JEE MAIN - 2008

MATHEMATICS

[Time: 3 hours] [Maximum Marks: 246]

General Instructions :

(i) **Section I** contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **only one** is correct.

(ii) **Section II** contains 4 multiple correct answer type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **one or more answers** are correct.

(iii) **Section III** contains 4 Reasoning type questions. Each question contains STATEMENT–1 and STATEMENT–2.

Bubble (A) if both the statements are TRUE and STATEMENT-2 is the correct explanation of STATEMENT-1

Bubble (B) if both the statements are TRUE but STATEMENT-2 is NOT the correct explanation of STATEMENT- 1

Bubble (C) if STATEMENT-1 is TRUE and STATEMENT-2 is FALSE.

Bubble (D) if STATEMENT-1 is FALSE and STATEMENT-2 is TRUE.

(iv) **Section IV** contains 3 sets of Linked Comprehension type questions. Each set consists of a paragraph followed by 3 questions. Each question has 4 choices (A), (B), (C) and (D), out of which **only one** is correct.

Marking Scheme:

(i) For each question in **Section I**, you will be awarded **3 Marks** if you have darkened only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (– 1) mark** will be awarded.

(ii) For each question in **Section II**, you will be awarded **4 Marks** if you have darkened all the bubble(s) corresponding to the correct answer and **zero mark** for all other cases. It may be noted that there is **no negative marking** for **wrong answer**.

(iii) For each question in **Section III**, you will be awarded **3 Marks** if you have darkened only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (– 1) mark** will be awarded.

(iv) For each question in **Section IV**, you will be awarded **4 Marks** if you have darkened only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (– 1) mark** will be awarded.

SECTION – I

This section contains 6 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

1. Let a and b be non-zero real numbers. Then, the equation $\left(ax^2+by^2+c\right)\left(x^2-5xy+6y^2\right)=0$ represents

(A) four straight lines, when $c=0$ and a,b are of the same sign

(B) two straight lines and a circle, when $a=b$, and c is of sign opposite to that of a

(C) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a

(D) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a

2. The total number of local maxima and local minima of the function $f(x)=\begin{cases}(2+x)^3, & -3<x\le -1\\ x^{2/3}, & -1<x<2\end{cases}$ is

(A) 0

(B) 1

(C) 2

(D) 3

3. Let $g(x)=\dfrac{(x-1)^n}{\log\cos^m(x-1)};0<x<2$, m and n are integers, $m\neq 0, n>0$, and let p be the left hand derivative of $|x-1|$ at $x=1$. If $\lim\limits_{x\to 1^+} g(x)=p$, then

(A) $n=1,\ m=1$

(B) $n=1,\ m=-1$

(C) $n=2,\ m=2$

(D) $n>2,\ m=n$

4. If $0<x<1$, then $\sqrt{1+x^2}\left[\left\{x\cos\left(\cot^{-1}x\right)+\sin\left(\cot^{-1}x\right)\right\}^2-1\right]^{1/2}$ is equal to

(A) $\dfrac{x}{\sqrt{1+x^2}}$

(B) x

(C) $x\sqrt{1+x^2}$

(D) $\sqrt{1+x^2}$

5. Consider the two curves $C_1: y^2=4x, C_2: x^2+y^2-6x+1=0$. Then,

(A) C_1 and C_2 touch each other only at one point

(B) C_1 and C_2 touch each other exactly at two points

(C) C_1 and C_2 intersect (but do not touch) at exactly two points

(D) C_1 and C_2 neither intersect nor touch each other

6. The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a},\hat{b},\hat{c}$ such that $\hat{a}\cdot\hat{b}=\hat{b}\cdot\hat{c}=\hat{c}\cdot\hat{a}=1/2$. Then the volume of the parallelopiped is

(A) $\dfrac{1}{\sqrt{2}}$

(B) $\dfrac{1}{2\sqrt{2}}$

(C) $\dfrac{\sqrt{3}}{2}$

(D) $\dfrac{1}{\sqrt{3}}$

SECTION – II

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE OR MORE** is/are correct.

