# JEE MAIN-2007

## MATHEMATICS

45. **Sol. (A)**

The given ellipse is $\frac{x^2}{4}+\frac{y^2}{3}=1$

$\Rightarrow a=2,\ b=\sqrt{3}\Rightarrow 3=4\left(1-e^2\right)\Rightarrow e=\frac{1}{2}$

so that $ae=1$

Hence the eccentricity $e_1$, of the hyperbola is given by

$1=e_1\sin\theta\Rightarrow e_1=\operatorname{cosec}\theta$

$\Rightarrow b^2\sin^2\theta\left(\operatorname{cosec}^2\theta-1\right)=\cos^2\theta$

Hence the hyperbola is $\frac{x^2}{\sin^2\theta}+\frac{y^2}{\cos^2\theta}=1$ or $x^2\operatorname{cosec}^2\theta-y^2\sec^2\theta=1$

46. **Sol. (A)**

Slope of the line joining the points $\left(c-1,\ e^{c-1}\right)$ and $\left(c+1,\ e^{c+1}\right)$ is equal to

$\Rightarrow$ tangent to the curve $y=e^x$ will intersect the given line to the left of the line $x=c$.

**Alternative**

The equation of the tangent to the curve $y=e^x$ at $\left(c,e^c\right)$ is

$$y-e^c=e^c\left(x-c\right)\quad \ldots(1)$$

Equation of the line joining the given points is

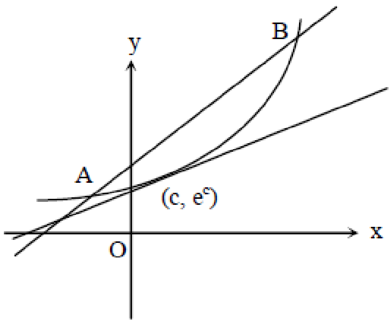
$$y-e^{c-1}=\frac{e^c\left(e-e^{-1}\right)}{2}\left[x-\left(c-1\right)\right]\quad \ldots(2)$$

Eliminating $y$ from $(1)$ and $(2)$, we get

$$\left[x-\left(c-1\right)\right]\left[2-\left(e-e^{-1}\right)\right]=2e^{-1}$$

$$\text{or } x-c\frac{e+e^{-1}-2}{2-\left(e-e^{-1}\right)}<0\Rightarrow x<c$$

$\Rightarrow$ the line $(1)$ and $(2)$ meet on the left of the line $x=c$.



47. **Sol. (D)**

Let $OA=3$, so that the complex number

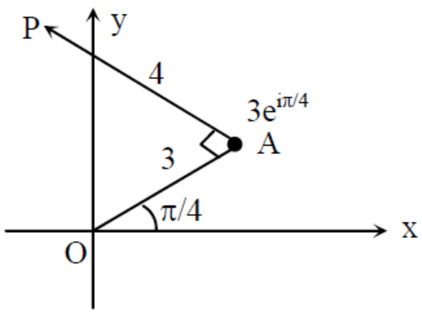
associated with $A$ is $3e^{i\pi/4}$.

If $z$ is the complex number associated with $P$,

then $\frac{z-3e^{i\pi/4}}{0-3e^{i\pi/4}}=\frac{4}{3}e^{-i\pi/2}=\frac{4i}{3}$

$\Rightarrow 3z-9e^{i\pi/4}=12e^{i\pi/4}$

$\Rightarrow z=(3+4i)e^{i\pi/4}$




48. **Sol. (A)**

$$\lim_{t\to x}\frac{t^2f(x)-x^2f(t)}{t-x}=1$$

$\Rightarrow x^2f'(x)-2xf(x)+1=0$

$\Rightarrow f(x)=cx^2+\frac{1}{3x}$ also $f(1)=1$

$\Rightarrow c\frac{2}{3}.$

Hence $f(x)=\frac{2}{3}x^2+\frac{1}{3x}$

49. **Sol. (C)**

$$2\sin^2\theta - \cos 2\theta = 0 \Rightarrow \sin^2\theta = \frac{1}{4}$$

$$\text{also } 2\cos^2\theta = 3\sin\theta \Rightarrow \sin\theta = \frac{1}{2}$$

$\Rightarrow$ two solutions in $[0, 2\pi]$.

50. **Sol. (D)**

The equation $x^2 - px + r = 0$ has roots $(\alpha, \beta)$ and the equation

$x^2 - qx + r = 0$ has roots $\left(\frac{\alpha}{2}, 2\beta\right)$.

$$\Rightarrow r = \alpha\beta \text{ and } \alpha + \beta = p \text{ and } \frac{\alpha}{2} + 2\beta = q$$

$$\Rightarrow \beta = \frac{2q - p}{3} \text{ and } \alpha = \frac{2(2p - q)}{3}$$

$$\Rightarrow \alpha\beta = r = \frac{2}{9}(2q - p)(2p - q)$$

51. **Sol. (C)**

$$\begin{vmatrix} -\lambda^2 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0 \Rightarrow \lambda^6 - 3\lambda^2 - 2 = 0$$

$$\Rightarrow (1 + \lambda^2)(\lambda^2 - 2) = 0 \Rightarrow \lambda = \pm\sqrt{2}$$

