JEE MAIN-2005

MATHEMATICS

[Time: 2 Hours]

General Instructions:

Note: Question number 1 to 8 carries ***2 marks*** each, 9 to 16 carries ***4 marks*** each and 17 to 18 carries ***6 marks*** each.

Q1. A person goes to office either by car, scooter, bus or train probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. Given that he reached office in time, then what is the probability that he travelled by a car.

Q2. Find the range of values of t for which $2\sin t = \frac{1-2x+5x^2}{3x^2-2x-1}, t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Q3. Circles with radii 3, 4 and 5 touch each other externally if P is the point of intersection of tangents to these circles at their points of contact. Find the distance of P from the points of contact.

Q4. Find the equation of the plane containing the line $2x - y + z - 3 = 0, 3x + y + z = 5$ and at a distance of $\frac{1}{\sqrt{6}}$ from the point $(2, 1, -1)$.

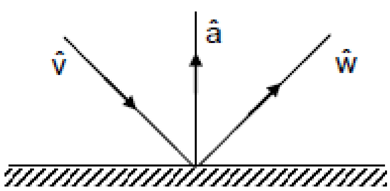
Q5. If $\left|f(x_1)-f(x_2)\right|<(x_1-x_2)^2$ for all $x_1, x_2 \in R$. Find the equation of tangent to the curve $y=f(x)$ at the point $(1,2)$.

Q6. If total number of runs scored in n matches is $\left(\frac{n+1}{4}\right)\left(2^{n+1}-n-2\right)$ where $n>1$, and the runs scored in the k^{th} match are given by $k.\ 2^{n+1-k}$, where $1 \le k \le n$. Find n.

Q7. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through $P(h,k)$ with the lines $y=x$ and $x+y=2$ is $4h^2$. Find the locus of the point P.

Q.8. Evaluate $\int_0^{\pi} e^{|\cos x|}\left(2\sin\left(\frac{1}{2}\cos x\right)+3\cos\left(\frac{1}{2}\cos x\right)\right)\sin x\, dx.$

Q9. Incident ray is along the unit vector $\hat{v}$ and the reflected ray is along the unit vector $\hat{w}$. The normal is along unit vector $\hat{a}$ outwards. Express $\hat{w}$ in terms of $\hat{a}$ and $\hat{v}$.



Q10. Tangents are drawn from any point on the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ to the circle $x^2+y^2=9$. Find the locus of mid – point of the chord of contact.

Q11. Find the equation of the common tangent in 1^{st} quadrant to the circle $x^2 + y^2 = 16$ and the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$. Also find the length of the intercept of the tangent between the coordinate axes.

Q12. If length of tangent at any point on the curve $y = f(x)$ intercepted between the point and the x-axis is of length 1. Find the equation of the curve.

Q13. Find the area bounded by the curves $x^2 = y, x^2 = -$ and $y^2 = 4x - 3$.

Q14. If one of the vertices of the square circumscribing the circle $|z-1| = \sqrt{2}$ is $2 + \sqrt{3}i$. Find the other vertices of square.

Q15. If $f(x-y) = f(x).g(y) - f(y).g(x)$. and $g(x-y) = g(x).g(y) + f(x).f(y)$ for all $x, y \in R$. If right hand derivative at $x = 0$ exists for $f(x)$. Find derivative of $g(x)$ at $x = 0$.

Q16. If $p(x)$ be a polynomial of degree 3 satisfying $p(-1) = 10,\ p(1) = -6$ and $p(x)$ has maximum at $x = -1$ and $p'(x)$ has minima at $x = 1$. Find the distance between the local maximum and local minimum of the

Q17. $f(x)$ is a differentiable function and $g(x)$ is a double differentiable function such that $|f(x)| \le 1$ and $f'(x) = g(x)$. If $f^2(0) = 9$. Prove that there exists some $c \in (-3, 3)$ such that $g(c).\ g''(c) < 0$.

Q18. If $\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix}\begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix}=\begin{bmatrix} 3a^2 & + & 3a \\ 3b^2 & + & 3b \\ 3c^2 & + & 3c \end{bmatrix}$, $f(x)$ is a quadratic function and its maximum value occurs at a point V. A is a point of intersection of $y=f(x)$ with x-axis and point B is such that chord AB subtends a right angle at V. Find the area enclosed by $f(x)$ and chord AB.