

AIEEE-2002

MATHEMATICS

1. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having α/β and β/α as its roots is

- (a) $3x^2 - 19x + 3 = 0$
- (b) $3x^2 - 19x - 3 = 0$
- (c) $3x^2 - 19x - 3 = 0$
- (d) $x^2 - 5x + 3 = 0$

2. If $y = (x + \sqrt{1+x^2})^2$, then $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is

- (a) $n^2 y$
- (b) $-n^2 y$
- (c) $-y$
- (d) $2x^2 y$

3. If $1, \log_9(3^{1-x} + 2), \log_3(4 \cdot 3^x - 1)$ are in *A.P.* then x equals

- (a) $\log_3 4$
- (b) $1 + \log_3 4$
- (c) $1 - \log_4 3$
- (d) $\log_4 3$

4. A problem in mathematics is given to three students A, B, C and their respective probability of solving the problem is $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. Probability that the problem is solved is

(a) $\frac{3}{4}$

(b) $\frac{1}{2}$

(c) $\frac{2}{3}$

(d) $\frac{1}{3}$

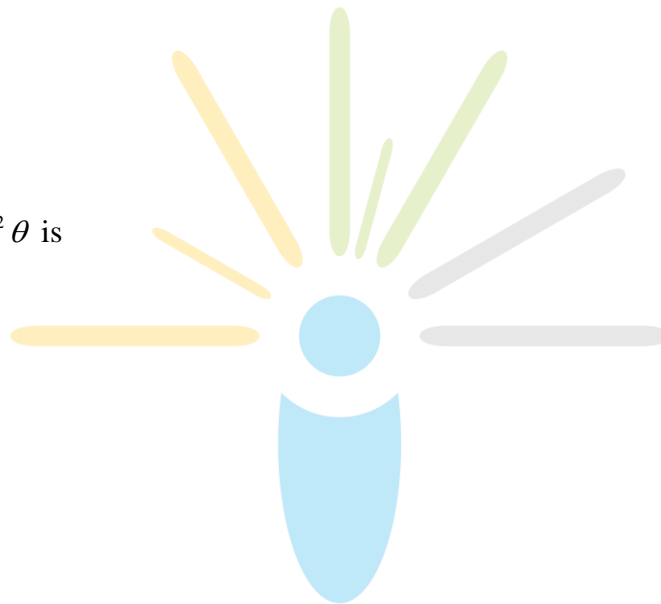
5. The period of $\sin^2 \theta$ is

(a) π^2

(b) π

(c) 2π

(d) $\pi/2$



6. l, m, n are the $p^{\text{th}}, q^{\text{th}}$ and r^{th} term of a $G.P.$ all positive, then $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals

(a) -1

(b) 2

(c) 1

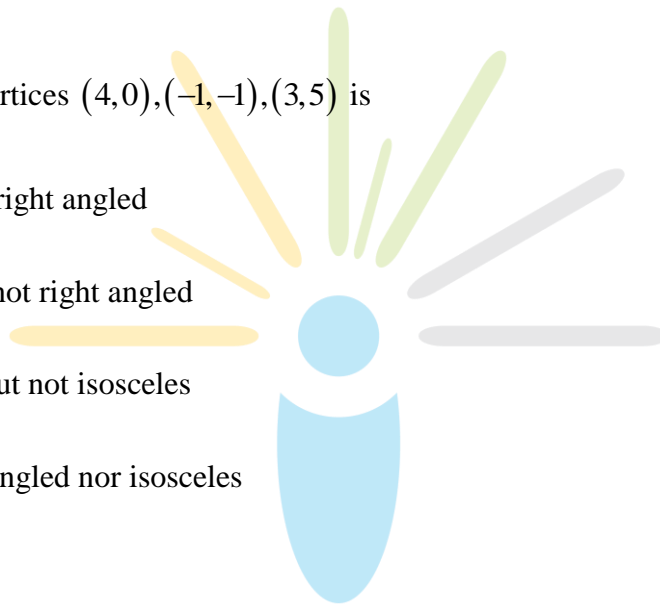
(d) 0

7. $\lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos 2x}}{\sqrt{2x}}$ is

- (a) 1
- (b) -1
- (c) zero
- (d) does not exist

8. A triangle with vertices $(4,0), (-1,-1), (3,5)$ is

- (a) isosceles and right angled
- (b) isosceles but not right angled
- (c) right angled but not isosceles
- (d) neither right angled nor isosceles



9. In a class of 100 students there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class is 72, then what is the average of the girls?