52. **Solution. (C)**

Let $E =$ event when each American man is seated adjacent to his wife

$A =$ event when Indian man is seated adjacent to his wife

Now, $n(A \cap E) = (4!) \times (2!)^5$

Even when each American man is seated adjacent to his wife

Again $n(E) = (5!) \times (2!)^4 \Rightarrow$

$$\Rightarrow P\left(\frac{A}{E}\right) = \frac{n(A \cap E)}{n(E)} = \frac{(4!) \times (2!)^5}{(5!) \times (2!)^4} = \frac{2}{5}$$

**Alternative**

Fixing four American couples and one Indian man in between any two couples; we have $5$ different ways in which his wife can be seated, of which $2$ cases are favorable.

$\therefore$ required probability $= \frac{2}{5}$

53.**Sol. (A)**

$$\lim_{x \to \frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t)\,dt}{x^2 - \frac{\pi^2}{16}} \quad \left(\frac{0}{0} \text{form}\right)$$

$$\text{Let } L = \lim_{x \to \frac{\pi}{4}} \frac{f(\sec^2 x)\, 2 \sec x \sec x \tan x}{2x}$$

$$\therefore L = \frac{2f(2)}{\pi/4} = \frac{8f(2)}{\pi}$$

54. **Sol.** (C)

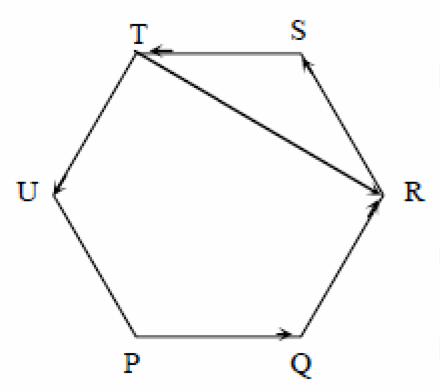
Since $\overrightarrow{PQ} И \overrightarrow{TR}$ $\therefore$ $\overrightarrow{TR}$ is resultant of $\overrightarrow{SR}$ and $\overrightarrow{ST}$ vector.

$\Rightarrow \overrightarrow{PQ} \times \left(\overrightarrow{RS} + \overrightarrow{ST}\right) \neq 0$

But for statement $2$, we have $\overrightarrow{PQ} \times \overrightarrow{RS} = \vec{0}$

which is not possible as $\overrightarrow{PQ} И \overrightarrow{TR}$

Hence, statement $1$ is true and statement $2$ is false.




55. **Sol. (D)**

$$F(x) = \int \sin^2 x dx = \int \frac{1 - \cos 2x}{2} dx$$

$$\Rightarrow F(x) = \frac{1}{4}(2x - \sin 2x) + c$$

Since, $F(x + \pi) \neq F(x)$

Hence statement 1 is false.

But statement 2 is true as $\sin^2 x$ is periodic with period $\pi$.

56. **Sol. (D)**

Statement : 1

If $P(H_i \cap E) = 0$ for some $i$ , then $P\left(\frac{H_i}{E}\right) = P\left(\frac{E}{H_i}\right) = 0$

If $P(H_i \cap E) \neq 0$ for $\forall\ i = 1, 2 \ldots n,$ , then

$$P\left(\frac{H_i}{E}\right) = P\frac{P(H_i \cap E)}{P(H_i)} \times \frac{P(H_i)}{P(E)}$$

$$= \frac{P\left(\frac{E}{H_i}\right) \times P(H_i)}{P(E)} > P\left(\frac{E}{H_i}\right) \cdot P(H_i) \quad [\text{as} < P(E) < 01]$$

Hence statement 1 may not always be true.

Statement : 2

Clearly $H_1 \cup H_2 \ldots H_n = S$ (sample space)

$\Rightarrow P(H_1) + P(H_2) + \ldots + P(H_n) = 1$ .

57. **Sol. (A)**

Since the tangents are perpendicular $\Rightarrow$ locus of perpendicular tangents to circle

$x^2 + y^2 = 169$ is a director circle having equation $x^2 + y^2 = 338$ .

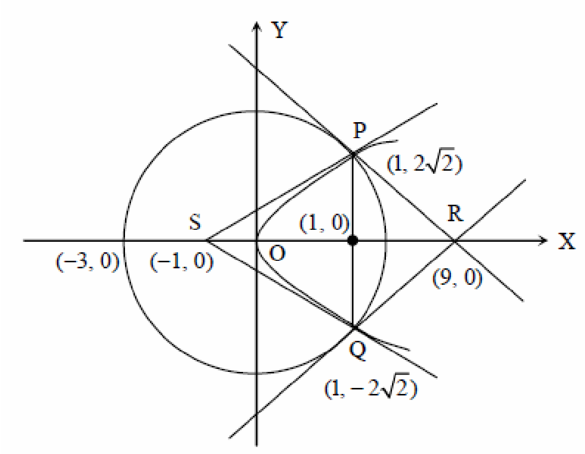
58. **Sol.** (C)

Coordinates of $P$ and $Q$ are $\left(1,+2\sqrt{2}\right)$ and $\left(1,-2\sqrt{2}\right)$ .

