IITJEE–2006

Mathematics

Section – A (Single Option Correct)

1.Correct Answer: (C)

$$\lim_{x\to 0}\left((\sin x)^{1/x}+\left(\frac{1}{x}\right)^{\sin x}\right)$$

$$0+e^{\lim_{x\to 0}\sin x\ln\left(\frac{1}{x}\right)}=1 \text{ (using } L\text{' Hospital's rule).}$$

2.Correct Answer: (D)

$$\int \frac{\left(\frac{1}{x^3}-\frac{1}{x^5}\right)dx}{\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}}$$

$$\text{Let } 2-\frac{2}{x^2}+\frac{1}{x^4}=z\Rightarrow \frac{1}{4}\int\frac{dz}{\sqrt{z}}$$

$$\Rightarrow \frac{1}{2}\times\sqrt{z}+c\Rightarrow \frac{1}{2}\sqrt{2-\frac{2}{x^2}+\frac{1}{x^4}}+c.$$

3. Correct Answer: (C)

$$\Delta=\frac{\sqrt{3}}{4}b^2 \quad \ldots(1)$$

$$\text{Also } \frac{\sin 120^\circ}{a}=\frac{\sin 30^\circ}{b}\Rightarrow a=\sqrt{3}b$$

$$\text{and } \Delta=\sqrt{3}s \text{ and } s=\frac{1}{2}(a+2b)$$

$$\Rightarrow \Delta = \frac{\sqrt{3}}{2}(a+2b) \ \ldots(2)$$

From (1) and (2), we get $\Delta = \left(12+7\sqrt{3}\right)$.

4. Correct Answer: (A)

$$2\sin^2\theta - 5\sin\theta + 2 > 0$$
$$\Rightarrow (\sin\theta - 2)(2\sin\theta - 1) > 0$$
$$\Rightarrow \sin\theta < \frac{1}{2}$$
$$\Rightarrow \theta \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$$

5. Correct Answer: (D)

$$\frac{w - \overline{w}z}{1-z} = \frac{\overline{w} - w\overline{z}}{1-\overline{z}}$$
$$\Rightarrow (z\overline{z} - 1)(\overline{w} - w) = 0$$
$$\Rightarrow z\overline{z} = 1 \Rightarrow |z|^2 = 1 \Rightarrow |z| = 1.$$

6. Correct Answer: (A)

$$D \geq 0$$
$$\Rightarrow 4(a+b+c)^2 - 12\lambda(ab+bc+ca) \geq 0$$
$$\Rightarrow \lambda \leq \frac{a^2+b^2+c^2}{3(ab+bc+ca)} + \frac{2}{3}$$
$$\sin ce|a-b| < c \Rightarrow a^2+b^2-2ab < c^2 \quad \ldots(1)$$
$$|b-c| < a \Rightarrow b^2+c^2-2bc < a^2 \quad \ldots(2)$$
$$|c-a| < b \Rightarrow c^2+a^2-2ac < b^2 \quad \ldots(3)$$

From (1), (2) and (3), we get $\frac{a^2+b^2+c^2}{ab+bc+ca} < 2.$

Hence $\lambda < \frac{2}{3} + \frac{2}{3} \Rightarrow \lambda < \frac{4}{3}$.

7. Correct Answer: (A)

$$f'(x) = -f(x) \text{ and } f'(x) = g(x)$$
$$\Rightarrow f'(x).f'(x) + f(x).f'(x) = 0$$
$$\Rightarrow f(x)^2 + (f'(x))^2 = c \Rightarrow \left(f(x)^2 + g(x)\right)^2 = c$$
$$\Rightarrow F(x) = c \Rightarrow F(10) = 5.$$

8. Correct Answer: (C)

Required number of ordered pair (p, q) is $(2\times3-1)\ (2\times5-1)\ (2\times3-1) = 225$.

9. Correct Answer: (B)

Given $\theta \in \left(0, \frac{\pi}{4}\right)$, then $\tan\theta < 1$ and $\cot\theta > 1$.

Let $\tan\theta = 1-\lambda_1$ and $\cot\theta = 1+\lambda_2$ where λ_1 and λ_2 are very small and positive.

then $t_1 = (1-\lambda_1)^{1-\lambda_1}, t_2 = (1-\lambda_1)^{1+\lambda_2}$

$t_3 = (1+\lambda_2)^{1-\lambda_1}$ and $t_4 = (1+\lambda_2)^{1+\lambda_2}$

Hence $t_4 > t_3 > t_1 > t_2$.

