

IITJEE–2006

Mathematics

[Time allowed: 2 hours]

Note: Question number 1 to 12 carries (3, -1) *marks* each, 13 to 20 carries (5, -1) *marks* each, 21 to 32 carries (5, -2) *marks* each and 33 to 40 carries (6, 0) *marks* each.

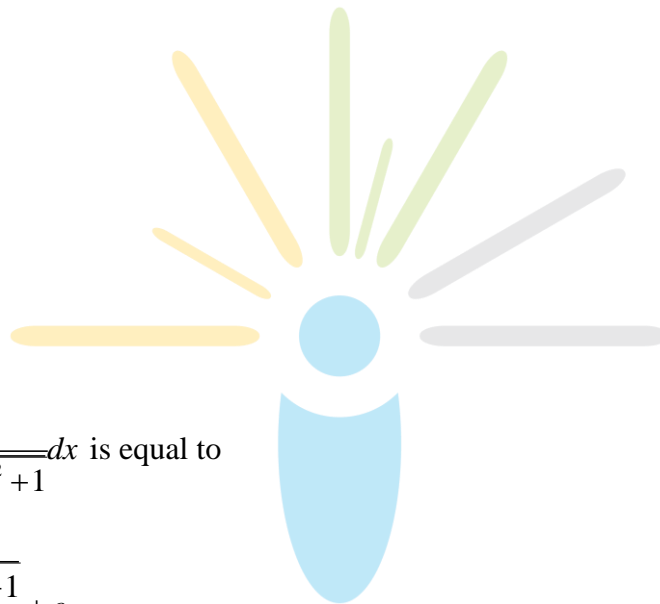
Section – A (Single Option Correct)

1. For $x > 0$, $\lim_{x \rightarrow 0} \left((\sin x)^{1/x} + (1/x)^{\sin x} \right)$ is

- (A) 0
- (B) -1
- (C) 1
- (D) 2

2. $\int \frac{x^2 - 1}{x^3 \sqrt{2x^4 - 2x^2 + 1}} dx$ is equal to

- (A) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^2} + c$
- (B) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^3} + c$
- (C) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x} + c$
- (D) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + c$

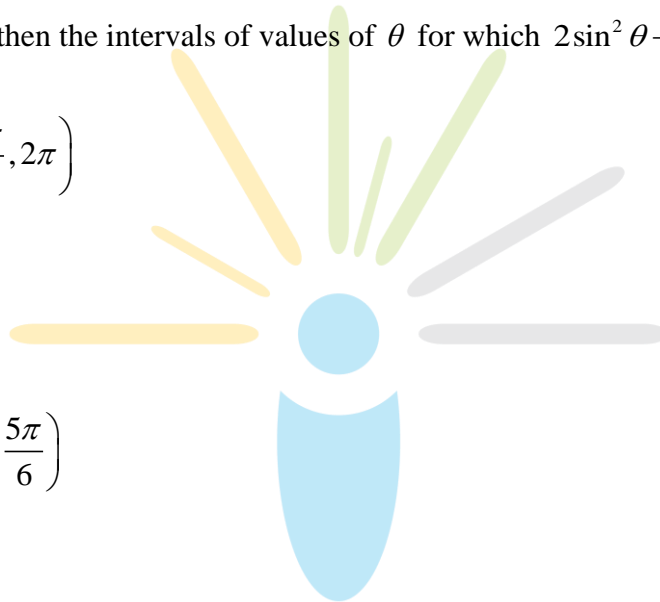


3. Given an isosceles triangle, whose one angle is 120° and radius of its incircle $= \sqrt{3}$. Then the area of the triangle in sq. units is

- (A) $7+12\sqrt{3}$
- (B) $12-7\sqrt{3}$
- (C) $12+7\sqrt{3}$
- (D) 4π

4. If $0 < \theta < 2\pi$, then the intervals of values of θ for which $2\sin^2 \theta - 5\sin \theta + 2 > 0$, is

- (A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$
- (B) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$
- (C) $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$
- (D) $\left(\frac{41\pi}{48}, \pi\right)$