Area of $\Delta PQR=\frac{1}{2}\cdot 4\sqrt{2}\cdot 8=16\sqrt{2}$

Area of $\Delta PQS=\frac{1}{2}\cdot 4\sqrt{2}\cdot 2=4\sqrt{2}$

Ratio of area of triangle $PQS$ and $PQR$ is $1:4$.




59. **Sol.** (B)

Equation of circumcircle of $\Delta PRS$ is $(x+1)(y-9)+y^2+\lambda y=0$

It will pass through $\left(1,2\sqrt{2}\right)$, then $-16+8+\lambda.2\sqrt{2}=0$

$\lambda\frac{8}{2\sqrt{2}}=2\sqrt{2}$

Equation of circumcircle is $x^2+y^2-x8+2\sqrt{2}y-9=0$.

Hence its radius is $3\sqrt{3}$.

**Alternative**

Let $\angle PSR = \theta$

$$\Rightarrow \sin\theta \frac{2\sqrt{2}}{2\sqrt{3}}$$

$$\Rightarrow PR = 6\sqrt{2} = 2R \cdot \sin\theta \Rightarrow R3\sqrt{3}$$

60. **Sol.** (D)

Radius of incircle is $r = \frac{\Delta}{s}$

as $\Delta = 16\sqrt{2}$

$$s = \frac{6\sqrt{2} + 6\sqrt{2} + 4\sqrt{2}}{2} = 8\sqrt{2}$$

$$r = \frac{16\sqrt{2}}{8\sqrt{2}} = 2$$

61. **Sol.** (B)

$$V_r = \frac{r}{2}\left[2r + (r-1)(2r-1)\right] = \frac{1}{2}\left(2r^3 - r^2 + r\right)$$

$$\sum V_r = \frac{1}{12} n(n+1)\left(3n^2 + n + 2\right)$$

62. **Sol. (D)**

$$V_{r+1}-V_r=(r+1)^3-r^3-\frac{1}{2}\left[(r+1)^2-r^2\right]+\frac{1}{2}(1)$$

$$=3r^2+2r+1$$

$$T_r=3r^2+2r-1=(r+1)(3r-1)$$

which is a composite number.

63. **Sol. (B)**

$$T_r=3r^2+2r-1$$

$$T_{r+1}=3(r+1)^2+2(r+1)-1$$

$$Q_r=T_{r+1}-T_r=3[2r+1]+2[1]$$

$$Q_r=6r+5$$

$$Q_{r+1}=6(r\ +\ 1)+5$$

Common difference $=Q_{r+1}-Q_r=6$.

**64. Sol. A – r B – q C – p D – s**

$$\Delta=\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}=-\frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$$

(A). If $a+b+c \neq 0$ and $a^2+b^2+c^2 = ab+bc+ca$

$\Rightarrow \Delta = 0$ and $a = b = c \neq 0$

$\Rightarrow$ the equations represent identical planes.

(B). $a+b+c = 0$ and $a^2+b^2+c^2 \neq ab+bc+ca$

$\Rightarrow \Delta \neq 0 \Rightarrow$ the equation represent planes meeting at only one point.

$ax+by = (a+b)z$

$bx+cy = (b+c)z$

$\Rightarrow (b^2-ac)y = (b^2-ac)z \Rightarrow y = z$

$\Rightarrow ax+by+cy = 0 \Rightarrow ax = ay \Rightarrow x = y = z.$

(C). $a+b+c \neq 0$ and $a^2+b^2+c^2 \neq ab+bc+ca$

$\Rightarrow \Delta \neq 0 \Rightarrow$ the equation represent whole of the three dimensional space.

(D). $a+b+c = 0$ and $a^2+b^2+c^2 = ab+bc+ca \Rightarrow a = b = c = 0$

$\Rightarrow$ the equation represent whole of the three dimensional space.

**65. Sol. A – s B – s C – p D – r**

(A). $\int_{-1}^{1} \frac{dx}{1+x^2} = \frac{\pi}{2}$

(B) $\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}} = \frac{\pi}{2}$

(C). $\int_{2}^{3} \frac{dx}{1-x^2} = \frac{1}{2}\ln\frac{2}{3}$

(D). $\int_{1}^{2} \frac{dx}{x\sqrt{x^2-1}} = \frac{\pi}{3}$

**66. Sol. A – p, q, r B – p, s C – r, s D – p, q**

(A). $x|x|$ is continuous, differentiable and strictly increasing in $(-1,1)$.

(B). $\sqrt{|x|}$ is continuous in $(-1,1)$ and not differentiable at $x=0$.

(C). $x+[x]$ is strictly increasing in $(-1,1)$ and discontinuous at $x=0 \Rightarrow$ not differentiable at $x=0$.

(D). $|x-1|+|x+1| = 2 \text{ in } (-1,1)$

the function is continuous and differentiable in $(-1,1)$.