10. Correct Answer: (D)

Equation of directrix is $x + y = 0$

Hence equation of the parabola is

$$\frac{x+y}{\sqrt{2}}=\sqrt{(x-2)^2+(y-2)^2}$$

Hence equation of parabola is

$$(x-y)^2=8(x+y-2).$$

11. Correct Answer: (D)

The plane is a $(x-1)+b(y+2)+c(z-1)=0$

where $2a-2b+c=0$ and $a-b+2c=0$

$$\Rightarrow \frac{a}{1}=\frac{b}{1}=\frac{c}{0}$$

So, the equation of plane is

$$x+y+1=0$$

$\therefore$ Distance of the plane from the point $(1,2,2)=\dfrac{1+2+1}{\sqrt{1^2+1^2}}=2\sqrt{2}$.

12. Correct Answer: (A)

Vector lying in the plane of $\vec{a}$ and $\vec{b}$ is $\vec{r}=\lambda_1\vec{a}+\lambda_2\vec{b}$ and its projection on $\vec{c}$ is $\dfrac{1}{\sqrt{3}}$

$$\Rightarrow\left[(\lambda_1+\lambda_2)\hat{i}-(2\lambda_1-\lambda_2)\hat{j}+(\lambda_1+\lambda_2)\hat{k}\right]\cdot\frac{\left[\hat{i}-\hat{j}-\hat{k}\right]}{\sqrt{3}}=\frac{1}{\sqrt{3}}$$

$$\Rightarrow 2\lambda_1-\lambda_2=-1\Rightarrow \vec{r}=(3\lambda_1+1)\hat{i}-\hat{j}+(3\lambda_1+1)\hat{k}$$

Hence the required vector is $4\hat{i}-\hat{j}+4\hat{k}$

Alternate:

Vector lying in the plane of $\vec{a}$ and $\vec{b}$ is $\vec{a}+\lambda\vec{b}$, and its projection on C is $\dfrac{1}{\sqrt{3}}$.

$$\Rightarrow\left((1+\lambda)\hat{i}+(2-\lambda)\hat{j}+(1+\lambda)\hat{k}\cdot\frac{(\hat{i}-\hat{j}-\hat{k})}{\sqrt{3}}\right)=\frac{1}{\sqrt{3}}$$

$\Rightarrow \lambda=3$

Hence the required vector is $4\hat{i}-\hat{j}+4\hat{k}$

Section – B (May have more than one option correct)

13. Correct Answer: (A,B)

Equation of tangent to $x^2=y$ is

$y=mx-\frac{1}{4}m^2 \qquad \ldots(1)$

Equation of tangent to $(x-2)^2=-y$ is

$y=m(x-2)+\frac{1}{4}m^2 \qquad \ldots(2)$

(1) and (2) are identical.

$\Rightarrow m=0 \text{ or } 4$

$\therefore$ Common tangents are $y=0$ and $y=4x-4$.

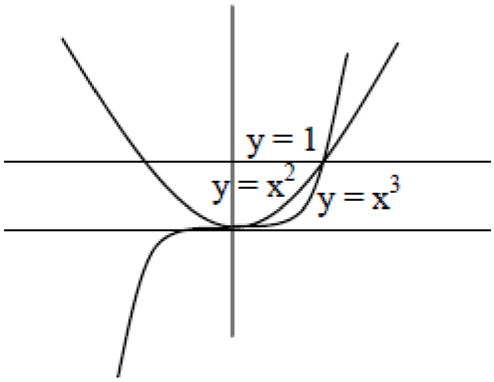
14. Correct Answer: (A,C)

$f(x)=\min\{1,x^2,x^3\}$

$$\Rightarrow f(x)=\begin{cases} x^3, & x\le 1 \\ 1, & x>1 \end{cases}$$

$\Rightarrow f(x)$ is continuous $\forall x\in R$

and non-differentiable at $x = 1$.