5. If $w = \alpha + i\beta$, where $\beta \neq 0$ and $z \neq 1$, satisfies the condition that $\left(\frac{w - \bar{w}z}{1 - z}\right)$ is purely real,

then the set of values of z is

- (A) $\{z : |z| = 1\}$
- (B) $\{z : z = \bar{z}\}$

(C) $\{z : z \neq 1\}$

(D) $\{z : |z|=1, z \neq 1\}$

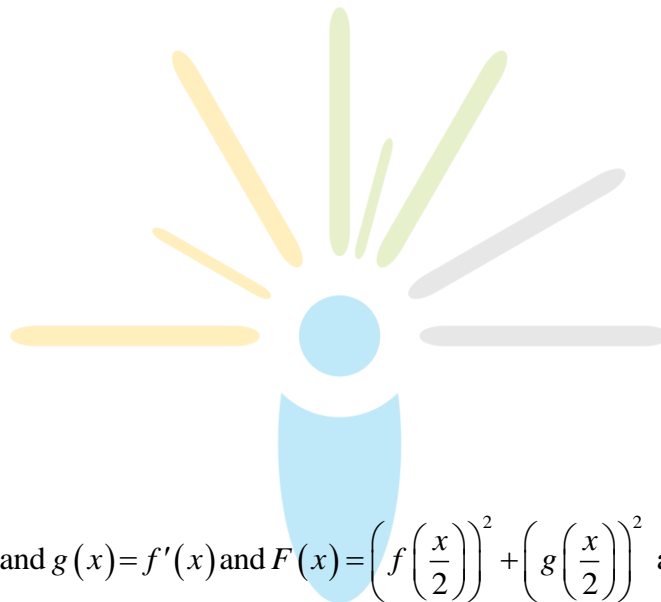
6. Let a, b, c be the sides of a triangle. No two of them are equal and $\lambda \in R$. If the roots of the equation $x^2 + 2(a+b+c)x + 3\lambda(ab+bc+ca) = 0$ are real, then

(A) $\lambda < \frac{4}{3}$

(B) $\lambda < \frac{5}{3}$

(C) $\lambda \in \left(\frac{1}{3}, \frac{5}{3}\right)$

(D) $\lambda \in \left(\frac{4}{3}, \frac{5}{3}\right)$



7. If $f'(x) = -f(x)$ and $g(x) = f'(x)$ and $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and given that

$F(5) = 5$, then $F(10)$ is equal to

(A) 5

(B) 10

(C) 0

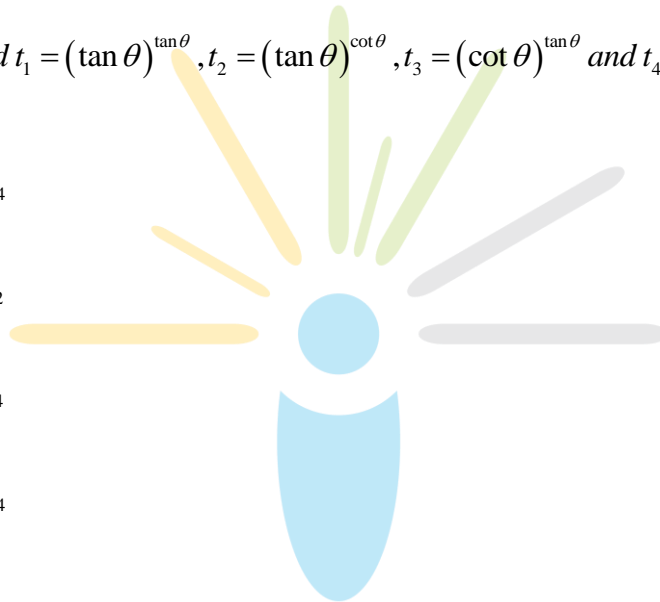
(D) 15

8. If r, s, t are prime numbers and p, q are the positive integers such that the LCM of p, q is $r^2 t^4 s^2$, then the number of ordered pair (p, q) is