- (a) 73
- (b) 65
- (c) 68
- (d) 74

10. $\cot^{-1}(\sqrt{\cos \alpha}) = \tan^{-1}(\sqrt{\cos \alpha}) = x$, then $\sin x =$

(a) $\tan^2\left(\frac{\alpha}{2}\right)$

(b) $\cot^2\left(\frac{\alpha}{2}\right)$

(c) $\tan \alpha$

(d) $\cot\left(\frac{\alpha}{2}\right)$

11. The order and degree of the differential equation $\left(1 + 3\frac{dy}{dx}\right)^{2/3} = 4\frac{d^3y}{dx^3}$ are

(a) $\left(1, \frac{2}{3}\right)$

(b) (3,1)

(c) (3,3)

(d) (1,2)

12. A plane which passes through the point (3, 2, 0) and the line $\frac{x-4}{1} = \frac{y-7}{5} = \frac{z-4}{4}$ is

(a) $x - y + z = 1$

(b) $x + y + z = 5$

(c) $x + 2y - z = 1$

(d) $2x - y + z = 5$

13. The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$

(a) $\frac{e^{-2x}}{4}$

(b) $\frac{e^{-2x}}{4} + cx + d$

(c) $\frac{1}{4}e^{-2x} + cx^2 + d$

(d) $\frac{1}{4}e^{-4x} + cx + d$

14. $\lim_{x \rightarrow \infty} \frac{x^2 + 5x + 3^{\frac{1}{x}}}{x^2 + x + 3}$

(a) e^4

(b) e^2

(c) e^3

(d) 1



15. The domain of $\sin^{-1}[\log_3(x/3)]$ is

(a) $[1,9]$

(b) $[-1,9]$

(c) $[-9,1]$

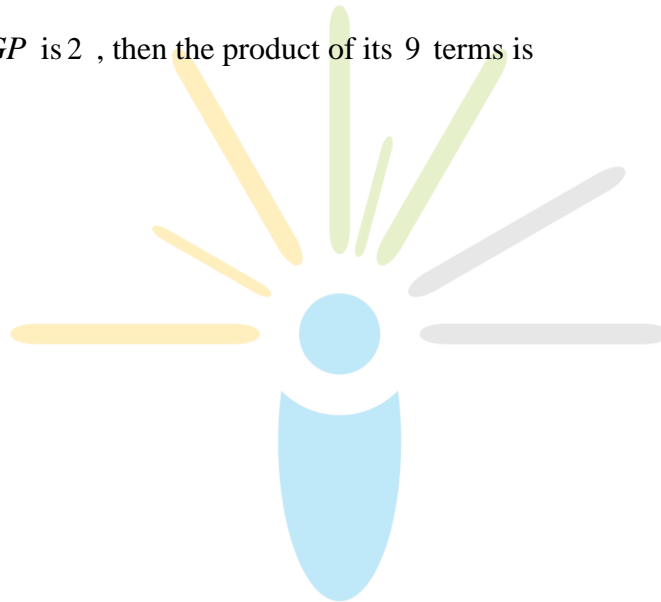
(d) $[-9,-1]$

16. The value of $2^{1/4}, 4^{1/8}, 8^{1/6} + K K \infty$ is

- (a) 1
- (b) 2
- (c) $3/2$
- (d) 4

17. Fifth term of a *GP* is 2 , then the product of its 9 terms is

- (a) 256
- (b) 512
- (c) 1024
- (d) none of these



18. $\int_0^{10\pi} |\sin x| dx$ is

- (a) 20
- (b) 8
- (c) 10
- (d) 18

19. $I_2 = \int_0^{\pi/4} \tan^n x \, dx$ then equals $\lim_{n \rightarrow \infty} n [I_n + I_{n-2}]$

- (a) $\frac{1}{2}$
- (b) 1
- (c) ∞
- (d) zero

20. $\int_0^{\sqrt{2}} [X^2] \, dx$ is

- (a) $2 - \sqrt{2}$
- (b) $2 + \sqrt{2}$
- (c) $\sqrt{2} - 1$
- (d) $\sqrt{2} - 2$



21. $\int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} \, dx$ is

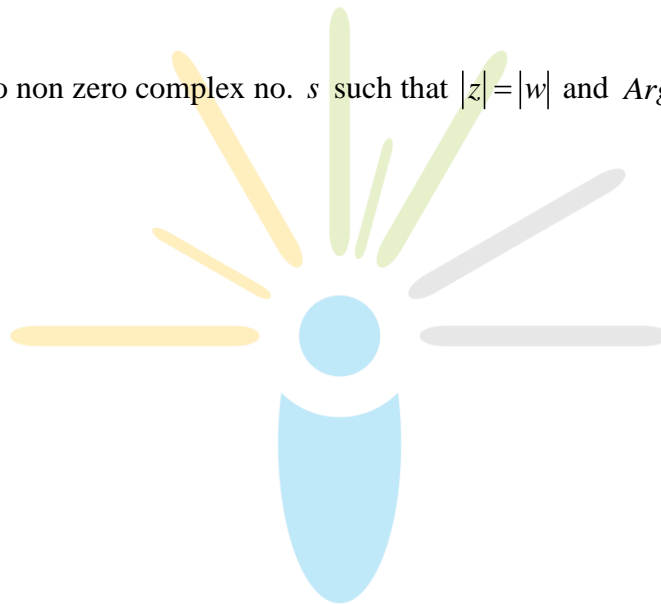
- (a) $\frac{\pi^2}{4}$
- (b) π^2
- (c) zero
- (d) $\frac{\pi}{2}$

22. Let $f(x) = 4$ and $f'(x) = 4$. Then $\lim_{x \rightarrow 2} \frac{xf(2) - 2f(x)}{x-2}$ is given by

- (a) 2
- (b) -2
- (c) -4
- (d) 3

23. z and w are two non zero complex no. s such that $|z| = |w|$ and $\text{Arg } z + \text{Arg } w = \pi$ z equals

