AIEE-2002

MATHEMATICS

1. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having α/β and β/α as its roots is

(a) $3x^2 - 19x + 3 = 0$

(b) $3x^2 - 19x - 3 = 0$

(c) $3x^2 - 19x - 3 = 0$

(d) $x^2 - 5x + 3 = 0$

2. If $y = \left(x + \sqrt{1 + x^2}\right)^2$, then $\left(1 + x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is

(a) $n^2 y$

(b) $-n^2 y$

(c) $-y$

(d) $2x^2 y$

3. If $1,\ \log_9\left(3^{1-x} + 2\right), \log_3\left(4.3^x - 1\right)$ are in $A.P.$ then x equals

(a) $\log_3 4$

(b) $1 + \log_3 4$

(c) $1 - \log_4 3$

(d) $\log_4 3$

4. A problem in mathematics is given to three students A, B, C and their respective probability of solving the problem is $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. Probability that the problem is solved is

(a) $\frac{3}{4}$

(b) $\frac{1}{2}$

(c) $\frac{2}{3}$

(d) $\frac{1}{3}$

5. The period of $\sin^2\theta$ is

(a) π^2

(b) π

(c) 2π

(d) $\pi/2$

6. l, m, n are the p^{th}, q^{th} and r^{th} term of a $G.P.$ all positive, then $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals

(a) –1

(b) 2

(c) 1

(d) 0

7. $\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is

(a) 1

(b) –1

(c) zero

(d) does not exist

8. A triangle with vertices $(4,0),(-1,-1),(3,5)$ is

(a) isosceles and right angled

(b) isosceles but not right angled

(c) right angled but not isosceles

(d) neither right angled nor isosceles

9. In a class of 100 students there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class is 72, then what is the average of the girls?

(a) 73

(b) 65

(c) 68

(d) 74

10. $\cot^{-1}\left(\sqrt{\cos\alpha}\right)=\tan^{-1}\left(\sqrt{\cos\alpha}\right)=x$, then $\sin x=$

(a) $\tan^2\left(\frac{\alpha}{2}\right)$

(b) $\cot^2\left(\frac{\alpha}{2}\right)$

(c) $\tan\alpha$

(d) $\cot\left(\frac{\alpha}{2}\right)$

11. The order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{2/3}=4\frac{d^3y}{dx^3}$ are

(a) $\left(1,\frac{2}{3}\right)$

(b) $(3,1)$

(c) $(3,3)$

(d) $(1,2)$

12. A plane which passes through the point $(3,2,0)$ and the line $\frac{x-4}{1}=\frac{y-7}{5}=\frac{z-4}{4}$ is

(a) $x-y+z=1$

(b) $x+y+z=5$

(c) $x+2y-z=1$

(d) $2x-y+z=5$

13. The solution of the equation $\frac{d^2y}{dx^2}=e^{-2x}$

(a) $\frac{e^{-2x}}{4}$

(b) $\frac{e^{-2x}}{4}+cx+d$

(c) $\frac{1}{4}e^{-2x}+cx^2+d$

(d) $\frac{1}{4}e^{-4x}+cx+d$

14. $\lim\limits_{x\to\infty}\frac{x^2+5x+3}{x^2+x+3}^{\frac{1}{x}}$

(a) e^4

(b) e^2

(c) e^3

(d) 1

15. The domain of $\sin^{-1}\left[\log_3(x/3)\right]$ is

(a) $[1,9]$

(b) $[-1,9]$

(c) $[-9,1]$

(d) $[-9,-1]$

16. The value of $2^{1/4}, 4^{1/8}, 8^{1/6} + \mathrm{K\ K}\ \infty$ is

(a) 1

(b) 2

(c) $3/2$

(d) 4

17. Fifth term of a GP is 2, then the product of its 9 terms is

(a) 256

(b) 512

(c) 1024

(d) none of these

18. $\int_{0}^{10\pi} |\sin x| dx$ is

(a) 20

(b) 8

(c) 10

(d) 18

19. $l_2 = \int_0^{\pi/4} \tan^n x\,dx$ then equals $\lim_{n\to\infty} n\left[l_n + l_{n-2}\right]$

(a) $\frac{1}{2}$

(b) 1

(c) ∞

(d) zero

20. $\int_0^{\sqrt{2}} \left[X^2\right] dx$ is

(a) $2-\sqrt{2}$

(b) $2+\sqrt{2}$

(c) $\sqrt{2}-1$

(d) $\sqrt{2}-2$

21. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is

(a) $\frac{\pi^2}{4}$

(b) π^2

(c) zero

(d) $\frac{\pi}{2}$

22. Let $f(x)=4$ and $f'(x)=4$. Then $\lim\limits_{x\to 2}\dfrac{xf(2)-2f(x)}{x-2}$ is given by

(a) 2

(b) –2

(c) –4

(d) 3

23. z and w are two non zero complex no. s such that $|z|=|w|$ and $Arg\ z+Arg\ w=\pi$ z equals

