rightarrow \quad e^{\frac{Rt}{2L}} = \frac{1}{2} \,\&\, t = \frac{2L}{R}\ell n2$$

$$\Rightarrow \quad \text{and } i_{\max} = \frac{V}{4R}$$

5. Ans: (A,B,D)

Sol.

Magnetic field due to ring at origin

$$= \frac{\mu_0 \times I \times R^2}{2 \times 8R^3}\left(-\hat{K}\right) = \frac{\mu_0 I}{16R}\left(-\hat{K}\right)$$

Magnetic field at origin due to wires

$$= \left(\frac{\mu_0 I_1}{2\pi R} - \frac{\mu_0 I_2}{2\pi R}\right)\hat{K}$$

Here I_1 and I_2 will be substituted with sign

If $I_1 = I_2$ then $\vec{B}_0 = \frac{\mu_0 I}{16R}\left(-\hat{K}\right)$

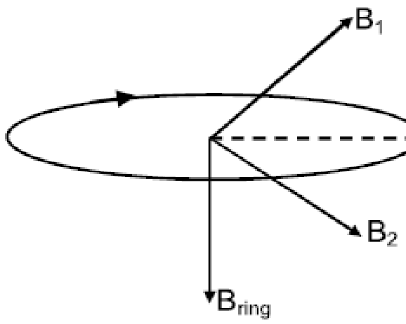
It can be zero

If $I_1 < 0, I_2 > 0$

then $\vec{B}_0 = -\left[\frac{\mu_0\left(I_1 + I_2\right)}{2\pi R} + \frac{\mu J}{16R}\right]\hat{K}$

It cannot be zero

(D)



$B_1 = B_2$

magnetic field along z-axis is only due to ring

$B = \frac{\mu_0 i}{2R}$ in $-z$ direction

6. **Ans: (B,C,D)**

Sol. Process II is isothermal expansion

$\Rightarrow$ heat is positive

Process IV is isothermal compression

$\Rightarrow$ heat is negative

: 0 In all other cases.

7. **Ans:** 2.00

Sol.

$\vec{A} = a\hat{i}$ and $\vec{B} = a\cos\omega t\hat{i} + a\sin\omega t\hat{j}$

$|\vec{A}+\vec{B}| = \sqrt{3}|\vec{A}-\vec{B}|$

$\sqrt{(a+a\cos\omega t)^2 + (a\sin\omega t)^2} = \sqrt{3}\sqrt{(a-a\cos\omega t)^2 + (a\sin\omega t)^2}$

$$\Rightarrow 2\cos\frac{\omega t}{2} = \pm\sqrt{3}\times 2\sin\frac{\omega t}{2}$$

$$\tan\frac{\omega t}{2} = \pm\frac{1}{\sqrt{3}}$$

$$\frac{\omega t}{2} = n\pi \pm \frac{\pi}{6}$$

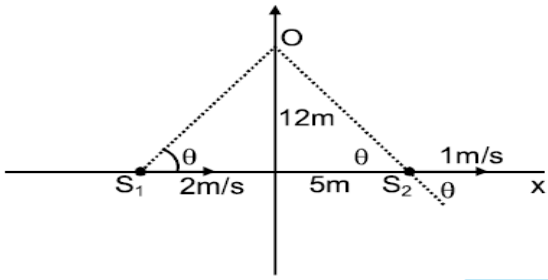
$$\frac{\pi}{12}t = n\pi \pm \frac{\pi}{6}$$

$$t = (12n \pm 2)s$$

$$= 2s, 10s, 14s\ldots$$

8. **Ans:** 5.00

Sol.



$$f_1 = f\left(\frac{v}{v-2\cos\theta}\right) \quad f_2 = f\left(\frac{v}{v+\cos\theta}\right)$$

$$\text{Beat frequency} = f_1 - f_2$$

$$= \frac{fvx3\cos\theta}{(v-2\cos\theta)(v+2\cos\theta)} \approx \frac{2f\cos\theta}{v} = 5$$

9. **Ans:** 0.75

Sol.