- (a) \bar{w}
- (b) $-\bar{w}$
- (c) w
- (d) $-w$



24. If $|z-4| < |z-2|$, its solution is given by

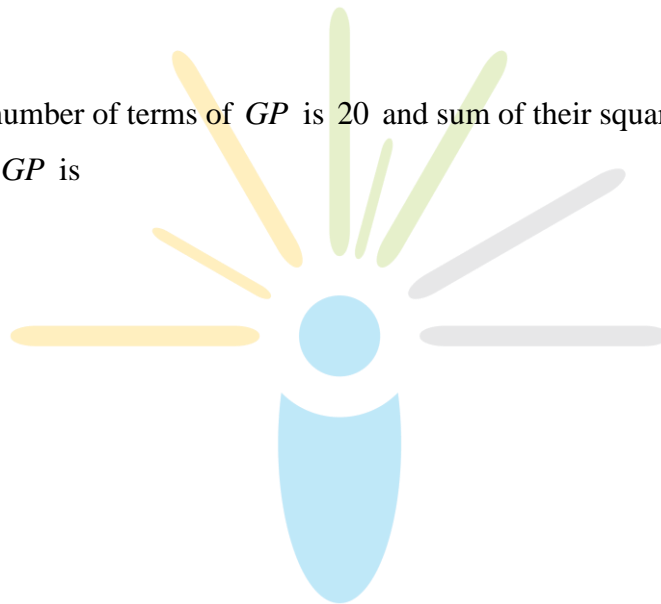
- (a) $\text{Re}(z) > 0$
- (b) $\text{Re}(z) < 0$
- (c) $\text{Re}(z) > 3$
- (d) $\text{Re}(z) > 2$

25. The locus of the centre of a circle which touches the circle $|z - z_1| = a$ and $|z - z_2| = b$ externally (z , z_1 and z_2 are complex numbers) will be

- (a) an ellipse
- (b) a hyperbola
- (c) a circle
- (d) none of these

26. Sum of infinite number of terms of GP is 20 and sum of their square is 100 .The common ratio of GP is

- (a) 5
- (b) $3/5$
- (c) $8/5$
- (d) $1/5$



27. $1^3 - 2^3 + 3^3 - 4^3 + K + 9^3 =$

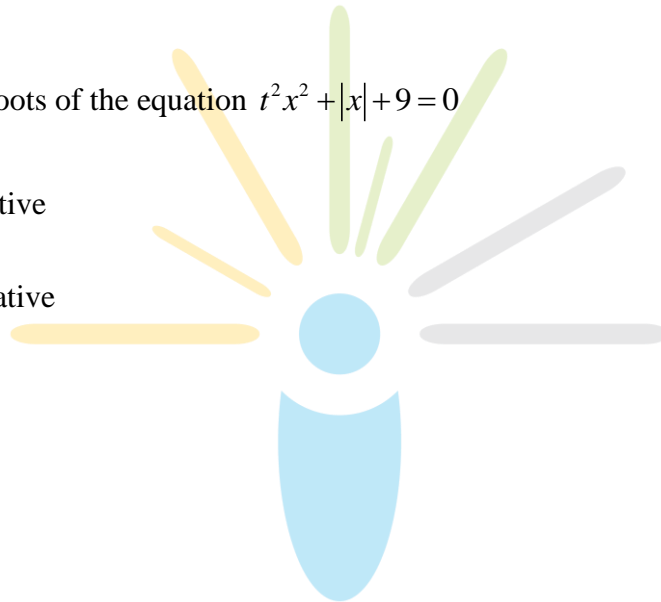
- (a) 425
- (b) -425
- (c) 475
- (d) -475

28. Difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then

- (a) $a + b + 4 = 0$
- (b) $a + b - 4 = 0$
- (c) $a - b - 4 = 0$
- (d) $a - b + 4 = 0$

29. Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$

- (a) is always positive
- (b) is always negative
- (c) does not exist
- (d) none of these



30. If p and q are the roots of the equation $x^2 + px + q = 0$, then

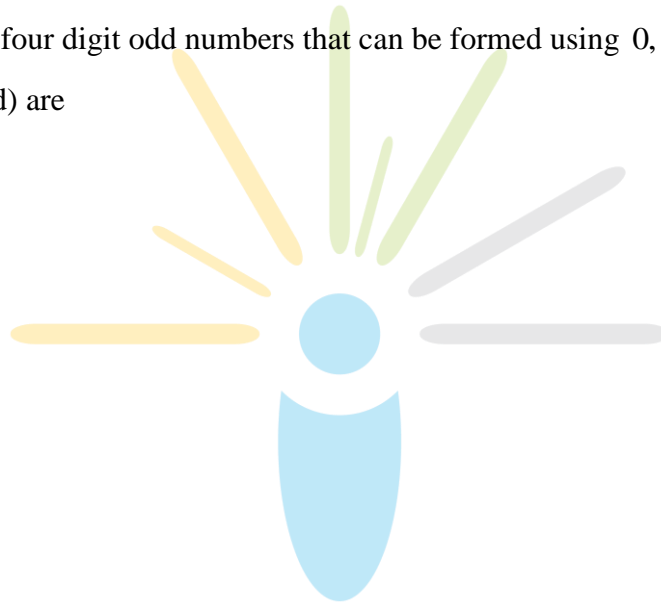
- (a) $p = 1, q = -2$
- (b) $p = 0, q = 1$
- (c) $p = -2, q = 0$
- (d) $p = -2, q = 1$

31. If a, b, c are distinct +ve real numbers and $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ is

- (a) less than 1
- (b) equal to 1
- (c) greater than 1
- (d) any real no.

32. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 (using repetition allowed) are

- (a) 216
- (b) 375
- (c) 400
- (d) 720



33. Number greater than 1000 but less than 4000 is formed using the digits 0, 1, 2, 3, 4 (repetition allowed) is

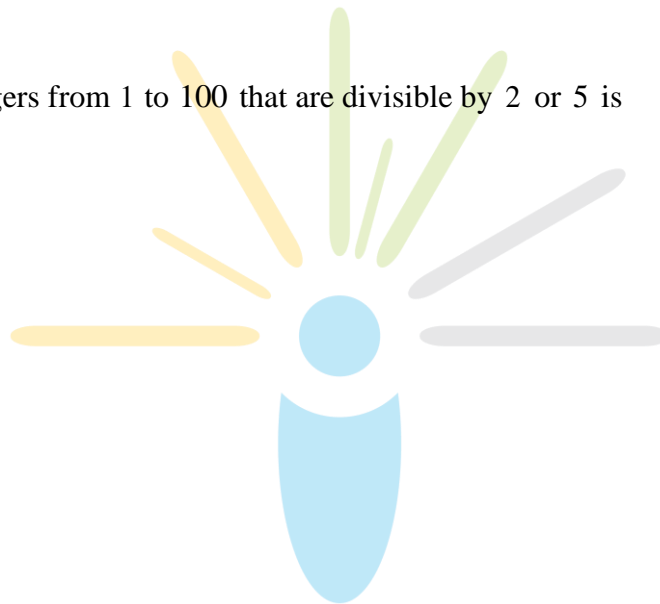
- (a) 125
- (b) 105
- (c) 375
- (d) 625

34. Five digit number divisible by 3 is formed using 0, 1, 2, 3, 4, 6 and 7 without repetition. Total number of such numbers are

- (a) 312
- (b) 3125
- (c) 120
- (d) 216

35. The sum of integers from 1 to 100 that are divisible by 2 or 5 is

- (a) 3000
- (b) 3050
- (c) 3600
- (d) 3250



36. The coefficients of x^p and x^q in the expansion of $(1+x)^{p+q}$ are

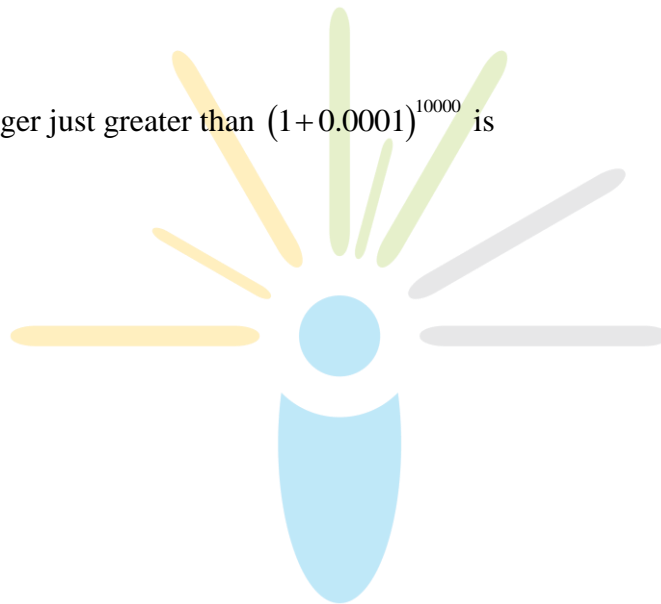
- (a) equal
- (b) equal with opposite signs
- (c) reciprocals of each other
- (d) none of these

37. If the sum of the coefficients in the expansion of $(a+b)^n$ is 4096 then the greatest coefficient in the expansion is

- (a) 1594
- (b) 792
- (c) 924
- (d) 2924

38. The positive integer just greater than $(1+0.0001)^{10000}$ is

- (a) 4
- (b) 5
- (c) 2
- (d) 3



39. r and n are positive integers $r > 1, n > 2$ and coefficient of $(r+2)^{\text{th}}$ term and $3r^{\text{th}}$ term in the expansion of $(1+x)^{2n}$ are equal, then n equals