- (A) 252
- (B) 254
- (C) 225
- (D) 224

9. Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan \theta)^{\tan \theta}, t_2 = (\tan \theta)^{\cot \theta}, t_3 = (\cot \theta)^{\tan \theta}$ and $t_4 = (\cot \theta)^{\cot \theta}$, then

- (A) $t_1 > t_2 > t_3 > t_4$
- (B) $t_4 > t_3 > t_1 > t_2$
- (C) $t_3 > t_1 > t_2 > t_4$
- (D) $t_2 > t_3 > t_1 > t_4$



10. The axis of a parabola is along the line $y = x$ and the distance of its vertex from origin is $\sqrt{2}$ and that from its focus is $2\sqrt{2}$. If vertex and focus both lie in the first quadrant, then the equation of the parabola is

- (A) $(x + y)^2 = (x - y - 2)$
- (B) $(x - y)^2 = (x + y - 2)$
- (C) $(x - y)^2 = 4(x + y - 2)$
- (D) $(x - y)^2 = 8(x + y - 2)$

11. A plane passes through $(1, -2, 1)$ and is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$. The distance of the plane from the point $(1, 2, 2)$ is
- (A) 0
 (B) 1
 (C) $\sqrt{2}$
 (D) $2\sqrt{2}$

12. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. A vector in the plane of \vec{a} and \vec{b} whose projection on \vec{c} is $\frac{1}{\sqrt{3}}$, is

- (A) $4\hat{i} - \hat{j} + 4\hat{k}$
 (B) $3\hat{i} + \hat{j} - 3\hat{k}$
 (C) $2\hat{i} + \hat{j} - 2\hat{k}$
 (D) $4\hat{i} + \hat{j} - 4\hat{k}$

Section – B (May have more than one option correct)

13. The equations of the common tangents to the parabola $y = x^2$ and $y = -(x - 2)^2$ is/are
- (A) $y = 4(x - 1)$
 (B) $y = 0$
 (C) $y = -4(x - 1)$
 (D) $y = -30x - 50$

14. If $f(x) = \min\{1, x^2, x^3\}$, then

- (A) $f(x)$ is continuous $\forall x \in R$
- (B) $f(x) > 0, \forall x > 1$
- (C) $f(x)$ is not differentiable but continuous $\forall x \in R$
- (D) $f(x)$ is not differentiable for two values of x

15. A tangent drawn to the curve $y = f(x)$ at $P(x, y)$ cuts the x -axis and y -axis at A and B respectively such that $BP : AP = 3 : 1$, given that $f(1) = 1$, then

- (A) equation of curve is $x \frac{dy}{dx} - 3y = 0$
- (B) normal at $(1, 1)$ is $x + 3y = 4$
- (C) curve passes through $(2, 1/8)$
- (D) equation of curve is $x \frac{dy}{dx} + 3y = 0$

16. If a hyperbola passes through the focus of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with the major and minor axes of the ellipse, and the product of eccentricities is 1, then

- (A) the equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(B) the equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{25} = 1$

(C) focus of hyperbola is $(5, 0)$

(D) focus of hyperbola is $(5\sqrt{3}, 0)$

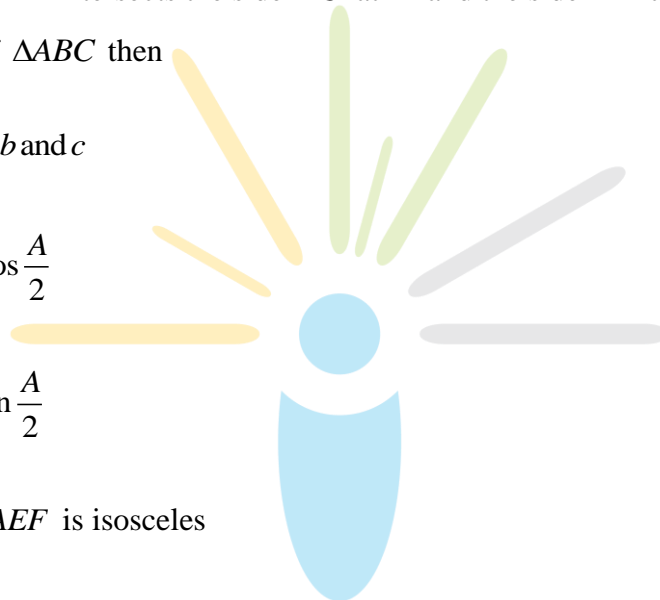
17. Internal bisector of $\angle A$ of triangle ABC meets side BC at D . A line drawn through D perpendicular to AD intersects the side AC at E and the side AB at F . If a, b, c represent sides of $\triangle ABC$ then