$$a_{cm} = \frac{g\sin\theta}{1+\frac{K^2}{R^2}} = \frac{\frac{\sqrt{3}g}{2}}{1+\frac{K^2}{R^2}}$$

$$a_{\text{ring}} = \frac{\frac{\sqrt{3}g}{2}}{1+1} = \frac{\sqrt{3}g}{4}$$

$$a_{\text{disc}} = \frac{\frac{\sqrt{3}g}{2}}{1+\frac{1}{2}} = \frac{g}{\sqrt{3}}$$

$$t_{\text{ring}} = \sqrt{\frac{2\cdot\frac{2h}{\sqrt{3}}}{\frac{\sqrt{3}g}{4}}} = \sqrt{\frac{4h}{\sqrt{3}}\frac{4}{\sqrt{3}g}} = \sqrt{\frac{16h}{3g}}$$

$$t_{\text{disc}} = \sqrt{\frac{2\cdot\frac{2h}{\sqrt{3}}}{\frac{g}{\sqrt{3}}}} = \sqrt{\frac{4h}{\sqrt{3}}\frac{\sqrt{3}}{g}} = \sqrt{\frac{4h}{g}}$$

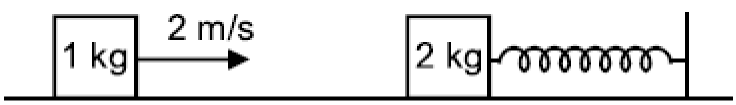
$$\Delta t = \sqrt{\frac{16h}{3g}} - \sqrt{\frac{4h}{g}} = \sqrt{\frac{h}{g}}\left(\sqrt{\frac{16}{3}} - \sqrt{4}\right) = \sqrt{\frac{h}{g}}\left(\frac{4}{\sqrt{3}} - 2\right)$$

$$= 2\sqrt{\frac{h}{g}}\left(\frac{2-\sqrt{3}}{\sqrt{3}}\right) = 2\sqrt{\frac{h}{3}} = 1$$

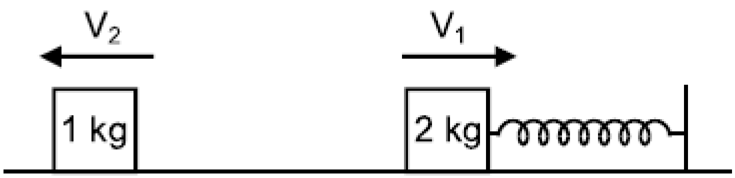
$$= \sqrt{\frac{h}{3}} = \frac{1}{2} \quad \Rightarrow \quad h = \frac{3}{4} = 0.75\text{m}$$

10. **Ans:** 2.09

Sol. Just Before Collision



Just After Collision



$$V_1 + V_2 = 2 \quad \ldots(1)$$
$$2V_1 - V_2 = 4 \quad \ldots(2)$$

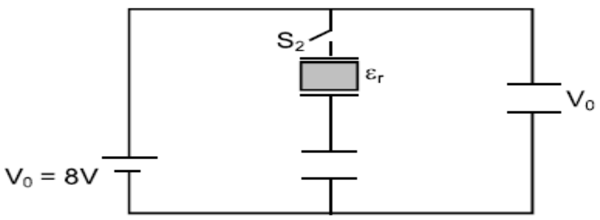
$$3V_1 = 4 \Rightarrow V_1 = \frac{4}{3} \quad V_2 = 2 - \frac{4}{3} = \frac{2}{3}\,\text{m/s}$$

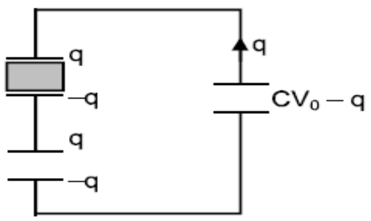
$$\Delta t = \frac{T}{2} = \frac{2\pi}{2}\sqrt{\frac{2}{2}} = \pi\,\text{sec}.$$

$$\text{Distance} = \pi V_2 = \left(\frac{2}{3}\right)\left(\frac{22}{7}\right)\text{m} = \frac{44}{21}\,\text{m} = 2.09$$