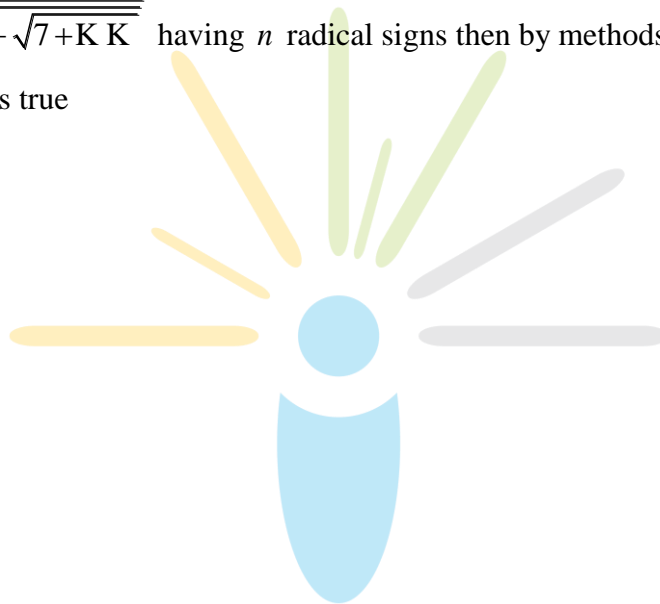
- (a) $3r$
- (b) $3r+1$
- (c) $2r$
- (d) $2r+1$

40. If $a > 0$ discriminant of $ax^2 + 2bx + c$ is -ve, then $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is

- (a) +ve
- (b) $(ac - b^2)(ax^2 + 2bx + c)$
- (c) -ve
- (d) 0

41. If $a_n = \sqrt{7 + \sqrt{7 + \sqrt{7 + \dots + K}}}$ having n radical signs then by methods of mathematical induction which is true

- (a) $a_n > 7 \forall n \geq 1$
- (b) $a_n > 4 \forall n \geq 1$
- (c) $a_n < 4 \forall n \geq 1$
- (d) $a_n < 3 \forall n \geq 1$



42. The sides of a triangle are $3x + 4y$, $4x + 37$ and $5x + 57$ where $x, y > 0$ then the triangle is

- (a) right angled
- (b) obtuse angled
- (c) equilateral
- (d) none of these

43. Locus of mid point of the portion between the axes of $x \cos \alpha + y \sin \alpha = p$ where p is constant is

(a) $x^2 + y^2 = \frac{4}{p^2}$

(b) $x^2 + y^2 = 4p^2$

(c) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{2}{p^2}$

(d) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$

44. If the pair of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ intersect on the y -axis then

(a) $2fgh = bg^2 + ch^2$

(b) $bg^2 \neq ch^2$

(c) $abc = 2fgh$

(d) none of these

45. The point of lines represented by $3ax^2 + 5xy + (a^2 - 2)y^2 = 0$ and perpendicular to each other for

(a) two values of a

(b) $\forall a$

(c) for one value of a

(d) for no values of a

46. If the chord $y = mx + 1$ of the circle $x^2 + y^2 = 1$ subtends an angle of measure 45° at the major segment of the circle then value of m is

- (a) $2 \pm \sqrt{2}$
- (b) $-2 \pm \sqrt{2}$
- (c) $-1 \pm \sqrt{2}$
- (d) none of these

47. The centres of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is

- (a) $4 \leq x^2 + y^2 \leq 64$
- (b) $x^2 + y^2 \leq 25$
- (c) $x^2 + y^2 \geq 25$
- (d) $3 \leq x^2 + y^2 \leq 9$

48. The centre of the circle passing through $(0, 0)$ and $(1, 0)$ and touching the circle $x^2 + y^2 = 9$ is

- (a) $\left(\frac{1}{2}, \frac{1}{2}\right)$
- (b) $\left(\frac{1}{2}, -\sqrt{2}\right)$
- (c) $\left(\frac{3}{2}, \frac{1}{2}\right)$
- (d) $\left(\frac{1}{2}, \frac{3}{2}\right)$

49. The equation of a circle with origin as a centre and passing through equilateral triangle whose median is of length $3a$ is

