

IIT-JEE 2009

MATHS

PART -1

[Time allowed: 3 hours] [Maximum Marks: 240]

A. Question paper format:

1. The question paper consists of 3 parts (Chemistry, Mathematics and Physics). Each part consists of 4 sections.
2. **Section I** contains **8** multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **only one is correct**.
3. **Section II** contains **4** multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **one or more is/are correct**.
4. **Section III** contains **2** groups of questions. Each group has 3 questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **only one is correct**.
5. **Section IV** contains 2 questions. Each question has four statements (A, B, C and D) given in column I and five statements (p, q, r, s and t) in Column II. Any given statement in column I can have correct matching with **one or more** statements(s) given in column II. For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

B. Marking scheme

6. For each question in **Section I** you will be **awarded 3 marks** if you darken the bubble corresponding to the correct answer and **zero** mark if no bubbles is darkened. In case of bubbling of incorrect answer, **minus (-1)** mark will be awarded.
7. For each question in **Section II**, you will be **awarded 4 marks** if you darken the bubble (s) corresponding to the correct choice(s) for the answer, and **zero** mark if no bubble is darkened. In all other cases, **Minus (-1)** mark will be awarded.
8. For each question in **Section III**, you will be **awarded 4 marks** if you darken the bubble (s) corresponding to the correct answer, and **zero** mark if no bubble is darkened. In all other cases, **minus one (-1)** mark will be awarded.
9. For each question in **Section IV**, you will be **awarded 2 marks** for each row in which you have darkened the bubble(s) corresponding to the correct answer. Thus, each question in this section carries a maximum of **8 marks**. There is **no negative marking** for incorrect answer(s) for this section.

SECTION -1

Single Correct Choice Type

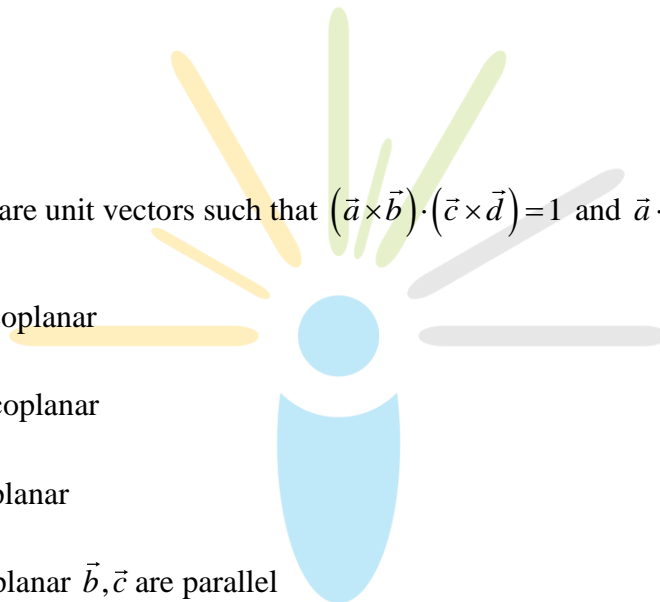
This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C), and (D) for its answer, out of which ONLY ONE is correct.

21. Let $z = x + iy$ be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation $\bar{z}z^3 + z\bar{z}^3 = 350$ is

- (A) 48
- (B) 32
- (C) 40
- (D) 80

22. If $\vec{a}, \vec{b}, \vec{c}$ and \vec{d} are unit vectors such that $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 1$ and $\vec{a} \cdot \vec{c} = \frac{1}{2}$, then

- (A) $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar
- (B) $\vec{b}, \vec{c}, \vec{d}$ are non-coplanar
- (C) \vec{b}, \vec{d} are non-coplanar
- (D) \vec{a}, \vec{d} are non-coplanar \vec{b}, \vec{c} are parallel



23. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M . Then the area of the triangle with vertices at A , M and the origin O is

(A) $\frac{31}{10}$

(B) $\frac{29}{10}$

(C) $\frac{21}{10}$

(D) $\frac{27}{10}$

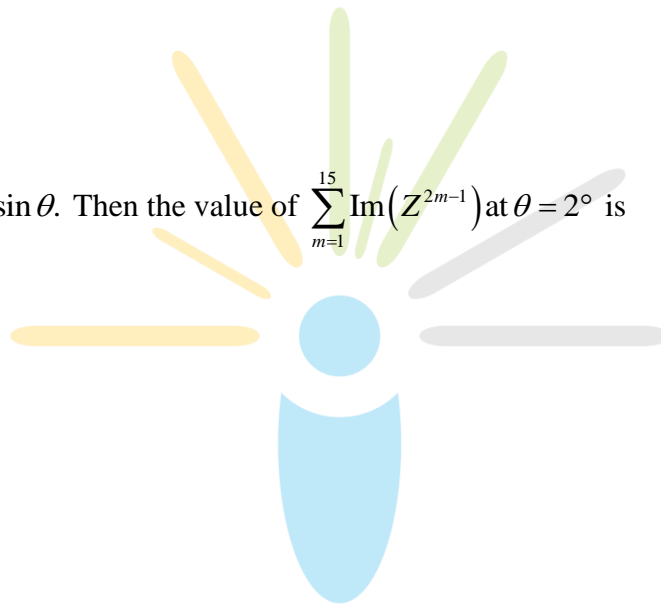
24. Let $z = \cos \theta + i \sin \theta$. Then the value of $\sum_{m=1}^{15} \text{Im}(Z^{2m-1})$ at $\theta = 2^\circ$ is

