# IIT-JEE 2009

## MATHS

## PART -1

**21 Correct Answer: (A)**

$z\bar{z}\left(\bar{z}^2 + z^2\right) = 35$

Put $z = x + iy$

$\left(x^2 + y^2\right)\left(x^2 - y^2\right) = 175$

$\left(x^2 + y^2\right)\left(x^2 - y^2\right) = 5 \cdot 5 \cdot 7$

$x^2 + y^2 = 25$

$x^2 - y^2 = 7$

$x = \pm 4, y = \pm 3$

$x, y \in I$

Area $= 8 \times 6 = 48$sq.

**22 Correct Answer: (C)**

$\left(\vec{a} \times \vec{b}\right) \cdot \left(\vec{c} \times \vec{d}\right) = 1$ possible only when $\left|\vec{a} \times \vec{b}\right| = \left|\vec{c} \times \vec{d}\right| = 1$

and $\left(\vec{a} \times \vec{b}\right) \| \left(\vec{c} \times \vec{d}\right)$

Since $\vec{a} \cdot \vec{c} = 1/2$ and $\vec{b} \| \vec{d}$, then $\left|\vec{c} \times \vec{d}\right| \neq 1$.

**23 Correct Answer: (D)**

Equation of line $AM$ is $x + 3y - 3 = 0$

Perpendicular distance of line from origin $= \dfrac{3}{\sqrt{10}}$

Length of $AM = 2\sqrt{9 - \dfrac{9}{10}} = 2 \times \dfrac{9}{\sqrt{10}}$

$\Rightarrow$ Area $= \dfrac{1}{2} \times 2 \times \dfrac{9}{\sqrt{10}} \times \dfrac{3}{\sqrt{10}} = \dfrac{27}{10}$ sq. Units.

**24 Correct Answer: (D)**

$$x = \sin\theta + \sin\theta + \ldots + \sin 29\theta$$

$$2(\sin\theta)X = 1 - \cos 2\theta + \cos 2\theta - \cos 4\theta + \ldots + \cos 28\theta - \cos 30\theta$$

$$X = \frac{1 - \cos 30\theta}{2\sin\theta} = \frac{1}{4\sin 2°}$$

**25 Correct Answer: (A)**

Any point on the line can be taken as

$$Q \equiv \{(1-3\mu),(\mu-1),(5\mu+2)\}$$
$$\overrightarrow{PQ} = \{-3\mu-2, \mu-3, 5\mu-4\}$$
$$\text{Now}, 1(-3\mu-2) - 4(\mu-3) + 3(5\mu-4) = 0$$
$$\Rightarrow -3\mu - 2 - 4\mu + 12 + 15\mu - 12 = 0$$
$$8\mu = 2 \Rightarrow \mu = 1/4.$$

**26 Correct Answer: (C)**

Coefficient of $x^{10}$ in $\left(x + x^2 + x^3\right)^7$

Coefficient of $x^3$ in $\left(1 + x + x^2\right)^7$

Coefficient of $x^3$ in $\left(1 - x^3\right)^7 (1-x)$

$$= {}^{7+3-1}C_3 - 7$$

$$= {}^{9}C_3 - 7$$

$$= \frac{9\times 8\times 7}{6} - 7 = 77.$$

Alternate:

The digits are $1,1,1,1,1,2,3$

or $1,1,1,1,2,2,2$

Hence number of seven digit numbers formed

$$=\frac{7!}{5!}+\frac{7!}{4!3!}=77.$$

**27 Correct Answer: (C)**

$$f'=\pm\sqrt{1-f^2}$$
$$\Rightarrow f(x)=\sin x \text{ or } f(x)=s-\text{in } x$$
$$\Rightarrow f(x)=\sin x$$
Also $x>\sin x \forall x>0.$

**28 Correct Answer: (B)**

The centre of the circle is $C(3,2)$.

Since $CA$ and $CB$ are perpendicular to $PA$ and $PB, CP$ is die diameter of the circumcircle of triangle $PAB$. Its equation is

$$(x-3)(x-1)+(y-2)(y-8)=0$$
$$\text{or } x^2+y^2-4x-10y+19=0$$

## SECTION-II

### Multiple Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer,

out of which ONE OR MORE is/are correct

**29 Correct Answer: (B,C)**

$$2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)=4\sin^2\frac{A}{2}$$

$$\cos\left(\frac{B-C}{2}\right)=2\sin\left(A/2\right)$$

$$\Rightarrow \frac{\cos\left(\frac{B-C}{2}\right)}{\sin\left(A/2\right)}=2$$

$$\Rightarrow \frac{\sin B+\sin C}{\sin A}=2$$

$$\Rightarrow b+c=2a$$

**30 Correct Answer: (A, B)**

$$\frac{\sin^4 x}{2}+\frac{\cos^4 x}{3}=\frac{1}{5}$$

$$3\sin^4 x+2\left(1-\sin^2 x\right)^2=\frac{6}{5}$$

$$\Rightarrow 25\sin^4 x-20\sin^2 x+4=0$$

$$\Rightarrow \sin^2 x=\frac{2}{5}\text{and}\cos^2 x=\frac{3}{5}$$

$$\therefore \tan^2 x=\frac{2}{3}\text{and}\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{1}{125}$$