15. Correct Answer: (C,D)

Equation of the tangent is

$$Y - y = \frac{dy}{dx}(X - x)$$

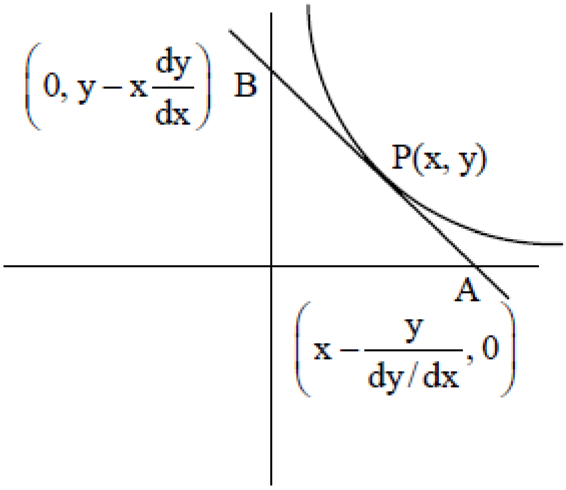
Given $\frac{BP}{AP} = \frac{3}{1}$

$$\Rightarrow \frac{dx}{x} = -\frac{dy}{3y} \Rightarrow x\frac{dy}{dx} + 3y = 0$$

$$\Rightarrow \ln x = -\frac{1}{3}\ln y - \ln c \Rightarrow \ln x^3 = -(\ln cy)$$

$$\Rightarrow \frac{1}{x^3} = cy.\text{Given } f(1) = 1 \Rightarrow c = 1$$

$$\therefore y = \frac{1}{x^3}.$$



16. Correct Answer: (A, C)

Eccentricity of ellipse $= \frac{3}{5}$

Eccentricity of hyperbola $= \frac{5}{3}$ and it passes through $(\pm 3, 0)$

$\Rightarrow$ its equation $\frac{x^2}{9} - \frac{y^2}{b^2} = 1$

where $1 + \frac{b^2}{9} = \frac{25}{9} \Rightarrow b^2 = 16$

$\Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$ and its foci are $(\pm 5, 0)$

17. Correct Answer: (A,B,C,D)

We have $\Delta ABC = \Delta ABD + \Delta ACD$

$$\Rightarrow \frac{1}{2} bc \sin \mathrm{A} = \frac{1}{2} cAD \sin \frac{A}{2} + \frac{1}{2} b \times AD \sin \frac{A}{2}$$

$$\Rightarrow AD = \frac{2bc}{b+c} \cos \frac{A}{2}$$

Again $AE = AD \sec \frac{A}{2}$

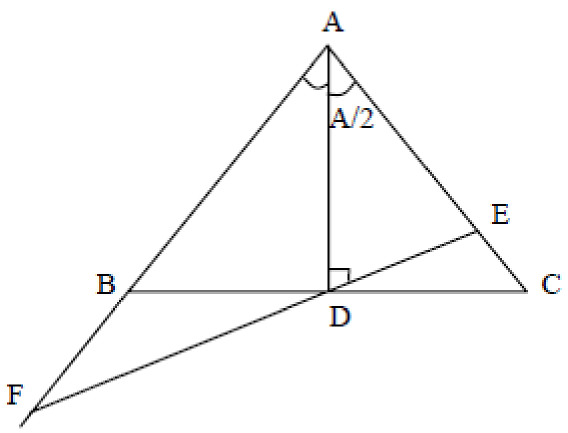
$= \frac{2bc}{b+c} \Rightarrow AE$ is HM of b and c.

$$EF = ED + DF = 2DE = 2 \times AD \tan \frac{A}{2} = \frac{2 \times 2bc}{b+c} \times \cos \frac{A}{2} \times \tan \frac{A}{2}$$

$$= \frac{4bc}{b+c} \sin \frac{A}{2}$$

As $AD \perp EF$ and $DE = DF$ and AD is bisector $\Rightarrow AEF$ is isosceles.

Hence A, B, C and D are correct answers.



18. Correct Answer: (B,C)

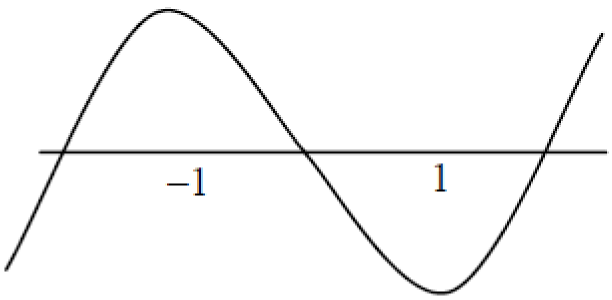
The required polynomial which satisfy the condition

is $f(x)=\frac{1}{4}\left(19x^3-57x+34\right)$

$f(x)$ has local maximum at $x=-1$ and local

minimum at $x=1$

Hence $f(x)$ is increasing for $x\in\left[1,2\sqrt{5}\right]$.