7. Let $f(x)$ be a non-constant twice differentiable function defined on $(-\infty,\infty)$ such that $f(x)=f(1-x)$ and $f'\left(\frac{1}{4}\right)=0$. Then

(A) $f'(x)$ vanishes at least twice on $[0,1]$

(B) $f'\left(\frac{1}{2}\right)=0$

(C) $\int_{-1/2}^{1/2} f\left(x+\frac{1}{2}\right)\sin x\,dx=0$

(D) $\int_{0}^{1/2} f(t)e^{\sin\pi t}\,dt=\int_{1/2}^{1} f(1-t)e^{\sin\pi t}dt$

8. A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then

(A) $\frac{1}{PS}+\frac{1}{ST}<\frac{2}{\sqrt{QS\times SR}}$

(B) $\frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\times SR}}$

(C) $\frac{1}{PS}+\frac{1}{ST}<\frac{4}{QR}$

(D) $\frac{1}{PS}+\frac{1}{ST}>\frac{4}{QR}$

9. Let $P(x_1, y_1)$ and $Q(x_2, y_2), y_1 < 0, y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are

(A) $x^2 + 2\sqrt{3}y = 3 + \sqrt{3}$

(B) $x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$

(C) $x^2 + 2\sqrt{3}y = 3 + \sqrt{3}$

(D) $x^2 - 2\sqrt{3}y = 3 - \sqrt{3}$

10. Let $S_n = \sum_{k=1}^{n} \frac{n}{n^2 + kn + k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$ for $n = 1, 2, 3, \ldots$. Then,

(A) $S_n < \frac{\pi}{3\sqrt{3}}$

(B) $S_n > \frac{\pi}{3\sqrt{3}}$

(C) $T_n < \frac{\pi}{3\sqrt{3}}$

(D) $T_n > \frac{\pi}{3\sqrt{3}}$

SECTION – III

This section contains 4 reasoning type questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

11. Consider the system of equations $ax+by=0, cx+dy=0$, where $a,b,c,d \in \{0,1\}$.

STATEMENT – 1: The probability that the system of equations has a unique solution is $3/8$

and

STATEMENT – 2: The probability that the system of equations has a solution is 1.

(A) Statement-1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement -1 is True, Statement -2 is False

(D) Statement -1 is False, Statement -2 is True

12. Consider the system of equations

$$x-2y+3z=-1$$
$$-x+y-2z=k$$
$$x-3y+4z=1$$

STATEMENT -1 : The system of equations has no solution for $k \neq 3$.

and

STATEMENT -2 : The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$.

(A) Statement-1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement -1 is True, Statement -2 is False

(D) Statement -1 is False, Statement -2 is True

13. Let f and g be real valued functions defined on interval $(-1,1)$ such that $g''(x)$ is continuous, $g(0) \neq 0, g'(0) = 0, g''(0) \neq 0$, and $f(x) = g(x)\sin x$.

STATEMENT -1 : $\lim\limits_{x \to 0}\left[g(x)\cot x - g(0)\operatorname{cosec} x\right] = f''(0)$.

and

STATEMENT -2 : $f'(0) = g(0)$.

(A) Statement-1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement -1 is True, Statement -2 is False

(D) Statement -1 is False, Statement -2 is True

14. Consider three planes

$P_1 : x - y + z = 1$
$P_2 : x + y - z = -1$
$P_3 : x - 3y + 3z = 2.$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, and P_1 and P_2, respectively.

STATEMENT -1 : At least two of the lines L_1, L_2 and L_3 are non-parallel.

and

STATEMENT -2 : The three planes do not have a common point.

(A) Statement-1 is True, Statement -2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement -1 is True, Statement -2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement -1 is True, Statement -2 is False

(D) Statement -1 is False, Statement -2 is True

SECTION – IV

This section contains 3 paragraphs. Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 15 to 17

Consider the functions defined implicitly by the equation $y^3-3y+x=0$ on various intervals in the real line. If $x\in(-\infty,-2)\cup(2,\infty)$, the equation implicitly defines a unique real valued differentiable function $y=f(x)$. If $x\in(-2,-2)$, the equation implicitly defines a unique real valued differentiable function $y=g(x)$ satisfying $g(0)=0$.