(a) $x^2 + y^2 = 9a^2$

(b) $x^2 + y^2 = 16a^2$

(c) $x^2 + y^2 = 4a^2$

(d) $x^2 + y^2 = a^2$

50. Two common tangents to the circle $x^2 + y^2 = 2a^2$ and parabola $y^2 = 8ax$ are

(a) $x = \pm(y + 2a)$

(b) $y = \pm(x + 2a)$

(c) $x = \pm(y + a)$

(d) $y = \pm(x + a)$

51. In a triangle with sides a, b, c , $r_1 > r_2 > r_3$ (which are the ex- radii) then

(a) $a > b > c$

(b) $a < b < c$

(c) $a > b$ and $b < c$

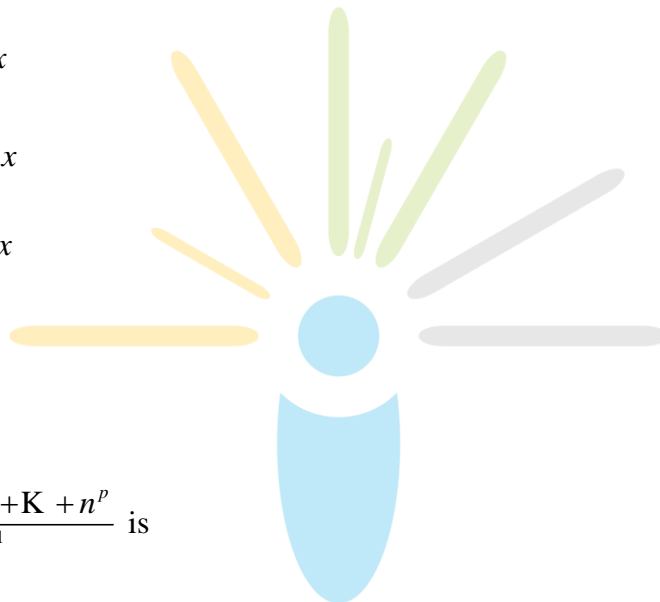
(d) $a < b$ and $b > c$

52. The number of solution of $\tan x + \sec x = 2 \cos x$ in $(0, 2\pi)$ is

- (a) 2
- (b) 3
- (c) 0
- (d) 1

53. Which one is not periodic

- (a) $|\sin 3x| + \sin^2 x$
- (b) $\cos \sqrt{x} + \cos^2 x$
- (c) $\cos 4x + \tan^2 x$
- (d) $\cos 2x + \sin x$



54. $\lim_{n \rightarrow \infty} \frac{1^p + 2^p + 3^p + \dots + n^p}{n^{p+1}}$ is

- (a) $\frac{1}{p+1}$
- (b) $\frac{1}{1-p}$
- (c) $\frac{1}{p} - \frac{1}{p-1}$
- (d) $\frac{1}{p+2}$

55. $\lim_{n \rightarrow 0} \frac{\log x^n - [x]}{[X]}$, $n \in N$ ($[x]$ denotes greatest integer less than or equal to x)

- (a) has value -1
- (b) has value 0
- (c) has value 1
- (d) does not exist

56. If $f(1) = 1, f'(1) = 2$, then $\lim_{x \rightarrow 1} \frac{\sqrt{f(x)} - 1}{\sqrt{x} - 1}$ is

- (a) 2
- (b) 4
- (c) 1
- (d) $1/2$

57. f is defined in $[-5, 5]$ as $f(x) = x$ if x is rational and $= -x$ if x is irrational. Then

- (a) $f(x)$ is continuous at every x , except $x = 0$
- (b) $f(x)$ is discontinuous at every x , except $x = 0$
- (c) $f(x)$ is continuous everywhere
- (d) $f(x)$ is discontinuous everywhere

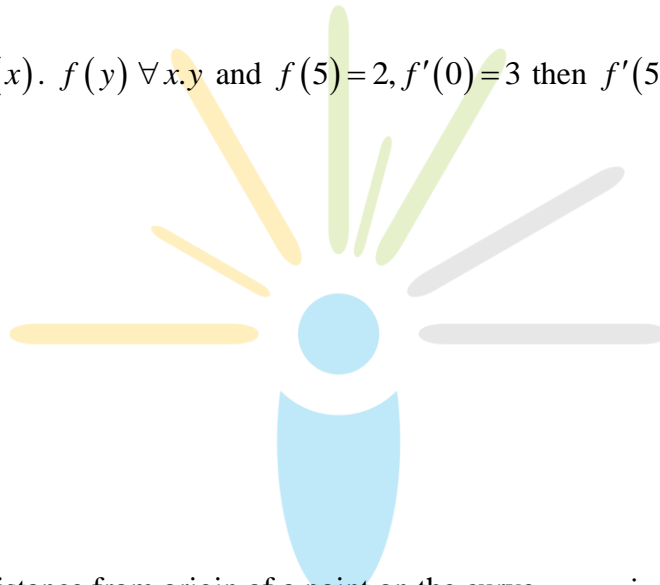
58. $f(x)$ and $g(x)$ are two differentiable functions on $[0, 2]$ such that $f''(x) - g''(x) = 0$

$f'(1) = 2g'(1) = 4f(2) = 3g(2) = 9$ then $f(x) - g(x)$ at $x = 3/2$ is

- (a) 0
- (b) 2
- (c) 10
- (d) 5

59. If $f(x+y) = f(x) \cdot f(y) \forall x, y$ and $f(5) = 2, f'(0) = 3$ then $f'(5)$ is

- (a) 0
- (b) 1
- (c) 6
- (d) 2



60. The maximum distance from origin of a point on the curve $x = a \sin t - b \sin\left(\frac{at}{b}\right)$

$y = a \cos t - b \cos\left(\frac{at}{b}\right)$, both $a, b > 0$ is

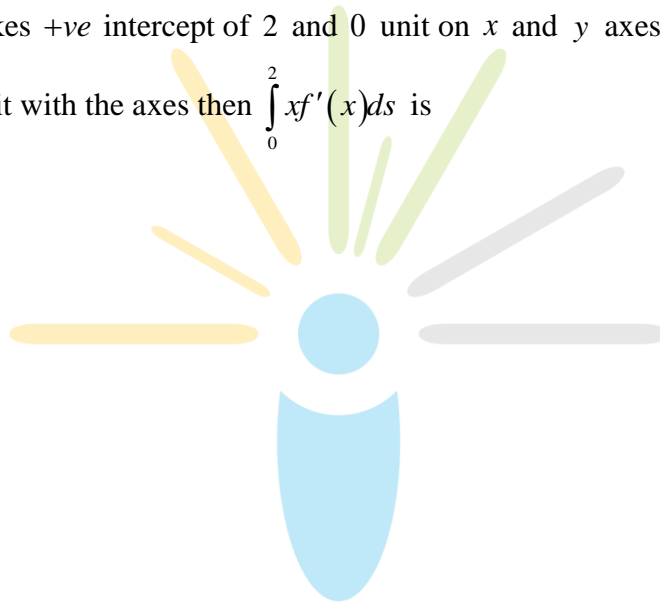
- (a) $a - b$
- (b) $a + b$
- (c) $\sqrt{a^2 + b^2}$
- (d) $\sqrt{a^2 - b^2}$