19. Correct Answer: (B,D)

Vector AB is parallel to $\left[\left(2\hat{i}+3\hat{k}\right)\times(4)-3\hat{k}\right]\times\left[\left(\hat{j}-\hat{k}\right)\times\left(3\hat{i}+3\hat{j}\right)\right]=54\left(\hat{j}-\hat{k}\right)$

Let θ is the angle between the vector, then

$$\cos\theta=\pm\left(\frac{54+108}{3.54\sqrt{2}}\right)=\pm\frac{1}{\sqrt{2}}$$

Hence $\theta=\frac{\pi}{4},\frac{3\pi}{4}$.

20 .Correct Answer: (A,B)

$$g'(x) = f(x) = \begin{cases} e^x & 0 \le x \le 1 \\ 2 - e^{x-1} & 1 < x \le 2 \\ x - e & 2 < x \le 3 \end{cases}$$

$g'(x) = 0$, when $x = 1 + \ln 2$ and $x = e$

$$g''(x) = \begin{Bmatrix} -e^{x-1} & 1 < x \le 2 \\ x - e & 2 < x \le 3 \end{Bmatrix}$$

$g''(1+\ln 2) = -e^{\ln 2} < 0$ hence at $x = 1 + \ln 2$, $g(x)$ has a local maximum

$g''(e) = 1 > 0$ hence at $x = e$, $g(x)$ has local minimum.

$\because f(x)$ is discontinuous at $x = 1$, then we get local maxima at $x = 1$ and local minima at $x = 2$.

Section – C

Comprehension I

21. Correct Answer: (B)

$P(u_i) = ki$

$\sum P(u_i) = 1$

$\Rightarrow k = \dfrac{2}{n(n+1)}$

$$\lim_{n\to\infty} P(w) = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2i^2}{n(n+1)^2} = \lim_{n\to\infty} \frac{2n(n+1)(2n+1)}{n(n+1)^2 6} = \frac{2}{3}$$

22. Correct Answer: (A)

$$P\left(\frac{u_n}{w}\right)=\frac{c\left(\frac{n}{n+1}\right)}{c\left(\frac{\sum i}{(n+1)}\right)}=\frac{2}{n+1}.$$

23. Correct Answer: (B)

$$P\left(\frac{W}{E}\right)=\frac{2+4+6+\ldots n}{\frac{n(n+1)}{2}}=\frac{n+2}{2(n+1)}$$

Comprehension II

24. Correct Answer: (A)

$$\int_0^{\pi/2}\sin x\,dx=\frac{\frac{\pi}{2}+0}{4}\left(\sin(0)+\sin\left(\frac{\pi}{2}\right)+2\sin\left(\frac{0+\frac{\pi}{2}}{2}\right)\right)$$

$$=\frac{\pi}{8}\left(1+\sqrt{2}\right).$$

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25. Correct Answer: (A)

$$\left(F'(c)=(b-a)f'(c)+f(a)-f(b)\right.$$

$$F''(c)=f''(c)(b-a)<0$$

$$\Rightarrow F'(c)=0\Rightarrow F'(c)=\frac{f(b)-f(a)}{b-a}.$$

Comprehension III

26. Correct Answer: (A)

Let $A, B, C,$ and D be the complex numbers $\sqrt{2}, -\sqrt{2}, \sqrt{2}i$ and $-\sqrt{2}i$ respectively.

$$\Rightarrow \frac{PA^2+PB^2+PC^2+PD^2}{QA^2+QB^2+QC^2+QD^2} = \frac{\left|z_1-\sqrt{2}\right|^2+\left|z_1+\sqrt{2}\right|^2\left|z_1+\sqrt{2}i\right|^2+\left|z_1-\sqrt{2}i\right|^2}{\left|z_2+\sqrt{2}\right|^2+\left|z_2-\sqrt{2}\right|^2+\left|z_2-\sqrt{2}i\right|^2+\left|z_2+\sqrt{2}i\right|^2} = \frac{\left|z_1\right|^2+2}{\left|z_2\right|^2+2} = \frac{3}{4}.$$