(A) AE is HM of b and c

(B) $AD = \frac{2bc}{b+c} \cos \frac{A}{2}$

(C) $EF = \frac{4bc}{b+c} \sin \frac{A}{2}$

(D) the triangle AEF is isosceles



18. $f(x)$ is cubic polynomial which has local maximum at

$x = -1$. If $f(2) = 18$, $f(1) = -1$ and $f'(x)$ has local minima at $x = 0$, then

(A) the distance between $(-1, 2)$ and $(a, f(a))$, where $x = a$ is the point of local minima is $2\sqrt{5}$

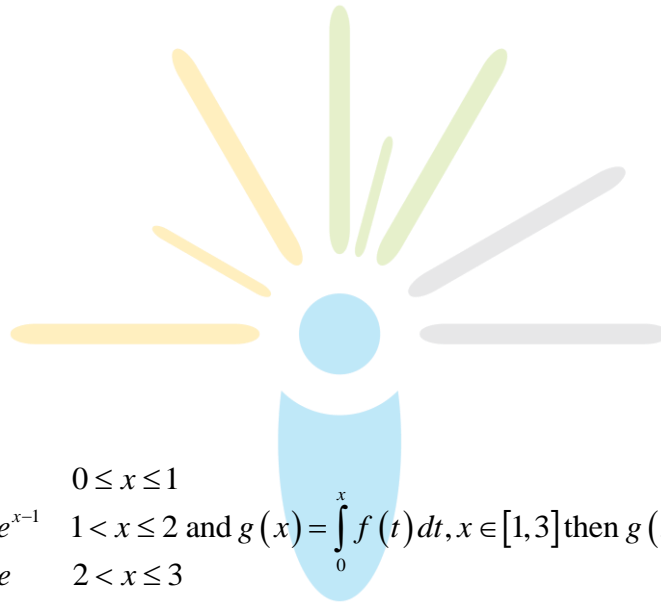
(B) $f(x)$ is increasing for $x \in [1, 2\sqrt{5}]$

(C) $f(x)$ has local minima at $x = 1$

(D) the value of $f(0) = 5$

19. Let \bar{A} be vector parallel to line of intersection of planes P_1 and P_2 through origin. P_1 is parallel to the vectors $2\hat{j} + 3\hat{k}$ and $4\hat{j} - 3\hat{k}$ and P_2 is parallel to $\hat{j} - \hat{k}$ and $3\hat{i} + 3\hat{j}$, then the angle between vectors \bar{A} and $2\hat{i} + \hat{j} - 2\hat{k}$ is

- (A) $\frac{\pi}{2}$
- (B) $\frac{\pi}{4}$
- (C) $\frac{\pi}{6}$
- (D) $\frac{3\pi}{4}$



20. $f(x) = \begin{cases} e^x & 0 \leq x \leq 1 \\ 2 - e^{x-1} & 1 < x \leq 2 \\ x - e & 2 < x \leq 3 \end{cases}$ and $g(x) = \int_0^x f(t) dt, x \in [1, 3]$ then $g(x)$ has

- (A) local maxima at $x = 1 + \ln 2$ and local minima at $x = e$
- (B) local maxima at $x = 1$ and local minima at $x = 2$
- (C) no local maxima
- (D) no local minima

Section – C

Comprehension I

There are n urns each containing $n+1$ balls such that the i th urn contains i white balls and $(n+1-i)$ red balls. Let u_i be the event of selecting i th urn, $i = 1, 2, \dots, n$ and w denotes the event of getting a white ball.