(A) $\frac{1}{\sin 2^\circ}$

(B) $\frac{1}{3 \sin 2^\circ}$

(C) $\frac{1}{3 \sin 2^\circ}$

(D) $\frac{1}{4 \sin 2^\circ}$



25. Let $P(3, 2, 6)$ be a point in space and Q be a point on the line

$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$ Then the value of μ for which the vector \overline{PQ} is parallel to the plane $x - 4y + 3z = 1$ is

(A) $\frac{1}{4}$

(B) $-\frac{1}{4}$

(C) $\frac{1}{8}$

(D) $-\frac{1}{8}$

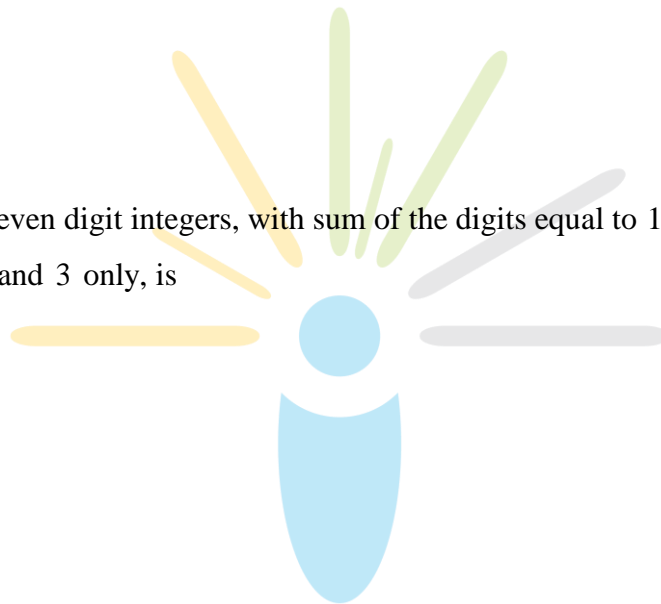
26. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is

(A) 55

(B) 66

(C) 77

(D) 88



27. Let f be a non-negative function defined on the interval $[0,1]$. If

$$\int_0^x \sqrt{1-(f'(t))^2} dt = \int_0^x f(t) dt, 0 \leq x \leq 1, \text{ and } f(0) = 0, \text{ then}$$

(A) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(B) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(C) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

(D) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

28. Tangents drawn from the point $P(1,8)$ to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B . The equation of the circumcircle of the triangle PAB is

(A) $x^2 + y^2 + 4x - 6y + 19 = 0$

(B) $x^2 + y^2 - 4x - 10y + 19 = 0$

(C) $x^2 + y^2 - 2x + 6y - 29 = 0$

(D) $x^2 + y^2 - 6x - 4y + 19 = 0$

SECTION-II

Multiple Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

29. In a triangle ABC with fixed base BC , the vertex A moves such that

$\cos B + \cos C = 4 \sin^2 \frac{A}{2}$ If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C , respectively, then

(A) $b + c = 4a$

(B) $b + c = 2a$

(C) locus of point A is an ellipse

(D) locus of point A is a pair of straight lines

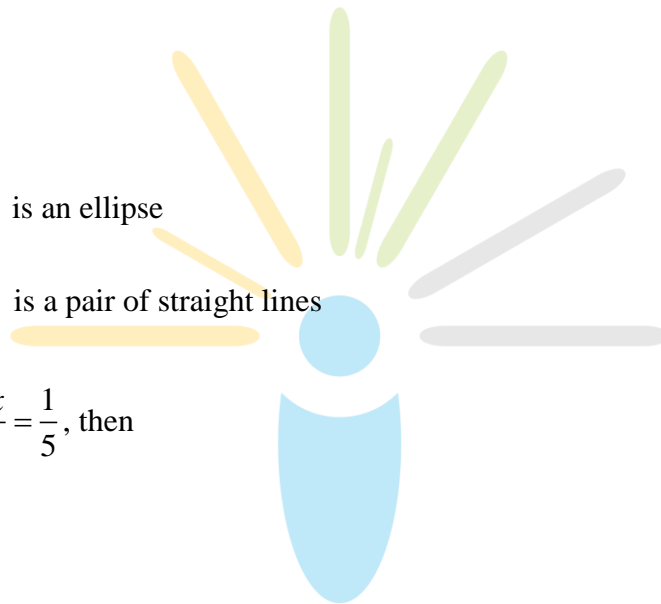
30. If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, then