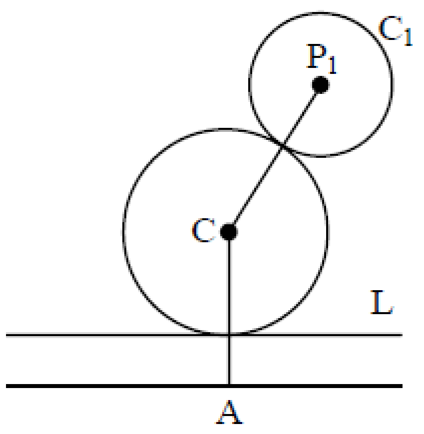
27. Correct Answer: (C)

Let C be the centre of the required circle.

Now draw a line parallel to L at a distance of r_1

(radius of C_1) from it.

Now $CP_1 = AC \Rightarrow C$ lies on a parabola.



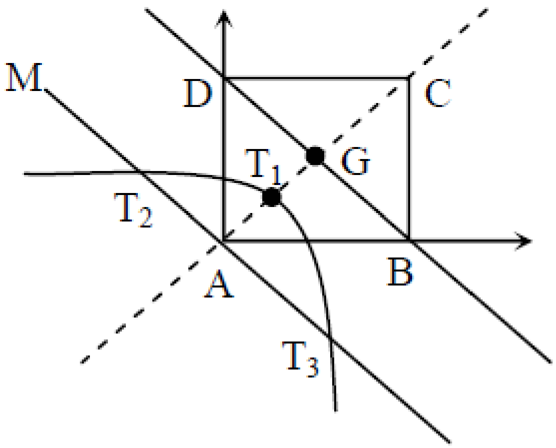
28. Correct Answer: (C)

$\because AG = \sqrt{2}$

$\therefore AT_1 = T_1G = \dfrac{1}{\sqrt{2}}$ [as A is the focus, T_1 is the vertex and BD is the directrix of parabola].

Also T_2T_3 is latus rectum $\therefore T_2T_3 = 4\times\frac{1}{\sqrt{2}}$

$$\therefore \text{Area of}\,\Delta T_1T_2T_3 = \frac{1}{2}\times\frac{1}{\sqrt{2}}\times\frac{4}{\sqrt{2}} = 1.$$



29. Correct Answer: (A)

Let U_1 be $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ so that

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Similarly $U_2 = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}, U_3 = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}.$

Hence $U = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix}$ and $|U| = 3.$

30. Correct Answer: (B)

Moreover adj $U = \begin{bmatrix} 1 & -2 & 0 \\ -7 & -5 & -3 \\ 9 & 6 & 3 \end{bmatrix}$.

Hence $U^{-1} = \dfrac{\text{adj}U}{3}$ and sum of the elements of $U^{-1} = 0$.

31. Correct Answer: (A)

The value of $\begin{bmatrix} 3 & 2 & 0 \end{bmatrix} U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 3 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} -1 & 4 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} = -3 + 8 = 5.$

Section – D

32. Correct Answer:

As $a + b = 10c$ and $c + d = 10a$

$ab = -11d, cd = -11b$

$\Rightarrow ac = 121$ and $(b + d) = 9(a + c)$

$a^2 - 10ac - 11d = 0$

$c^2 - 10ac - 11b = 0$

$\Rightarrow a^2 + c^2 - 20ac - 11(b + d) = 0$

$\Rightarrow (a + c)^2 - 22(121)$ or -22 (rejected)

$\therefore a + b + c + d = 1210.$

33. Correct Answer:

$$\frac{5050\int_0^1 \left(1-x^{50}\right)^{100} dx}{\int_0^1 \left(1-x^{50}\right)^{101} dx} = 5050\frac{I_{100}}{I_{101}}$$

$$I_{101} = \int_0^1 \left(1-x^{50}\right)\left(1-x^{50}\right)^{100} dx$$

$$= I_{100} - \int_0^1 x \cdot x^{49}\left(1-x^{50}\right)^{100} dx$$

$$= I_{100} - \left[\frac{-x\left(1-x^{50}\right)^{101}}{101}\right]_0^1 - \int_0^1 \frac{\left(1-x^{50}\right)^{101}}{5050}$$

$$I_{101} = I_{100} - \frac{I_{101}}{5050}$$

$$\Rightarrow 5050\frac{I_{100}}{I_{101}} = 5051.$$

34. Correct Answer:

$$a_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \cdots + (-1)^{x-1}\left(\frac{3}{4}\right)^n$$