**31 Correct Answer: (A,C**

$$L=\lim_{x\to 0}\frac{a-\sqrt{a^2-x^2}-\frac{x^2}{4}}{x^4}=L=\lim_{x\to 0}\frac{1}{x^2\left(a-\sqrt{a^2-x^2}\right)}-\frac{1}{4x^2}$$

$$=\lim_{x\to 0}\frac{(4-a)-\sqrt{a^2-x^2}}{4x^2\left(a+\sqrt{a^2-x^2}\right)}$$

numerator $\to 0$ if $a=2$ and then $L=\frac{1}{64}$

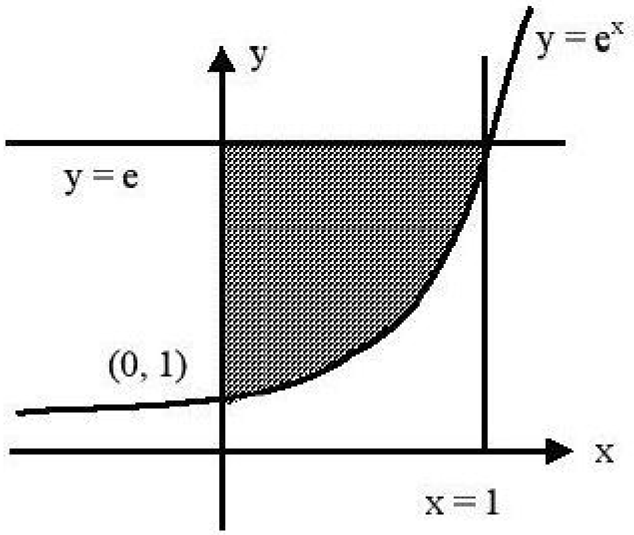
**32 Correct Answer: (B,C,D)**

Required Area $=\int_1^e \ln y\,dy$

$=\left(y\ln y-y\right)_1^e=(e-e)-\{-1\}=1.$

Also, $\int_1^e \ln y\,dy=\int_1^e \ln\left(e+1-y\right)dy$

Further the required area $=e\times 1e-\int_1^e e^x dx.$



## SECTION-III

### Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONLY ONE is correct.

**Paragraph for question** Nos. 33 **to** 35

**33 Correct Answer: (A)**

$$P(X=3)=\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\frac{1}{6}=\frac{25}{216}.$$

**34 Correct Answer: (B)**

$$P(X\le 2)=\frac{1}{6}+\frac{5}{6}\times\frac{1}{6}=\frac{11}{36}$$

Required probability $=1-\frac{11}{36}=\frac{25}{36}$.

**35 Correct Answer: (D)**

For $X\ge 6$, the probability is

$$\frac{5^5}{6^6}+\frac{5^4}{6^5}+\frac{5^5}{6^6}+\ldots\infty=\left(\frac{5}{6}\right)^3$$

Hence the conditional probability $\frac{(5/6)^6}{(5/6)^3}=\frac{25}{36}$.

**Paragraph for question** Nos. 36 **to** 38

**36 Correct Answer: (A)**

If two zero's are the entries in the diagonal, then

$^3C_2 \times {}^3C_1$

If all the entries in the principle diagonal is $1$, then

$^3C_1$

$\Rightarrow$Total matrix $=12$.

**37 Correct Answer: (B)**

$$\begin{bmatrix} 0 & a & b \\ a & 1 & c \\ b & c & 1 \end{bmatrix}$$

either $a=0$ or $c=0 \Rightarrow |A| \neq 0$

$$\begin{bmatrix} 0 & a & b \\ a & 1 & c \\ b & c & 0 \end{bmatrix}$$

either $a=0$ or $c=0 \Rightarrow |A| \neq 0$

$\Rightarrow 2$ matrices

$$\begin{bmatrix} 1 & a & b \\ a & 0 & c \\ b & c & 0 \end{bmatrix}$$

either $a=0$ or $b=0 \Rightarrow |A| \neq 0$

$\Rightarrow 2$ matrices

$$\begin{bmatrix} 1 & a & b \\ a & 1 & c \\ b & c & 1 \end{bmatrix}$$

If $a = b = 0 \Rightarrow |A| = 0$

If $a = c = 0 \Rightarrow |A| = 0$

If $b = c = 0 \Rightarrow |A| = 0$

$\Rightarrow$ there will be only $6$ matrices.

**38 Correct Answer: (B)**

The six matrix $A$ for which $|A| = 0$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow \text{inconsistent.}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow \text{inconsistent.}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow \text{infinite solutions.}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \text{inconsistent.}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \Rightarrow \text{inconsistent.}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow \text{infinite solutions.}$$