JEE MAIN-2007

MATHEMATICS

[Time: 3 hours] [Maximum Marks: 243]

A. General Instructions :

Note: (i) The question paper consists of 3 parts (Physics, Chemistry and Mathematics). Each part has 4 sections.

(ii) Section I contains 9 multiple choice questions which have only one correct answer. Each question carries **+3 marks** each for correct answer and **– 1 mark** for each wrong answer.

(iii) Section II contains 4 questions. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason).

Bubble (A) if both the statements are TRUE and STATEMENT-2 is the correct explanation of STATEMENT-1

Bubble (B) if both the statements are TRUE but STATEMENT-2 is NOT the correct explanation of STATEMENT- 1

Bubble (C) if STATEMENT-1 is TRUE and STATEMENT-2 is FALSE.

Bubble (D) if STATEMENT-1 is FALSE and STATEMENT-2 is TRUE.

carries **+3 marks** each for correct answer and **– 1 mark** for each wrong answer.

(iv) Section III contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has only one correct answer and carries **+4 marks** for correct answer and **– 1 mark** for wrong answer.

(v) Section IV contains 3 questions. Each question contains statements given in 2 columns. Statements in the first column have to be matched with statements in the second column and each question carries **+6 marks** and marks will be awarded if all the four parts are correctly matched. No marks will be given for any wrong match in any question. There is no negative marking.

SECTION I

This section contains 9 multiple choice questions numbered 23 to 31. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

45. A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2+4y^2=12$. Then its equation is

(A) $x^2 \cos ec^2\theta - y^2 \sec^2\theta = 1$

(B) $x^2\sec^2\theta - y^2 \operatorname{cosec}^2\theta = 1$

(C) $x^2 \sin^2\theta - y^2 \cos^2\theta = 1$

(D) $x^2 \cos^2\theta - y^2 \sin^2\theta = 1$

46. The tangent to the curve $y=e^x$ drawn at the point (c, e^c) intersects the line joining the points $(c-1, e^{c-1})$ and $(c+1, e^{c+1}))$

(A) on the left of $x=c$

(B) on the right of $x=c$

(C) at no point

(D) at all points

47. A man walks a distance of 3 units from the origin towards the north-east $(N45°E)$ direction. From there, he walks a distance of 4 units towards the north-west $(N45°W)$ direction to reach a point P. Then the position of P in the Argand plane is

(A) $3e^{i\pi/4}+4i$

(B) $(3-4i)e^{i\pi/4}$

(C) $(4+3i)e^{i\pi/4}$

(D) $(3+4i)e^{i\pi/4}$

48. Let $f(x)$ be differentiable on the interval $(0,\infty)$ such that $f(1)=1$, and

$$\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t-x} = 1$$

for each $x>0$. Then $f(x)$ is

(A) $\frac{1}{3x}+\frac{2x^2}{3}$

(B) $\frac{1}{3x}+\frac{4x^2}{3}$

(C) $-\frac{1}{x}+\frac{2}{x^2}$

(D) $\frac{1}{x}$

49. The number of solutions of the pair of equations

$2\sin^2\theta - \cos 2\theta = 0$

$2\cos^2\theta - 3\sin\theta = 0$

in the interval $[0,2\pi]$ is

(A) zero

(B) one

(C) two

(D) four

50. Let α, β be the roots of the equation $x^2 - px + r = 0$ and, $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is

(A) $\frac{2}{9}(p-q)(2p-q)$

(B) $\frac{2}{9}(q-p)(2p-q)$

(C) $\frac{2}{9}(q-2p)(2p-q)$

(D) $\frac{2}{9}(2p-q)(2p-q)$

51. The number of distinct real values of λ , for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar, is

(A) zero

(B) one

(C) two

(D) three

52. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is

(A) 1/2

(B) 1/3

(C) 2/5

(D) 1/5

53. $\lim_{x\to\frac{\pi}{4}} \frac{\int_{2}^{\sec^2 x} f(t)\,dt}{x^2 - \frac{\pi^2}{16}}$ equals

(A) $\frac{8}{\pi} f(2)$

(B) $\frac{2}{\pi} f(2)$

(C) $\frac{2}{\pi} f\left(\frac{1}{2}\right)$

(D) $4 f(2)$

SECTION –II

This section contains 4 questions numbered 54 to 57. Each question contains STATEMENT – 1 (Assertion) and STATEMENT -2(Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

54. Let the vectors $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RS}, \overrightarrow{ST}, \overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.

STATEMENT -1 : $\overrightarrow{PQ} \times \left(\overrightarrow{RS} + \overrightarrow{ST}\right) \neq \vec{0}$

because

STATEMENT -2 : $\overrightarrow{PQ} \times \overrightarrow{RS} \neq \vec{0}$ and $\overrightarrow{PQ} \times \overrightarrow{ST} \neq \vec{0}$

(A) Statement -1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement -1 is True, Statement -2 is False

(D) Statement -1 is False, Statement -2 is True

55. Let $F(x)$ be an indefinite integral of $\sin^2 x$.

STATEMENT -1 : The function $F(x)$ satisfies $F(x+\pi) = F(x)$ for all real x.

because

STATEMENT -2 : $\sin^2(x+\pi) = \sin^2 x$ for all real x.

(A) Statement -1 is True, Statement -2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement -1 is True, Statement -2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement -1 is True, Statement -2 is False

(D) Statement -1 is False, Statement -2 is True

56. Let $H_1, H_2, \ldots, H_n$ H1, be mutually exclusive and exhaustive events with $P(H_i) > 0, i = 1, 2, \ldots, n$. Let E be any other event with 0 < P(E) < 1.