(a) $\overline{W}$

(b) $-\overline{W}$

(c) w

(d) $-w$

24. If $|z-4|<|z-2|$, its solution is given by

(a) $\text{Re}(z)>0$

(b) $\text{Re}(z)<0$

(c) $\text{Re}(z)>3$

(d) $\text{Re}(z)>2$

25. The locus of the centre of a circle which touches the circle $|z - z_1| = a$ and $|z - z_2| = b$ externally (z , z_1 and z_2 are complex numbers) will be

(a) an ellipse

(b) a hyperbola

(c) a circle

(d) none of these

26. Sum of infinite number of terms of GP is 20 and sum of their square is 100 .The common ratio of GP is

(a) 5

(b) $3/5$

(c) $8/5$

(d) $1/5$

27. $1^3 - 2^3 + 3^3 - 4^3 + K + 9^3 =$

(a) 425

(b) -425

(c) 475

(d) -475

28. Difference between the corresponding roots of $x^2+ax+b=0$ and $x^2+bx+a=0$ is same and $a \neq b$, then

(a) $a+b+4=0$

(b) $a+b-4=0$

(c) $a-b-4=0$

(d) $a-b+4=0$

29. Product of real roots of the equation $t^2x^2+|x|+9=0$

(a) is always positive

(b) is always negative

(c) does not exist

(d) none of these

30. If p and q are the roots of the equation $x^2+px+q=0$, then

(a) $p=1, q=-2$

(b) $p=0,\ q=1$

(c) $p=-2,\ q=0$

(d) $p=-2,\ q=1$

31. If $a,\ b,\ c$ are distinct +ve real numbers and $a^2+b^2+c^2=1$ then $ab+bc+ca$ is

(a) less than 1

(b) equal to 1

(c) greater than 1

(d) any real no.

32. Total number of four digit odd numbers that can be formed using $0,\ 1,\ 2,\ 3,\ 5,\ 7$ (using repetition allowed) are

(a) 216

(b) 375

(c) 400

(d) 720

33. Number greater than 1000 but less than 4000 is formed using the digits $0,\ 1,\ 2,\ 3,\ 4$ (repetition allowed) is

(a) 125

(b) 105

(c) 375

(d) 625

34. Five digit number divisible by 3 is formed using $0,\ 1,\ 2,\ 3,\ 4,\ 6$ and 7 without repetition. Total number of such numbers are

(a) 312

(b) 3125

(c) 120

(d) 216

35. The sum of integers from 1 to 100 that are divisible by 2 or 5 is

(a) 3000

(b) 3050

(c) 3600

(d) 3250

36. The coefficients of x^p and x^p in the expansion of $(1+x)^{p+q}$ are

(a) equal

(b) equal with opposite signs

(c) reciprocals of each other

(d) none of these

37. If the sum of the coefficients in the expansion of $(a+b)^n$ is 4096 then the greatest coefficient in the expansion is

(a) 1594

(b) 792

(c) 924

(d) 2924

38. The positive integer just greater than $(1+0.0001)^{10000}$ is

(a) 4

(b) 5

(c) 2

(d) 3

39. r and n are positive integers $r>1, n>2$ and coefficient of $(r+2)^{\text{th}}$ term and $3r^{\text{th}}$ term in the expansion of $(1+x)^{2n}$ are equal, then n equals

(a) $3r$

(b) $3r+1$

(c) $2r$

(d) $2r+1$

40. If $a>0$ discriminant of $ax^2+2bx+c$ is $-$ve , then $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is

(a) $+$ve

(b) $\left(ac-b^2\right)\left(ax^2+2bx+c\right)$

(c) $-$ve

(d) 0

41. If $a_n=\sqrt{7+\sqrt{7+\sqrt{7+\mathrm{K\,K}}}}$ having n radical signs then by methods of mathematical induction which is true