15. If $f\left(-10\sqrt{2}\right)=2\sqrt{2}$, then $f''\left(-10\sqrt{2}\right)=$

(A) $\dfrac{4\sqrt{2}}{7^3 3^2}$

(B) $-\dfrac{4\sqrt{2}}{7^3 3^2}$

(C) $\dfrac{4\sqrt{2}}{7^3 3}$

(D) $-\dfrac{4\sqrt{2}}{7^3 3}$

16. The area of the region bounded by the curves $y=f(x)$, the x-axis , and the lines $x=a$ and $x=b$, where $-\infty<a<b<-2$, is

(A) $\displaystyle\int_a^b \frac{x}{3\left(\left(f(x)\right)^2-1\right)}dx+bf(b)-af(a)$

(B) $\displaystyle-\int_a^b \frac{x}{3\left(\left(f(x)\right)^2-1\right)}dx+bf(b)-af(a)$

(C) $\int_a^b \frac{x}{3\left(\left(f(x)\right)^2-1\right)}dx+bf(b)+af(a)$

(D) $-\int_a^b \frac{x}{3\left(\left(f(x)\right)^2-1\right)}dx-bf(b)+af(a)$

17. $\int_{-1}^{1} g'(x)\,dx =$

(A) $2g(-1)$

(B) 0

(C) $-2g(1)$

(D) $2g(1)$

Paragraph for Question Nos. 18 to 20

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F, respectively. The line PQ is given by the equation $\sqrt{3}x+y-6=0$ and the point D is $\left(\frac{3\sqrt{3}}{3},\frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

18. The equation of circle C is

(A) $\left(x-2\sqrt{3}\right)^2+(y-1)^2=1$

(B) $\left(x-2\sqrt{3}\right)^2+\left(y+\frac{1}{2}\right)^2=1$

(C) $\left(x-\sqrt{3}\right)^2+(y+1)^2=1$

(D) $\left(x-\sqrt{3}\right)^2+(y-1)^2=1$

19. Points E and F are given by

(A) $\left(\frac{\sqrt{3}}{2},\frac{3}{2}\right),\left(\sqrt{3},0\right)$

(B) $\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right),\left(\sqrt{3},0\right)$

(C) $\left(\frac{\sqrt{3}}{2},\frac{3}{2}\right),\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$

(D) $\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right),\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$

20. Equation of the sides QR, RP

(A) $y=\frac{2}{\sqrt{3}}x+1, y=-\frac{2}{\sqrt{3}}x-1$

(B) $y=\frac{1}{\sqrt{3}}x, y=0$

(C) $y=\frac{\sqrt{3}}{2}x+1, y=-\frac{\sqrt{3}}{2}x-1$

(D) $y=\sqrt{3}x, y=0$

Paragraph for Question Nos. 21 to 23

Let A, B, C be three sets of complex numbers as defined below

$$A = \{z : \text{Im}\, z \geq 1\}$$
$$B = \{z : |z - 2 - i| = 3\}$$
$$C = \{z : \text{Re}((1-i)z) = \sqrt{2}\}.$$

21. The number of elements in the set $A \cap B \cap C$ is

(A) 0

(B) 1

(C) 2

(D) ∞

22. Let z be any point in $A \cap B \cap C$. Then, $|z+1-i|^2 + |z+5-i|^2$ lies between

(A) 25 and 29

(B) 30 and 34

(C) 35 and 39

(D) 40 and 44

23. Let z be any point in $A \cap B \cap C$ and let w be any point satisfying $|w-2-i| < 3$. Then, $|z| - |w| + 3$ lies between

(A) -6 and 3

(B) -3 and 6

(C) -6 and 6

(D) -3 and 9