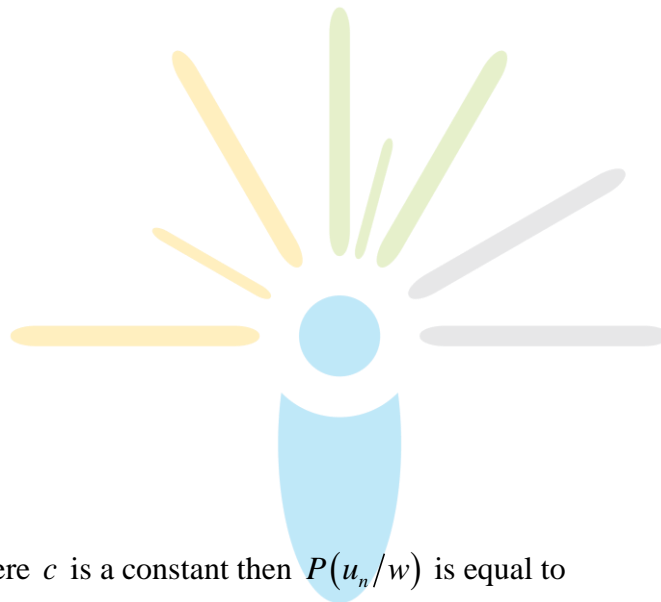
21. If $p(u_i) \propto i$, where $i = 1, 2, 3, \dots, n$, then $\lim_{n \rightarrow \infty} P(w)$ is equal to

(A) 1

(B) $\frac{2}{3}$

(C) $\frac{3}{4}$

(D) $\frac{1}{4}$



22. If $P(u_i) = c$ where c is a constant then $P(u_n/w)$ is equal to

(A) $\frac{2}{n+1}$

(B) $\frac{1}{n+1}$

(C) $\frac{n}{n+1}$

(D) $\frac{1}{2}$

23. If n is even and E denotes the event of choosing even numbered urn $\left(P(u_i) = \frac{1}{n}\right)$, then the value of $P(w/E)$ is

(A) $\frac{n+2}{2n+1}$

(B) $\frac{n+2}{2(n+1)}$

(C) $\frac{n}{n+1}$

(D) $\frac{1}{n+1}$

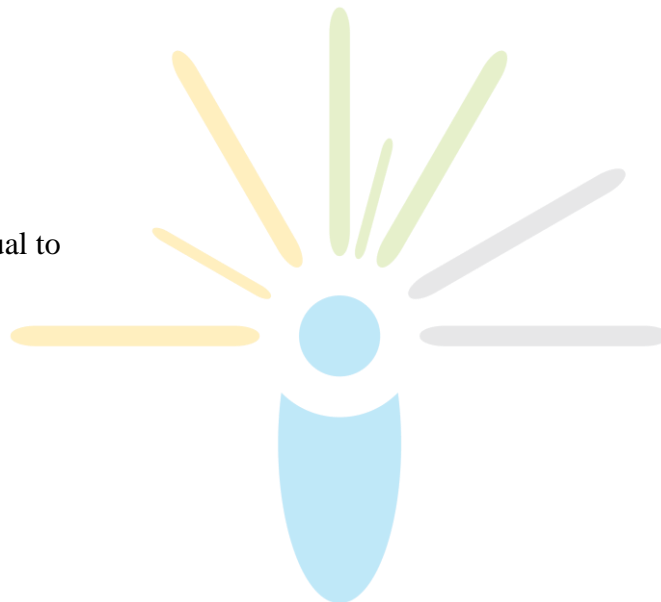
24. $\int_0^{\pi/2} \sin x dx$ is equal to

(A) $\frac{\pi}{8}(1+\sqrt{2})$

(B) $\frac{\pi}{4}(1+\sqrt{2})$

(C) $\frac{\pi}{8\sqrt{2}}$

(D) $\frac{\pi}{4\sqrt{2}}$



25. If $f'(x) < 0 \forall x \in (a, b)$ and c is a point such that $a < c < b$, and $(c, f(c))$ is the point lying on the curve for which $F(c)$ is maximum, then $f'(c)$ is equal to

- (A) $\frac{f(b) - f(a)}{b - a}$
- (B) $\frac{2(f(b) - f(a))}{b - a}$
- (C) $\frac{2f(b) - f(a)}{2b - a}$
- (D) 0

Comprehension III

Let ABCD be a square of side length 2 units. C_2 is the circle through vertices A, B, C, D and C_1 is the circle touching all the sides *of the square ABCD*. L is a line through A.