(a) $a_n>7\ \forall\ n\geq 1$

(b) $a_n>7\ \forall n\geq 1$

(c) $a_n<4\ \forall n\geq 1$

(d) $a_n<3\forall n\geq 1$

42. The sides of a triangle are $3x+4y, 4x+37$ and $5x+57$ where $x,\ y>0$ then the triangle is

(a) right angled

(b) obtuse angled

(c) equilateral

(d) none of these

43. Locus of mid point of the portion between the axes of $x\cos\alpha + y\sin\alpha = p$ where p is constant is

(a) $x^2 + y^2 = \frac{4}{p^2}$

(b) $x^2 + y^2 = 4p^2$

(c) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{2}{p^2}$

(d) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$

44. If the pair of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ intersect on the y-axis then

(a) $2fgh = bg^2 + ch^2$

(b) $bg^2 \neq ch^2$

(c) $abc = 2fgh$

(d) none of these

45. The point of lines represented by $3ax^2 + 5xy + (a^2 - 2)y^2 = 0$ and perpendicular to each other for

(a) two values of a

(b) $\forall a$

(c) for one value of a

(d) for no values of a

46. If the chord $y = mx + 1$ of the circle $x^2 + y^2 = 1$ subtends an angle of measure $45°$ at the major segment of the circle then value of m is

(a) $2 \pm \sqrt{2}$

(b) $-2 \pm \sqrt{2}$

(c) $-1 \pm \sqrt{2}$

(d) none of these

47. The centres of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is

(a) $4 \le x^2 + y^2 \le 64$

(b) $x^2 + y^2 \le 25$

(c) $x^2 + y^2 \ge 25$

(d) $3 \le x^2 + y^2 \le 9$

48. The centre of the circle passing through $(0,\ 0)$ and $(1,\ 0)$ and touching the circle $x^2 + y^2 = 9$ is

(a) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(b) $\left(\frac{1}{2}, -\sqrt{2}\right)$

(c) $\left(\frac{3}{2}, \frac{1}{2}\right)$

(d) $\left(\frac{1}{2}, \frac{3}{2}\right)$

49. The equation of a circle with origin as a centre and passing through equilateral triangle whose median is of length $3a$ is

(a) $x^2+y^2=9a^2$

(b) $x^2+y^2=16a^2$

(c) $x^2+y^2=4a^2$

(d) $x^2+y^2=a^2$

50. Two common tangents to the circle $x^2+y^2=2a^2$ and parabola $y^2=8ax$ are

(a) $x=\pm(y+2a)$

(b) $y=\pm(x+2a)$

(c) $x=\pm(y+a)$

(d) $y=\pm(x+a)$

51. In a triangle with sides $a,\ b,\ c,\ r_1>r_2>r_3$ (which are the ex- radii) then

(a) $a>b>c$

(b) $a<b<c$

(c) $a>b$ and $b<c$

(d) $a<b$ and $b>c$

52. The number of solution of $\tan x + \sec x = 2\cos x$ in $(0, 2\pi)$ is

(a) 2

(b) 3

(c) 0

(d) 1

53. Which one is not periodic

(a) $|\sin 3x| + \sin^2 x$

(b) $\cos\sqrt{x} + \cos^2 x$

(c) $\cos 4x + \tan^2 x$

(d) $\cos 2x + \sin x$

54. $\lim_{n\to\infty} \frac{1^p + 2^p + 3^p + \ldots + n^p}{n^{p+1}}$ is

(a) $\frac{1}{p+1}$

(b) $\frac{1}{1-p}$

(c) $\frac{1}{p} - \frac{1}{p-1}$

(d) $\frac{1}{p+2}$

55. $\lim\limits_{n\to 0}\dfrac{\log x^{n}-[x]}{[X]}$, $n\in N$ ($[x]$ denotes greatest integer less than or equal to x)

(a) has value –1

(b) has value 0

(c) has value 1

(d) does not exist

56. If $f(1)=1, f'(1)=2$, then $\lim\limits_{x\to 1}\dfrac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ is

(a) 2

(b) 4

(c) 1

(d) 1/2

57. f is defined in $[-5,5]$ as $f(x)=x$ if x is rational and $=-x$ is irrational. Then

(a) $f(x)$ is continuous at every x, except $x=0$

(b) $f(x)$ is discontinuous at every x, except $x=0$

(c) $f(x)$ is continuous everywhere

(d) $f(x)$ is discontinuous everywhere

58. $f(x)$ and $g(x)$ are two differentiable functions on $[0,2]$ such that $f''(x)-g''(x)=0$

