IIT-JEE-2007

PAPER-II

MATHEMATICS

[Time allowed: 3 hours] [Maximum Marks: 243]

GENERAL INSTRUCTIONS

1. **Section I** contains 9 multiple choice questions which have only one correct answer. Each question carries +3 marks each for correct answer and **-1 mark** for each wrong answer.

2. **Section II** contains 4 questions. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason).

   Bubble (A) if both the statements are TRUE and STATEMENT-2 is the correct explanation of STATEMENT-1

   Bubble (B) if both the statements are TRUE but STATEMENT-2 is NOT the correct explanation of STATEMENT-1

   Bubble (C) if STATEMENT-1 is TRUE and STATEMENT-2 is FALSE.

   Bubble (D) if STATEMENT-1 is FALSE and STATEMENT-2 is TRUE.

   Carries +3 marks each for correct answer and -1 mark for each wrong answer.

3. **Section III** contains 2 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has only one correct answer and carries +4 marks for correct answer and **-1 mark** for wrong answer**.**

4. **Section IV** contains 3 questions. Each question contains statements given in 2 columns. Statements in the first column have to be matched with statements in the second column and each question carries +6 marks and marks will be awarded if all the four parts are correctly matched. No marks will be given for any wrong match in any question. There is **no negative marking.**

SECTION – I

Straight Objective Type

This section contains 9 multiple choice questions numbered 45 to 53. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

45. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$. Which one of the following is correct?

(A) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}=\vec{0}$

(B) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq\vec{0}$

(C) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{a}\times\vec{c}=\vec{0}$

(D) $\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}$ are mutually perpendicular

46. Let $f(x)=\dfrac{x}{\left(1+x^n\right)^{1/n}}$ for n ≥ 2 and $g(x)=\underbrace{(fof\ldots of)}_{f\text{ occurs } n \text{ times}}(x)$. Then $\int x^{n-2}g(x)\,dx$ equals

(A) $\dfrac{x}{n(n-1)}\left(1+nx^n\right)^{1-\frac{1}{n}}+K$

(B) $\dfrac{x}{n-1}\left(1+nx^n\right)^{1-\frac{1}{n}}+K$

(C) $\dfrac{x}{n(n+1)}\left(1+nx^n\right)^{1+\frac{1}{n}}+K$

(D) $\dfrac{x}{n+1}\left(1+nx^n\right)^{1+\frac{1}{n}}+K$

47. $\frac{d^2x}{dy^2}$ equals

(A) $\left(\frac{d^2x}{dx^2}\right)^{-1}$

(B) $-\left(\frac{d^2x}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

(C) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

(D) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

***48.** The letters of the word $COCHIN$ are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word $COCHIN$ is

(A) 360

(B) 192

(C) 96

(D) 48

***49.** If $|z| = 1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1-z^2}$ lie on

(A) a line not passing through the origin

(B) $|z| = \sqrt{2}$

(C) the x-axis

(D) the y-axis

***50.** Let $ABCD$ be a quadrilateral with area 18, with side AB parallel to the side CD and $AB = 2CD$. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral $ABCD$ touching all the sides, then its radius is

(A) 3

(B) 2

(C) 3/2

(D) 1

***51.** Let $O(0,0), P(3,4), Q(6,0)$ be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are

(A) $\left(\frac{4}{3}, 3\right)$

(B) $\left(3, \frac{2}{3}\right)$

(C) $\left(3, \frac{4}{3}\right)$

(D) $\left(\frac{4}{3}, \frac{2}{3}\right)$

52. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with

(A) variable radii and a fixed centre at $(0,1)$

(B) variable radii and a fixed centre at $(0,-1)$

(C) fixed radius 1 and variable centres along the x-axis

(D) fixed radius 1 and variable centres along the y-axis

53. Let E^c denote the complement of an event E . Let E, F, G be pairwise independent events with $P(G) > 0$ and $P(E \cap F \cap G) = 0$. Then $P\left(E^C \cap F^C | G\right) = 0$ equals

(A) $P\left(E^C\right) + P\left(F^C\right)$

(B) $P\left(E^C\right) - P\left(F^C\right)$

(C) $P\left(E^C\right) - P(F)$

(D) $P(E) - P\left(F^C\right)$

SECTION –II

Assertion – Reason Type

54. Let $f(x) = 2 + \cos x$ f(x) = 2 + cosx for all real x.

STATEMENT -1 : For each real t, there exists a point c in $[t, t+\pi]$ such that $f'(c) = 0$.

because

STATEMENT -2 : $f(t) = f(t + 2\pi)$ for each real t.

(A) Statement -1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement -1 is True, Statement -2 is False

(D) Statement -1 is False, Statement -2 is True

55. Consider the planes $3x-6y-2z=15$ *and* $2x+y-2z=5$.

STATEMENT -1 : The parametric equations of the line of intersection of the given planes are $x=3+14t,\ y=1+2t,\ z=15t$

because

STATEMENT -2 : The vectors $14\hat{i}+2\hat{j}+15\hat{k}$ is parallel to the line of intersection of the given planes.

(A) Statement -1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement -1 is True, Statement -2 is False

(D) Statement -1 is False, Statement -2 is True

***56.** Lines $L_1 : y-x=0$ *and* $L_2 : 2x+y=0$ intersect the line $L_3 : y+2=0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

STATEMENT -1 : The ratio $PR : RQ$ equals $2\sqrt{2} : \sqrt{5}$.

because

STATEMENT -2 : In any triangle, bisector of an angle divides the triangle into two similar triangles.