$$= \frac{\frac{3}{4}\left(1-\left(-\frac{3}{4}\right)^n\right)}{+1\frac{3}{4}} = \frac{3}{7}\left(1-\left(-\frac{3}{4}\right)^n\right)$$

$$b_n > a_n \Rightarrow 2a_n < 1$$

$$\Rightarrow \frac{6}{7}\left(1-\left(-\frac{3}{4}\right)^n\right) < 1$$

$$\Rightarrow 1-\left(-\frac{3}{4}\right)^n < \frac{7}{6}$$

$$\Rightarrow -\frac{1}{6} < \left(-\frac{3}{4}\right)^n \Rightarrow \text{minimum natural number } n_0 = 6.$$

35. Correct Answer:

$$g(x)=\frac{d}{dx}\left(f(x)\cdot f'(x)\right)$$

to get the zero of $g(x)$ we take function

$$h(x)=f(x).f'(x)$$

between any two roots of $h(x)$ there lies at least one root of $h'(x)=0$

$$\Rightarrow g(x)=0$$

$$h(x)=0$$

$$\Rightarrow f(x)=0 \text{ or } f'(x)=0$$

$f(x)=0$ has 4 minimum solutions

$f'(x)=0$ minimum three solution

$h(x)=0$ minimum 7 solution

$\Rightarrow h'(x)=g(x)=0$ has minimum 6 solutions.

Section – E

36 Sol. As normal passes through (3, 0)

$$\Rightarrow 0=3m-2m-m^3$$

$$\Rightarrow m^3=m \Rightarrow m=0,\pm1$$

$$\therefore \text{Centroid} \equiv \left(\frac{\left(m_1^2+m_2^2+m_3^2\right)}{3},-\frac{2\left(m_1+m_2+m_3\right)}{3}\right)=\left(\frac{2}{3},0\right)$$

$$\text{Circum radius}=\left|\frac{-2m_1+2m_2}{2}\right|=2\,\text{units.}$$

$$Q\equiv\left(m_3^2,-2m_2\right)\equiv(1,-2)$$

$R \equiv \left(m_3^2, -2m_3\right) \equiv (1,2)$

Area of $\Delta PQR - \frac{1}{2} \times 4 \times 1 = 2\,sq.$units.

$R = \frac{QR}{2\sin \angle QPR} = \frac{4}{2\sin\left(2\tan^{-1} 2\right)}$

$\Rightarrow \frac{4}{2 \times \sin\left(\tan^{-1}\frac{4}{1-4}\right)} = \frac{4}{2 \times \frac{4}{5}} = \frac{5}{2}$

$\therefore$ circumcentre $\equiv \left(\frac{5}{2}.0\right)$.

37. Sol. (i) $I = \int_0^{\pi/2} (\sin x)^{\cos x}\left(\cos x \cdot \cot x - \log(\sin x)^{\sin x}\right) dx$

$\Rightarrow I = \int_0^{\pi/2} \frac{d}{dx}(\sin x)^{\cos x}\, dx = 1.$

(ii) The points of intersection of $-4y^2 = x$ and $x - 1 = -5y^2$ is $(-4,-1)$ and $(-4,1)$

Hence required area $= 3\left|\left[\int_0^1 \left(1-5y^2\right) dy - \int_0^1 -4y^2 dy\right]\right| = \frac{4}{3}.$

(iii) The point of intersection of $y = 3^{x-1}\log x$ and $y = x^x - 1$ is $(1,0)$

Hence $\frac{dy}{dx} = \frac{3^{x-1}}{x} + 3^{x-1}\log 3.\log x.\quad \left.\frac{dy}{dx}\right|_{(1,0)} = 1$

for $y = x^x - 1.\left.\frac{dy}{dx}\right|_{(1,0)} = 1$

If θ is the angle between the curve then $\tan\theta = 0 \Rightarrow \cos\theta = 1.$