STATEMENT -1 : $P(H_i|E) > P(E|H_i).P(H_i)$ for $i = 1, 2, \ldots, n$

because

STATEMENT -2 : $\sum_{i=1}^{n} P(H_i) = 1$

(A) Statement -1 is True, Statement -2 is true; Statement-2 is a correct explanation for Statement-1

(B) Statement -1 is True, Statement -2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement -1 is True, Statement -2 is False

(D) Statement -1 is False, Statement -2 is True

57. Tangents are drawn from the point $(17, 7)$ to the circle $x^2 + y^2 = 169$.

STATEMENT -1 : The tangents are mutually perpendicular.

because

STATEMENT -2 : The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$

(A) Statement -1 is True, Statement -2 is true; Statement-2 is a correct explanation for Statement-1

(B) Statement -1 is True, Statement -2 is true; Statement-2 is **NOT** a correct explanation for Statement-1

(C) Statement -1 is True, Statement -2 is False

(D) Statement -1 is False, Statement -2 is True

SECTION − III

This section contains 2 paragraphs M_{58-60} and M_{61-63}. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choice (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

M58−60 : Paragraph for question Nos. 58 to 60

Consider the circle $x^2+y^2=9$ and the parabola $y^2=8x$. They intersect at P and Q in the first and the fourth quadrants, respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at P.

58. The ratio of the areas of the triangles PQS and PQR is

(A) $1:\sqrt{2}$

(B) $1:2$

(C) $1:4$

(D) $1:8$

59. The radius of the circumcircle of the triangle PRS is

(A) 5

(B) $3\sqrt{3}$

(C) $3\sqrt{2}$

(D) $2\sqrt{3}$

60. The radius of the incircle of the triangle PQR is

(A) 4

(B) 3 3

(C) 8/3

(D) 2

M_{61-63} : Paragraph for Question Nos. 61 to 63

Let V_r denote the sum of the first r terms of an arithmetic progression $(A.P.)$ whose first term is r and the common difference is $(2r-1)$. Let

$T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for $r = 1, 2, \ldots$

61. The sum $V_1 + V_2 + \ldots + V_n$ V1 + V2 + … + Vn is

(A) $\frac{1}{12} n(n+1)(3n^2 - n + 1)$

(B) $\frac{1}{12} n(n+1)(3n^2 + n + 1)$

(C) $\frac{1}{2}n\left(2n^2-n+1\right)$

(D) $\frac{1}{3}\left(2n^3-2n+3\right)$

62. T_r is always

(A) an odd number

(B) an even number

(C) a prime number

(D) a composite number

63. Which one of the following is a correct statement?

(A) $Q_1, Q_2, Q_3, \ldots$ are in $A.P.$ with common difference 5

(B) $Q_1, Q_2, Q_3, \ldots$ are in $A.P.$ with common difference 6

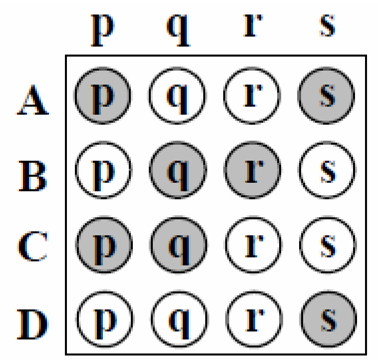
(C) $Q_1, Q_2, Q_3, \ldots$ are in $A.P.$ with common difference 11

(D) $Q_1 = Q_2 = Q_3 = \ldots$

SECTION – IV

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in Column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A−p, A−s, B−q, B−r, C−p, C−q and D−s, then the correctly bubbled 4 × 4 matrix should be as follows:



64. Consider the following linear equations

$$ax + by + cz = 0$$

$$bx + cy + az = 0$$

$$cx + ay + bz = 0$$

Match the conditions / expressions in Column I with statements in Column II and indicate your answers by darkening the appropriate bubbles in 4×4 matrix given in the *ORS*

Column I		Column II	
(A)	$a+b+c \neq 0$ and $a^2+b^2+c^2 = ab+bc+ca$	(p)	the equations represent planes meeting only at a single point.
(B)	$a+b+c = 0$ and $a^2+b^2+c^2 \neq ab+bc+ca$	(q)	the equations represent the line $x = y = z$
(C)	$a+b+c \neq 0$ and $a^2+b^2+c^2 \neq ab+bc+ca$	(r)	the equations represent identical planes.
(D)	$a+b+c = 0$ and $a^2+b^2+c^2 = ab+bc+ca$	(r)	the equations represent the whole of the three dimensional space.

65. Match the integrals in Column I with the values in Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

Column I		**Column II**	
(A)	$\int_{-1}^{1}\frac{dx}{1+x^2}$	(p)	$\frac{1}{2}\log\left(\frac{2}{3}\right)$
(B)	$\int_{0}^{1}\frac{dx}{\sqrt{1-x^2}}$	(q)	$2\log\left(\frac{2}{3}\right)$
(C)	$\int_{2}^{3}\frac{dx}{1-x^2}$	(r)	$\frac{\pi}{3}$
(D)	$\int_{1}^{2}\frac{dx}{x\sqrt{x^2-1}}$	(r)	$\frac{\pi}{2}$

66. In the following [x] denotes the greatest integer less than or equal to x.

Match the functions in Column I with the properties Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

	Column I		**Column II**
(A)	$x\lvert x\rvert$	(p)	continuous in $(-1,1)$
(B)	$\sqrt{\lvert x\rvert}$	(q)	differentiable in $(-1,1)$
(C)	$x+[x]$	(r)	strictly increasing in $(-1,1)$
(D)	$\lvert x-1\rvert+\lvert x+1\rvert$	(r)	not differentiable at least at one point in $(-1,1)$