61. If $2a+3b+6c=0$ ($a, b, c \in R$) then the quadratic equation $ax^2 + bx + c = 0$ has

- (a) at least one root in $[0,1]$
- (b) at least one root in $[2,3]$
- (c) at least one root in $[4,5]$
- (d) none of these

62. If $y = f(x)$ makes +ve intercept of 2 and 0 unit on x and y axes and encloses an area of $3/4$ square unit with the axes then $\int_0^2 xf'(x)ds$ is

- (a) $3/2$
- (b) 1
- (c) $5/4$
- (d) $-3/4$



63. The area bounded by the curves $y = \ln x, y = \ln|x|, y = |\ln x|, y = |\ln x|$ and $y = |\ln||x|$ is

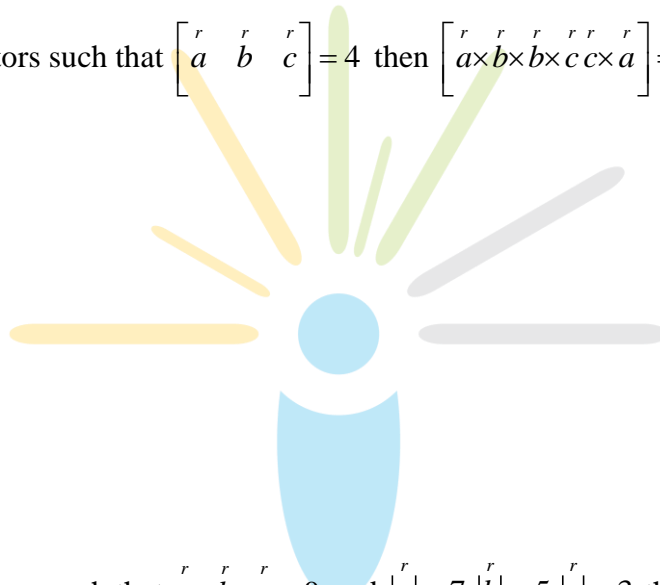
- (a) 4 sq . units
- (b) 6 sq . units
- (c) 10 sq . units
- (d) none of these

64. If $|\vec{a}|=4, |\vec{b}|=2$ and the angle between \vec{a} and \vec{b} is $\pi/6$ then $(\vec{a} \times \vec{b})^2 = 2$ is equal to

- (a) 48
- (b) 16
- (c) \vec{a}
- (d) none of these

65. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \end{vmatrix} = 4$ then $\begin{vmatrix} \vec{a} \times \vec{b} & \vec{b} \times \vec{c} & \vec{c} \times \vec{a} \end{vmatrix} =$

- (a) 16
- (b) 64
- (c) 4
- (d) 8



66. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}|=7, |\vec{b}|=5, |\vec{c}|=3$ then angle between vector \vec{b} and \vec{c} is

- (a) 60
- (b) 30°
- (c) 45°
- (d) 90°

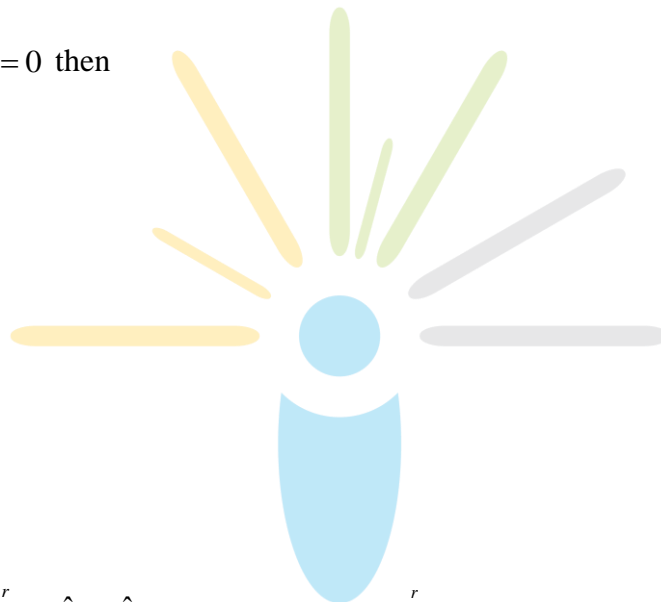
67. If $|a|=5, |b|=4, |c|=3$ thus what will be the value of $|a \cdot b + b \cdot c + c \cdot a|$, given that

$$\vec{a} + \vec{b} + \vec{c} = 0$$

- (a) 25
- (b) 50
- (c) -5
- (d) -50

68. $3\lambda \vec{c} + 2\mu (\vec{a} \times \vec{b}) = 0$ then

- (a) $3\lambda + 2\mu = 0$
- (b) $3\lambda = 2\mu$
- (c) $\lambda = \mu$
- (d) $\lambda + \mu = 0$