(A) Statement -1 is True, Statement -2 is true; Statement-2 **is** a correct explanation for Statement-1

(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement -1 is True, Statement -2 is False

(D) Statement -1 is False, Statement -2 is True

***57.** STATEMENT -1 : The curve y $\frac{-x^2}{2}+x+1$ is symmetric with respect to the line $x=1$.

because

STATEMENT -2 : A parabola is symmetric about its axis.

(A) Statement -1 is True, Statement -2 is true; Statement-2 **is** a correct explanation for Statement-1

(B) Statement -1 is True, Statement -2 is true; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement -1 is True, Statement -2 is False

(D) Statement -1 is False, Statement -2 is True

SECTION – III

Linked Comprehension Type

M58–60 : Paragraph for question Nos. 58 to 60

If a continuous f defined on the real line R, assumes positive and negative values in R then the equation $f(x)=0$ has a root in R.For example, if it is known that a continuous function f on R is positive at some point and its minimum values is negative then the equation $f(x)=0$ has a root in R.

Consider $f(x)=ke^x-x$ for all real x where k is a real constant.

58. The line $y = x$ meets $y = ke^x$ for $k \le 0$ at

(A) no point

(B) one point

(C) two points

(D) more than two points

59. The positive value of k for which $ke^x - x = 0$ has only one root is

(A) $1/e$

(B) 1

(C) e

(D) $\log_e 2$

60. For $k > 0$, the set of all values of k for which $k e^x - x = 0$ has two distinct roots is

(A) $\left(0, \frac{1}{e}\right)$

(B) $\left(\frac{1}{e}, 1\right)$

(C) $\left(\frac{1}{e}, \infty\right)$

(D) $(0,1)$

M61−63 : Paragraph for Question Nos. 61 to 63

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$,let A_{n-1} and H_{n-1} has arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

***61.** Which one of the following statements is correct?

(A) $G_1 > G_2 > G_3 > K$

(B) $G_1 < G_2 < G_3 < K$

(C) $G_1 = G_2 = G_3 = K$

(D) $G_1 < G_3 < G_5 < K$ and $G_2 > G_4 > G_6 > K$

***62.** Which of the following statements is correct?

(A) $A_1 > A_2 > A_3 > K$

(B) $A_1 < A_2 < A_3 < K$

(C) $A_1 > A_3 > A_5 > K$ and $A_2 < A_4 < A_6 < K$

(D) $A_1 < A_3 < A_5 < K$ and $A_2 > A_4 > A_6 > K$

***63.** Which of the following statements is correct?

(A) $H_1 > H_2 > H_3 > K$

(B) $H_1 < H_2 < H_3 < K$

(C) $H_1 > H_3 > H_5 > K$ and $H_2 < H_4 < H_6 < K$

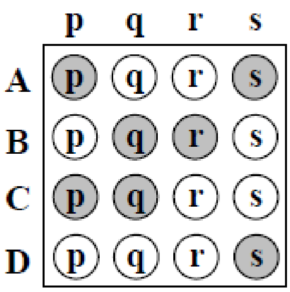
(D) $H_1 < H_3 < H_5 < K$ and $H_2 > H_4 > H_6 > K$

SECTION −IV

Matrix − Match Type

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in Column II. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A−p, A−s, B−q, B−r, C−p, C−q and D−s, then the correctly bubbled 4 × 4 matrix should be as follows:



64. Let $f(x)=\dfrac{x^2-6x+5}{x^2-5x+6}$

Match the conditions / expressions in **Column I** with statements in **Column II** and indicate your answers by darkening the appropriate bubbles in 4×4 matrix given in the *ORS*.

	Column I		**Column II**
(A)	If $-1<x<1$, then $f(x)$ satisfies	(p)	$0<f(x)<1$
(B)	If $1<x<2$, then $f(x)$ satisfies	(q)	$f(x)<0$
(C)	If $3<x<5$, then $f(x)$ satisfies	(r)	$f(x)>0$
(D)	If $x>5$, then $f(x)$ satisfies	(s)	$f(x)<1$

65. Let (x, y) be such that

$$\sin^{-1}(ax)+\cos^{-1}(y)+\cos^{-1}(bxy)=\frac{\pi}{2}$$

	Column I		
(A)	*If* $a=1$ *and* $b=0$, *then* (x, y)	(p)	lies on the circle $x^2+y^2=1$
(B)	*If* $a=1$ *and* $b=1$, *then* (x, y)	(q)	lies on the circle $(x^2-1)(y^2-1)=$
(C)	*If* $a=1$ *and* $b=2$, *then* (x, y)	(r)	lies on the circle $y=x$
(D)	*If* $a=2$ *and* $b=2$, *then* (x, y)	(s)	lies on $(4x^2-1)(y^2-1)=$

66. Match the statements in **Column I** with the statements in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the *ORS* .

	Column I		Column II
(A)	Two intersecting circles	(p)	have a common tangent
(B)	Two mutually external circles	(q)	have a common normal
(C)	two circles, one strictly inside the other	(r)	do not have a common tangent
(D)	two branches of a hyperbola	(s)	do not have a common normal