(A) $\tan^2 x = \frac{2}{3}$

(B) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$

(C) $\tan^2 x = \frac{1}{3}$

(D) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

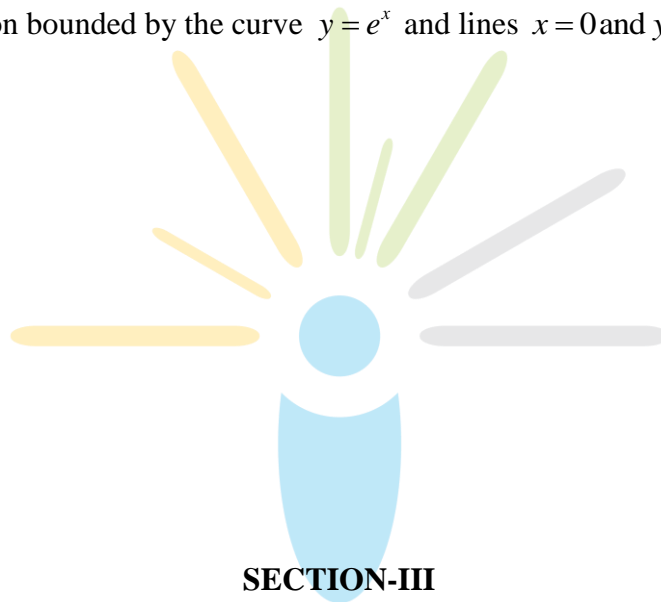


31. Let $L = \lim_{x \rightarrow 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, $a > 0$ If L is finite, then

- (A) $a = 2$
- (B) $a = 1$
- (C) $L = \frac{1}{64}$
- (D) $L = \frac{1}{32}$

32. Area of the region bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is

- (A) $e - 1$
- (B) $\int_1^e \ln(e + 1 - y) dy$
- (C) $e - \int_1^e e^x dx$
- (D) $\int_1^e \ln y dy$



SECTION-III

Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for question Nos. 33 to 35

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

33. The probability that $X = 3$ equals

(A) $\frac{25}{216}$

(B) $\frac{25}{36}$

(C) $\frac{5}{36}$

(D) $\frac{125}{216}$

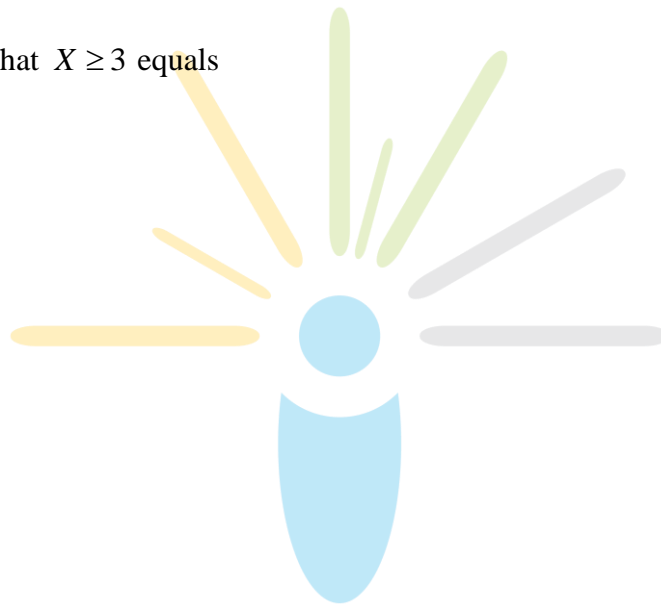
34. The probability that $X \geq 3$ equals

(A) $\frac{125}{216}$

(B) $\frac{25}{36}$

(C) $\frac{5}{36}$

(D) $\frac{25}{216}$



35. The conditional probability that $X \geq 6$ given $X > 3$ equals

(A) $\frac{125}{216}$

(B) $\frac{25}{216}$

(C) $\frac{5}{36}$

(D) $\frac{25}{36}$

Paragraph for question Nos. 36 to 38

Let \underline{A} be the set of all 3×3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

36. The number of matrices in \underline{A} is

- (A) 12
- (B) 6
- (C) 9
- (D) 3

37. The number of matrices A in \underline{A} for which the system of linear equations $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ has a unique solution, is

- (A) less than 4
- (B) at least 4 but less than 7
- (C) atleast 7 but less than 10
- (D) at least 10

38. The number of matrices A in \underline{A} for which the system of linear equations $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is inconsistent, is

- (A) 0
- (B) more than 2
- (C) 2
- (D) 1