69. $\vec{a} = 3\hat{i} - 5\hat{j}$ and $\vec{b} = 6\hat{i} + 3\hat{j}$ are two vectors and \vec{c} is a vector such that $\vec{c} = \vec{a} \times \vec{b}$ then

$$|\vec{a}| : |\vec{b}| : |\vec{c}|$$

- (a) $\sqrt{34} : \sqrt{45} : \sqrt{39}$
- (b) $\sqrt{34} : \sqrt{45} : 39$
- (c) $34 : 39 : 45$
- (d) $39 : 35 : 34$

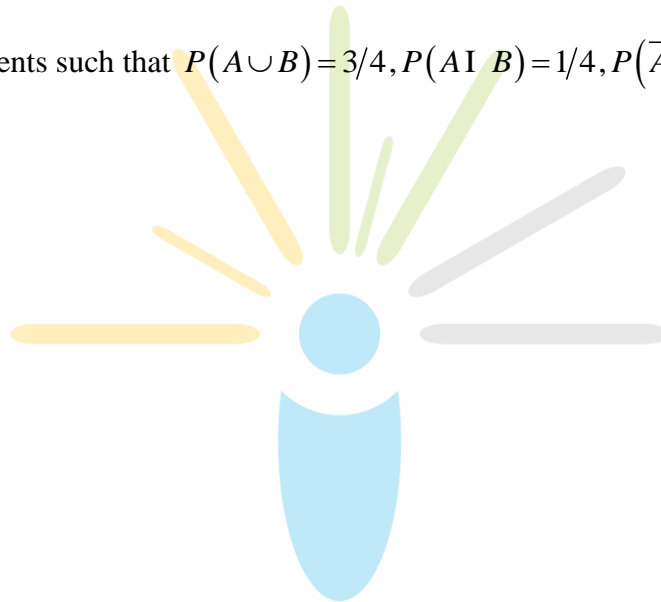
70. If $a^r \times b^r = b^r \times c^r = c^r \times a^r$ then $a^r + b^r + c^r =$

- (a) abc
- (b) -1
- (c) 0
- (d) 2

71. A and B are events such that $P(A \cup B) = 3/4$, $P(A \cap B) = 1/4$, $P(\bar{A}) = 2/3$ then

$P(\bar{A} \cap B)$

- (a) $5/12$
- (b) $3/8$
- (c) $5/8$
- (d) $1/4$



72. A die is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is

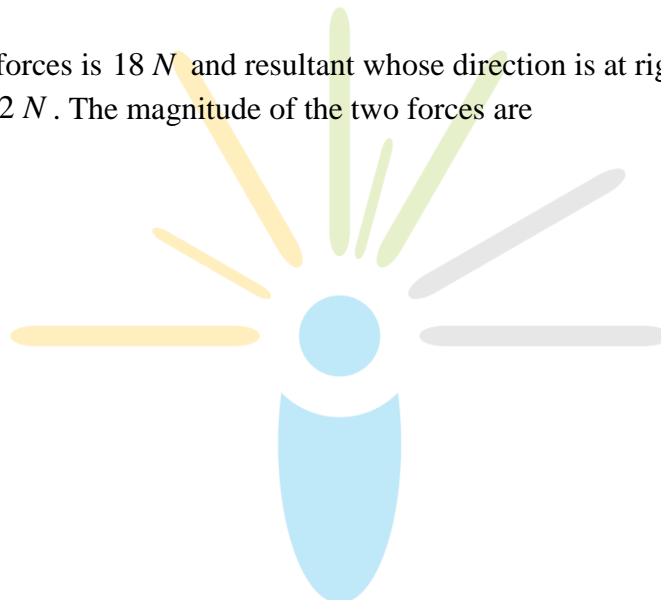
- (a) $8/3$
- (b) $3/8$
- (c) $4/5$
- (d) $5/4$

73. The *d.r.* of normal to the plane through $(1, 0, 0), (0, 1, 0)$ which makes an angle $\pi/4$ with plane $x + y = 3$ are

- (a) $1\sqrt{2}, 1$
- (b) $1, 1, \sqrt{2}$
- (c) $1, 1, 2$
- (d) $\sqrt{2}, 1, 1$

74. The sum of two forces is 18 N and resultant whose direction is at right angles to the smaller force is 12 N . The magnitude of the two forces are

- (a) $13, 5$
- (b) $12, 6$
- (c) $14, 4$
- (d) $11, 7$



75. A bead of weight w can slide on smooth circular wire in a vertical plane. The bead is attached by a light thread to the highest point of the wire and in equilibrium, the thread is taut and make an angle θ with the vertical then tension of the thread and reaction of the wire on the bead are

- (a) $T = w \cos \theta \quad R = w \tan \theta$
- (b) $T = 2w \cos \theta \quad R = w$
- (c) $T = w \quad R = w \sin \theta$
- (d) $T = w \sin \theta \quad R = w \cot \theta$