26. If P is a point on C_1 and Q is another point on C_2 , then $\frac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2}$ is equal to

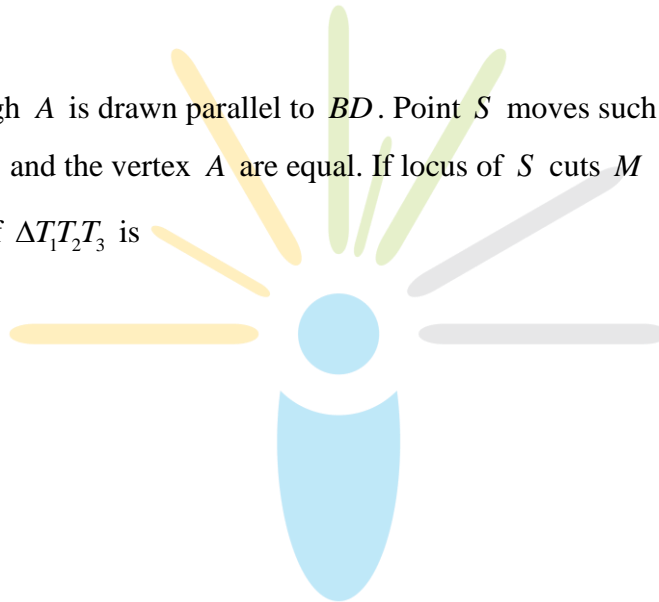
- (A) 0.75
- (B) 1.25
- (C) 1
- (D) 0.5

27. A circle touches the line L and the circle C_1 externally such that both the circles are on the same side of the line, then the locus of centre of the circle is

- (A) ellipse
- (B) hyperbola
- (C) parabola
- (D) parts of straight line

28. A line M through A is drawn parallel to BD . Point S moves such that its distances from the line BD and the vertex A are equal. If locus of S cuts M at T_2 and T_3 and AC at T_1 , then area of $\Delta T_1T_2T_3$ is

- (A) $\frac{1}{2}$ sq. units
- (B) $\frac{2}{3}$ sq. units
- (C) 1 sq. units
- (D) 2 sq. units



Comprehension IV

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}, \text{ if } U_1, U_2 \text{ and } U_3 \text{ are columns matrices satisfying.}$$

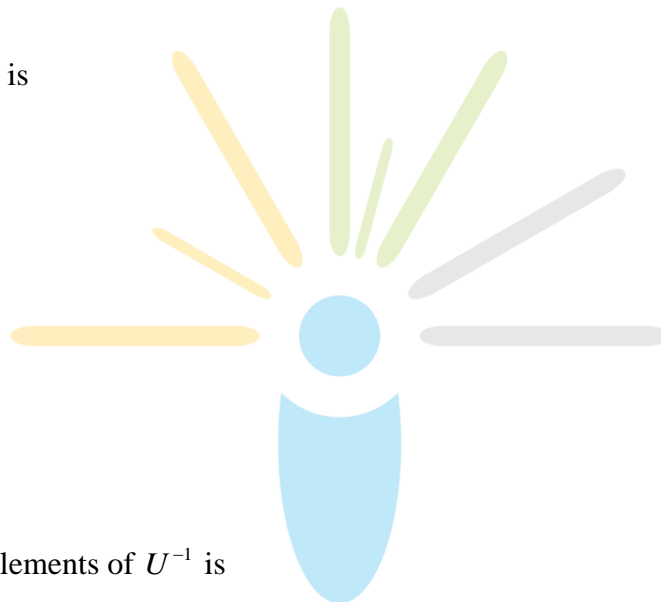
$AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ and U is 3×3 matrix whose columns are U_1, U_2, U_3 then answer the following questions