11. **Ans:** 1.50

Sol.





$$\frac{CV_0 - q}{C} - \frac{q}{\varepsilon_r C} - \frac{q}{C} = 0 \quad \frac{CV_0 - q}{C} = 5$$

$$5 = \frac{q}{C}\left(1 + \frac{1}{\varepsilon_r}\right) \qquad 8C - q = 5C$$

$$= 3\left(1 + \frac{1}{\varepsilon_r}\right) \qquad \Rightarrow q = 3C$$

$$\frac{5}{3} = 1 + \frac{1}{\varepsilon_r} \Rightarrow \frac{1}{\varepsilon_r} = \frac{2}{3}$$

$$\varepsilon_r = \frac{3}{2} = 1.5$$

12. **Ans:** 2.00

Sol. If average speed is considered along x-axis

$$R_1 = \frac{mv_0}{qB_1}, R_2 = \frac{mv_0}{qB_2} = \frac{mv_0}{4qB_1}$$

$$R_1 > R_2$$

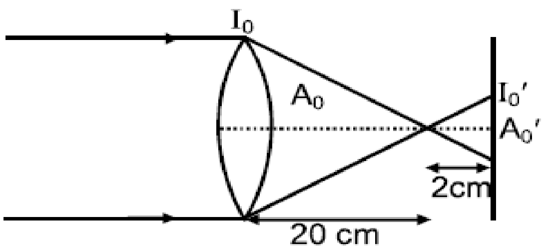
$$\text{distance along } x\text{-axis } \Delta x = 2\left(R_1 + R_2\right) = \frac{5mv_0}{2qB_1}$$

$$\text{Total time} = \frac{\pi m}{qB_1} + \frac{\pi m}{qB_2} = \frac{\pi m}{qB_1} + \frac{\pi m}{4qB_1} = \frac{5\pi m}{4qB_1}$$

$$\text{Magnitude of average speed} = \frac{\frac{5mv_0}{2qB_1}}{\frac{5\pi m}{4qB_1}} = 2\text{m/s}$$

13. **Ans:** 130.00

Sol.



$$\frac{A_0'}{A_0} = \left(\frac{2}{20}\right)^2 = \frac{1}{100} \Rightarrow A_0' = \frac{A_0}{100} \Rightarrow I_0' = \frac{I_0 A_0}{\frac{A_0}{100}} = 100 I_0 = 130\,\text{kW/m}^2$$

14. **Ans:** 4.00

Sol. since rate of heat flow is same, we can say

$$\frac{300-200}{R_1}=\frac{200-100}{R_2}$$

$$R_1=R_2 \quad \Rightarrow \frac{L_1}{K_1A_1}=\frac{L_2}{K_2A_2}$$

$$\Rightarrow \frac{K_1}{K_2}=\frac{A_2}{A_1}=4$$

15. **Ans :** (C)

Sol. In terms of dimension

$$qE=QvB$$

$$E=vB$$

$$[E]=[B][LT^{-1}]$$

16. **Ans :** (D)

Sol.

$$C=\frac{1}{\sqrt{\mu_0 \in_0}}$$

$$C^2=\frac{1}{\mu_0 \in_0}$$

$$\mu_0=\in_0 \cdot C^2$$

$$[\mu_0]=[\in_0]^{-1}L^{-2}T^2$$

17. **Ans : (B)**

Sol.

$$\frac{dr}{da}=\frac{(1+a)(-1)-(1-a)}{(1+a)^2}$$

$$\Rightarrow \quad dr = \frac{-2dr}{(1+a)^2}$$

18. **Ans : (C)**

Sol.

$$N = N_0 e^{-\lambda t}$$
$$\ell nN = \ell nN_0 - \lambda t$$

different w.r.t. λ

$$\frac{1}{N}\frac{dN}{d\lambda} = 0 - t$$
$$d\lambda = \frac{dN}{Nt} = \frac{40}{2000 \times 1} = 0.02$$