IIT-JEE 2009

MATHS

PART -1

[Time allowed: 3 hours] [Maximum Marks: 240]

A. Question paper format:

1. The question paper consists of 3 parts (Chemistry, Mathematics and Physics). Each part consists of 4 sections.

2. **Section I** contains **8** multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **only one is correct.**

3. **Section II** contains **4** multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **one or more is/are correct.**

4. **Section III** contains **2** groups of questions. Each group has 3 questions based on a paragraph. Each question has 4 choices (A), B), (C) and (D) for its answer, out of which **only one is correct.**

5. **Section IV** contains 2 questions. Each question has four statements (A, B, C and D) given in column I and five statements (p, q, r, s and t) in Column II. Any given statement in column I can have correct matching with **one or more** statements(s) given in column II. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

B. Marking scheme

6. For each question in **Section I** you will be **awarded 3 marks** if you darken the bubble corresponding to the correct answer and **zero** mark if no bubbles is darkened. In case of bubbling of incorrect answer, **minus (-1)** mark will be awarded.

7. For each question in **Section II,** you will be **awarded 4 marks** if you darken the bubble (s) corresponding to the correct choice(s) for the answer, and **zero** mark if no bubble is darkened. In all other cases, M**inus (-1)** mark will be awarded.

8. For each question in **Section III,** you will be **awarded 4 marks** if you darken the bubble (s) corresponding to the correct answer, and **zero** mark if no bubble is darkened. In all other cases, m**inus one (-1)** mark will be awarded.

9. For each question in **Section IV,** you will be **awarded 2 marks** for each row in which you have darkened the bubble(s) corresponding to the correct answer. Thus, each question in this section carries a maximum of **8** marks. There is **no negative marking** for incorrect answer(s) for this section.

SECTION -1

Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C), and (D) for its answer, out of which ONLY ONE is correct.

21. Let $z = x + iy$ be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation $\bar{z}z^3 + z\bar{z}^3 = 350$ is

(A) 48

(B) 32

(C) 40

(D) 80

22. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $\left(\vec{a} \times \vec{b}\right) \cdot \left(\vec{c} \times \vec{d}\right) = 1$ and $\vec{a} \cdot \vec{c} = \frac{1}{2}$, then

(A) $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar

(B) $\vec{b}, \vec{c}, \vec{d}$ are non-coplanar

(C) $\vec{b}, \vec{d}$ are non-coplanar

(D) $\vec{a}, \vec{d}$ are non-coplanar $\vec{b}, \vec{c}$ are parallel

23. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2+9y^2=9$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is

(A) $\frac{31}{10}$

(B) $\frac{29}{10}$

(C) $\frac{21}{10}$

(D) $\frac{27}{10}$

24. Let $z=\cos\theta+i\sin\theta$. Then the value of $\sum_{m=1}^{15}\text{Im}\left(Z^{2m-1}\right)$ at $\theta=2°$ is

(A) $\frac{1}{\sin 2°}$

(B) $\frac{1}{3\sin 2°}$

(C) $\frac{1}{3\sin 2°}$

(D) $\frac{1}{4\sin 2°}$

25. Let $P(3,2,6)$ be a point in space and Q be a point on the line

$\vec{r}=\left(\hat{i}-\hat{j}+2\hat{k}\right)+\mu\left(-3\hat{i}+\hat{j}+5\hat{k}\right)$ Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x-4y+3z=1$ is

(A) $\frac{1}{4}$

(B) $-\frac{1}{4}$

(C) $\frac{1}{8}$

(D) $-\frac{1}{8}$

26. The number of seven digit integers, with sum of the digits equal to 10 and fonned by using the digits $1,2$ and 3 only, is

(A) 55

(B) 66

(C) 77

(D) 88

27. Let f be a non-negative function defined on die interval $[0,1]$. If

$\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt, 0 \le x \le 1$, and $f(0)=0$, then

(A) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(B) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(C) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

(D) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

28. Tangents drawn from the point $P(1,8)$ to the circle $x^2+y^2-6x-4y-11=0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is

(A) $x^2+y^2+4x-6y+19=0$

(B) $x^2+y^2-4x-10y+19=0$

(C) $x^2+y^2-2x+6y-29=0$

(D) $x^2+y^2-6x-4y+19=0$

SECTION-II

Multiple Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

29. In a triangle ABC with fixed base BC, the vertex A moves such that $\cos B+\cos C=4\sin^2\frac{A}{2}$ If a,b and c denote the lengths of the sides of the triangle opposite to the angles A,B and C, respectively, then

(A) $b+c=4a$

(B) $b+c=2a$

(C) locus of point A is an ellipse

(D) locus of point A is a pair of straight lines

30. If $\frac{\sin^4 x}{2}+\frac{\cos^4 x}{3}=\frac{1}{5}$, then

(A) $\tan^2 x=\frac{2}{3}$

(B) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{1}{125}$

(C) $\tan^2 x=\frac{1}{3}$

(D) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{2}{125}$

31. Let $L=\lim_{x\to 0}\dfrac{a-\sqrt{a^2-x^2}-\dfrac{x^2}{4}}{x^4}, a>0$ If L is finite, then

(A) $a=2$

(B) $a=1$

(C) $L=\dfrac{1}{64}$

(D) $L=\dfrac{1}{32}$

32. Area of the region bounded by the curve $y=e^x$ and lines $x=0$ and $y=e$ is

(A) $e-1$

(B) $\int_1^e \ln(e+1-y)dy$

(C) $e-\int_1^e e^x dx$

(D) $\int_1^e \ln y dy$

SECTION-III

Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for question Nos. 33 to 35

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

33. The probability that $X = 3$ equals

(A) $\frac{25}{216}$

(B) $\frac{25}{36}$

(C) $\frac{5}{36}$

(D) $\frac{125}{216}$

34. The probability that $X \geq 3$ equals

(A) $\frac{125}{216}$

(B) $\frac{25}{36}$

(C) $\frac{5}{36}$

(D) $\frac{25}{216}$

35. The conditional probability that $X \geq 6$ given $X > 3$ equals

(A) $\frac{125}{216}$

(B) $\frac{25}{216}$

(C) $\frac{5}{36}$

(D) $\frac{25}{36}$

Paragraph for question Nos. 36 **to** 38

Let $\underline{|A}$ be the set of all 3×3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

36. The number of matrices in $\underline{A|}$ is

(A) 12

(B) 6

(C) 9

(D) 3

37. The number of matrices A in $\underline{A|}$ for which the system of linear equations $A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\1\end{bmatrix}$ has a unique solution, is

(A) less than 4

(B) at least 4 but less than 7

(C) atleast 7 but less than 10

(D) at least 10

38. The number of matrices A in $\underline{A|}$ for which the system of linear equations $A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$ is inconsistent, is

(A) 0

(B) more than 2

(C) 2

(D) 1