29. The value of $|U|$ is

- (A) 3
- (B) -3
- (C) $3/2$
- (D) 2

30. The sum of the elements of U^{-1} is

- (A) -1
- (B) 0
- (C) 1
- (D) 3



31. The value of $[3 \ 2 \ 0]U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ is

- (A) 5
- (B) $5/2$
- (C) 4
- (D) $3/2$

Section – D

32. If roots of the equation $x^2 - 10cx - 11d = 0$ are a, b and those of $x^2 - 10ax - 11b = 0$ are c, d , then the value of $a + b + c + d$ is (a, b, c and d are distinct numbers)

33. The value of $5050 \frac{\int_0^1 (1-x^{50})^{100} dx}{\int_0^1 (1-x^{50})^{101} dx}$ is

34. If $a_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \dots + (-1)^{n-1} \left(\frac{3}{4}\right)^n$ and $b_n = 1 - a_n$ then find the minimum natural number n_0 such that $b_n > a_n \forall n > n_0$

35. If $f(x)$ is a twice differentiable function such that

$f(a) = 0, f(b) = 2, f(c) = -, f(d) = 2, f(e) = 0$, where $a < b < c < d < e$, then the minimum number of zeroes of $g(x) = (f'(x))^2 + f'(x)f(x)$ in the interval $[a, e]$ is

Section – E

36. Match the following:

Normals are drawn at points P, Q and R lying on the parabola $y^2 = 4x$ which intersect at (3, 0). Then

- | | | | |
|-------|--|-----|------------|
| (i) | Area of ΔPQR | (A) | 2 |
| (ii) | Radius of circumcircle of ΔPQR | (B) | $5/2$ |
| (iii) | Centroid of ΔPQR | (C) | $(5/2, 0)$ |
| (iv) | Circumcentre of ΔPQR | (D) | $(2/3, 0)$ |

37. Match the following

- | | | | |
|-------|---|-----|-----------|
| (i) | $\int_0^{\pi/2} (\sin x)^{\cos x} (\cos x \cot x - \log(\sin x)^{\sin x}) dx$ | (A) | 1 |
| (ii) | Area bounded by $-4y^2 = x$ and $x - 1 = -5y^2$ | (B) | 0 |
| (iii) | Cosine of the angle of intersection of curves $y = 3^{x-1} \log x$ and $y = x^x - 1$ is | (C) | $6 \ln 2$ |
| (iv) | Data could not be retrieved. | (D) | $4/3$ |

38. Match the following

(i) Two rays in the first quadrant $x + y = |a|$ and $ax - y = 1$ (A) 2
intersects each other in the interval $a \in (a_0, \infty)$, the value
of a_0 is

(ii) Point (α, β, γ) lies on the plane $x + y + z = 2$. Let (B) $4/3$
 $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $\hat{k} \times (\hat{k} \times \vec{a}) = 0$, then $\gamma =$.

(iii) $\left| \int_0^1 (1 - y^2) dy \right| + \left| \int_1^0 (y^2 - 1) dy \right|$ (C) $\left| \int_0^1 \sqrt{1-x} dx \right| + \left| \int_{-1}^0 \sqrt{1+x} dx \right|$

(iv) If (D) 1
 $\sin A \sin B \sin C + \cos A \cos B = 1$, then the value of $\sin C =$

39. Match the following

(i) $\sum_{i=1}^{\infty} \tan^{-1} \left(\frac{1}{2i^2} \right) = t$, then $\tan t =$ (A) 0

(ii) Sides a, b, c of a triangle ABC are in AP and (B) 1
 $\cos \theta_1 = \frac{a}{b+c}$, $\cos \theta_2 = \frac{b}{a+c}$, $\cos \theta_3 = \frac{c}{a+b}$, then $\tan^2 \left(\frac{\theta_1}{2} \right) + \tan^2 \left(\frac{\theta_3}{2} \right) =$

(iii) A line is perpendicular to $x + 2y + 2z = 0$ and passes through $(0, 1, 0)$. (C) $\frac{\sqrt{5}}{3}$
The perpendicular distance of this line from the origin is

(iv) Data could not be retrieved. (D) $2/3$