$f'(1)=2g'(1)=4\,f(2)=3g(2)=9$ then $f(x)-g(x)$ at $x=3/2$ is

(a) 0

(b) 2

(c) 10

(d) 5

59. If $f(x+y)=f(x).\ f(y)\ \forall\, x.y$ and $f(5)=2, f'(0)=3$ then $f'(5)$ is

(a) 0

(b) 1

(c) 6

(d) 2

60. The maximum distance from origin of a point on the curve $x=a\sin t-b\sin\left(\frac{at}{b}\right)$

$y=a\cos t-b\cos\left(\frac{at}{b}\right)$, both $a,b>0$ is

(a) $a-b$

(b) $a+b$

(c) $\sqrt{a^2+b^2}$

(d) $\sqrt{a^2-b^2}$

61. If $2a+3b+6c=0\left(a,b,c\in R\right)$ then the quadratic equation $ax^2+bx+c=0$ has

(a) at least one root in $\left[0,1\right]$

(b) at least one root in $\left[2,3\right]$

(c) at least one root in $\left[4,5\right]$

(d) none of these

62. If $y=f\left(x\right)$ makes $+ve$ intercept of 2 and 0 unit on x and y axes and encloses an area of $3/4$ square unit with the axes then $\int_0^2 xf'\left(x\right)ds$ is

(a) $3/2$

(b) 1

(c) $5/4$

(d) $-3/4$

63. The area bounded by the curves $y=\ln x, y=\ln|x|, y=|\ln x|,\ y=|\ln x|\ and\ y=|\ln||x|$ is

(a) 4 sq . units

(b) 6 sq . units

(c) 10 sq . units

(d) none of these

64. If $|\vec{a}| = 4, |\vec{b}| = 2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$ than $\left(\vec{a}\times\vec{b}\right)^2 = 2$ is equal to

(a) 48

(b) 16

(c) $\vec{a}$

(d) none of these

65. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right] = 4$ then $\left[\vec{a}\times\vec{b} \times \vec{b}\times\vec{c} \, \vec{c}\times\vec{a}\right] =$

(a) 16

(b) 64

(c) 4

(d) 8

66. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a}+\vec{b}+\vec{c} = 0$ and $|\vec{a}| = 7, |\vec{b}| = 5, |\vec{c}| = 3$ then angle between vector $\vec{b}$ and $\vec{c}$ is

(a) 60

(b) 30°

(c) 45°

(d) 90°

67. If $|a|=5, |b|=4, |c|=3$ thus what will be the value of $|a.b+b.c+c.a|$, given that $\vec{a}+\vec{b}+\vec{c}=0$

(a) 25

(b) 50

(c) –5

(d) –50

68. $3\lambda\vec{c}+2\mu\left(\vec{a}\times\vec{b}\right)=0$ then

(a) $3\lambda+2\mu=0$

(b) $3\lambda=2\mu$

(c) $\lambda=\mu$

(d) $\lambda+\mu=0$

69. $\vec{a}=3\hat{i}-5\hat{j}$ and $\vec{b}=6\hat{i}+3\hat{j}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c}=\vec{a}\times\vec{b}$ then $|\vec{a}|:|\vec{b}|:|\vec{c}|$

(a) $\sqrt{34}:\sqrt{45}:\sqrt{39}$

(b) $\sqrt{34}:\sqrt{45}:39$

(c) $34:39:45$

(d) $39:35:34$

70. If $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ than $\vec{a}+\vec{b}+\vec{c}=$

(a) abc

(b) -1

(c) 0

(d) 2

71. A and B are events such that $P(A\cup B)=3/4, P(A\cap B)=1/4, P(\overline{A})=2/3$ than $P(\overline{A}\cap B)$

(a) $5/12$

(b) $3/8$

(c) $5/8$

(d) $1/4$

72. A die is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is

(a) $8/3$

(b) $3/8$

(c) $4/5$

(d) $5/4$

73. The $d.r.$ of normal to the plane through $(1,\ 0,\ 0),(0,\ 1,\ 0)$ which makes an angle $\pi/4$ with plane $x+y=3$ are

(a) $1\sqrt{2},1$

(b) $1,1,\sqrt{2}$

(c) $1,1,2$

(d) $\sqrt{2},1,1$

74. The sum of two forces is $18\ N$ and resultant whose direction is at right angles to the smaller force is $12\ N$. The magnitude of the two forces are

(a) $13,5$

(b) $12,6$

(c) $14,4$

(d) $11,7$

75. A bead of weight w can slide on smooth circular wire in a vertical plane. The bead is attached by a light thread to the highest point of the wire and in equilibrium, the thread is taut and make an angle θ with the vertical then tension of the thread and reaction of the wire on the bead are

(a) $T=w\cos\theta \quad R=w\tan\theta$

(b) $T=2w\cos\theta \quad R=w$

(c) $T=w \quad R=w\sin\theta$

(d) $T=w\sin\theta \quad R=w\cot\theta$