(iv) $\frac{dy}{dx} = \left(\frac{2}{x+y}\right)$

$$\Rightarrow \frac{dx}{dy}-\frac{x}{2}=\frac{y}{2}$$

$$\Rightarrow xe^{-y/2}=\frac{1}{2}\int y\cdot e^{-y/2}dt$$

$$\Rightarrow x+y+2=ke^{y/2}=3e^{y/2}.$$

38. Sol. (i) Solving the two equations of ray i.e. $x+y=|a|$ and $ax-y=1$

we get $x=\frac{|a|+1}{a+1}>0$ and $y=\frac{|a|-1}{a+1}>0$

when $a+1>0;$ we get $a>1 \quad \therefore a_0=1.$

(ii) We have $\vec{a}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}\Rightarrow\vec{a}\cdot\hat{k}=\gamma$

Now; $\hat{k}\times\left(\hat{k}\times\hat{a}\right)=\left(\hat{k}\cdot\vec{a}\right)\hat{k}-\left(\hat{k}\cdot\hat{k}\right)\vec{a}$

$=\gamma\hat{k}-\left(\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}\right)$

$=\alpha\hat{i}+\beta\hat{j}=\vec{0}\Rightarrow\alpha=\beta=0$

As $\alpha+\beta+\lambda=2\Rightarrow\gamma=2$

(iii) $\left|\int_0^1\left(1-y^2\right)dy\right|+\left|\int_1^0\left(y^2-1\right)dy\right|$

$=2\int_0^1\left(1-y^2\right)dy=\frac{4}{3}$

$\left|\int_0^1\sqrt{1-x}\,dx\right|+\left|\int_{-1}^0\sqrt{1+x}\,dx\right|=2\int_0^1\sqrt{1-x}\,dx$

$=2\int_0^1\sqrt{x}\,dx=2\cdot\frac{2}{3}\cdot x^{3/2}\Big|_0^1=\frac{4}{3}.$

(IV) $\sin A\sin B\sin C+\cos A\cos B\le\sin A\sin B\sin C+\cos A\cos B=\cos\left(A-B\right)$

$\Rightarrow \cos(A-B) \geq 1 \Rightarrow \cos(A-B)=1 \Rightarrow \sin C = 1.$

39. Sol. (i) $\sum_{i=1}^{\infty} \tan^{-1}\left(\frac{1}{2i^2}\right)=t$

Now; $\sum_{i=1}^{\infty} \tan^{-1}\left[\frac{2}{4i^2-1+1}\right]$

$=\sum_{i=1}^{\infty}\left[\tan^{-1}(2i+1)-\tan^{-1}(2i+1)\right]$

$=\left[\left(\tan^{-1}3-\tan^{-1}1\right)+\left(\tan^{-1}5-\tan^{-1}3\right)+\ldots+\tan^{-1}(2n+1)-\tan^{-1}(2n-1)\ldots\ldots\infty\right]$

$t=\tan^{-1}(2n+1)-\tan^{-1}1=\lim_{n\to\infty}\tan^{-1}\frac{2n}{1+(2n+1)}$

$\Rightarrow \tan t=\lim_{n\to\infty}\frac{n}{n+1} \Rightarrow t=\frac{\pi}{4}$

(ii) We have $\cos\theta_1=\dfrac{1-\tan^2\frac{\theta_1}{2}}{1+\tan^2\frac{\theta_1}{2}}=\dfrac{a}{b+c} \Rightarrow \tan^2\frac{\theta_1}{2}=\dfrac{b+c-a}{b+c+a}$

Also, $\cos\theta_3=\dfrac{1-\tan^2\frac{\theta_3}{2}}{1+\tan^2\frac{\theta_3}{2}}=\dfrac{c}{a+b} \Rightarrow \tan^2\frac{\theta_3}{2}=\dfrac{a+b-c}{a+b+c}$

$\therefore \tan^2\frac{\theta_1}{2}+\tan^2\frac{\theta_3}{2}=\frac{2b}{3b}=\frac{2}{3}$

(iii) Line through $(0,1,0)$ and perpendicular to plane $x+2y+2z=0$ is given by

$\frac{x-0}{1}=\frac{y-1}{2}=\frac{z-1}{2}=r.$

Let $P(r,2r+1,2r)$ be the foot of perpendicular on the straight line then

$r\times 1+(2r+1)2+2\times 2r=0\Rightarrow r=\frac{2}{9}$

$\therefore$ Point is given by $\left(-\frac{2}{9},\frac{5}{9},-\frac{4}{9}\right)$

$\therefore$ Required perpendicular distance $=\sqrt{\frac{4+25+16}{81}}=\sqrt{\frac{5}{2}}$ units.

(iv) Data could not be retrieved.