IIT-JEE-2009

PAPER– II

MATHEMATICS

[Time allowed: 3 hours] [Maximum Marks: 240]

General Instruction:

A. Question paper format:

1. **Section I** contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

2. **Section II** contains **5 multiple choice questions.** Each question has four choice (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

3. **Section III** contains **2 questions**. Each question has four statements (A, B, C and D) given in column I and five statements (p, q, r, s and t) in Column II. Any given statement in column I can have correct matching with one or more statements(s) given in column II. For example, if for example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, daken the bubbles corresponding to q and r in the ORS.

4. **Section IV** contains **8 questions.** The answer to each question is a **single digit integer,** ranging from 0 to 9. The answer will have to be appropriately bubbled in the ORS as per the instructions given at the beginning of the section.

B. Marking scheme:

5. For each question in **Section I,** you will be **awarded 3 marks** if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. In all other cases, **minus one (-1) mark** will be awarded.

6. For each question in **Section II,** you will be **awarded 4 marks** if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. In all other cases, **minus one (-1) mark** will be awarded.

13. For each question in **Section III,** you will be **awarded 2 marks** if you darken only the bubble corresponding to the correct answer. Thus, each question in this section carries a maximum of 8 marks. There is **no negative marking** for incorrect answer(s) for this section.

14. For each question in **Section IV,** you will be **awarded 4 marks** if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. In all other cases, **minus one (-1) mark** will be awarded.

SECTION – I

Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONLY ONE is correct.

20. The normal at a point P on the ellipse $x^2+4y^2=16$ meets the x-axis at Q. If M is the mid point of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the points

(A) $\left(\pm\frac{3\sqrt{5}}{2},\pm\frac{2}{7}\right)$

(B) $\left(\pm\frac{3\sqrt{5}}{2},\pm\frac{\sqrt{19}}{4}\right)$

(C) $\left(\pm2\sqrt{3},\pm\frac{1}{7}\right)$

(D) $\left(\pm2\sqrt{3},\pm\frac{4\sqrt{3}}{7}\right)$

21. The locus of the orthocentre of the triangle formed by the lines $(1+p)x-py+p(1+p)=0,(1+q)x-qy+q(1+q)=0$ and $y=0$, where $p\neq q$, is

(A) a hyperbola

(B) a parabola

(C) an ellipse

(D) a straight line

22. A line with positive direction cosines passes through the point $P(2, -1, 2)$ and makes equal angles with the coordinate axes. The line meets the plane $2x + y + z = 9$ at point Q. The length of the line segment PQ equals

(A) 1

(B) $\sqrt{2}$

(C) $\sqrt{3}$

(D) 2

23. If the sum of first n terms of an A. P. is en^2, then the sum of squares of these n terms is

(A) $\frac{n(4n^2-1)c^2}{6}$

(B) $\frac{n(4n^2+1)c^2}{3}$

(C) $\frac{n(4n^2-1)c^2}{3}$

(D) $\frac{n(4n^2+1)c^2}{6}$

SECTION-II

Multiple Correct Choice Type

This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONE OR MORE is/are correct

24. The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose

(A) vertex is $\left(\frac{2a}{3}, 0\right)$

(B) directrix is $x = 0$

(C) latus rectum is $\frac{2a}{3}$

(D) focus is $(a, 0)$

25. For function $f(x) = x\cos\frac{1}{x}, x \geq 1,$

(A) for atleast one x in interval $[1,8)$, $f(x+2) - f(x) < 2$

(B) $\lim_{x\to\infty} f'(x) = 1$

(C) for all x in the interval $[1,8)$, $f(x+2) - f(x) > 2$

(D) $f'(x)$ f (x) is strictly decreasing in die interval $[1,8)$

26. For $0 < \theta > \frac{\pi}{2}$, the solution(s) of $\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}\,\text{is}\,(\text{are})$

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{12}$

(A) $\frac{5\pi}{12}$

27. An ellipse intersects the hyperbola $2x - 2y = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinates axes, then

(A) equation of ellipse is $x^2 + 2y^2 = 2$

(B) The foci of ellipse are $(\pm 1, 0)$

(C) equation of ellipse is $x^2 + 2y^2 = 4$

(D) the foci of ellipse are $\left(\pm\sqrt{2}, 0\right)$

28. $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx, n = 0,1,2,\text{K}$, then

(A) $I_n = I_{n+2}$

(B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$

(C) $\sum_{m=1}^{10} I_{2m} = 0$

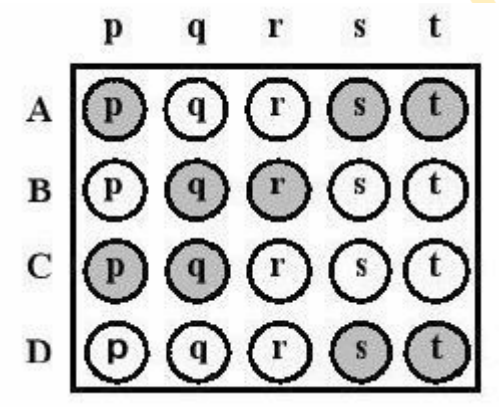
(D) $I_n = I_{n+1}$

SECTION - III

Matrix — Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statement in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with ONE OR MORE statement (s) in **Column II.** The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are A - p, s and t; B - q and r; C - p and q; and D -s and t; then the correct darkening of bubbles will look like the following.



29. Match the statements/expressions in **Column I** with the values given in **Column II.**

Column-I	**Column-II**
(A) The number of solutions of the equation $xe^{\sin x}-\cos x=0$ in the interval $\left(0,\frac{\pi}{2}\right)$	(p) 1
(B) Value(s) of k for which the planes $kx+4y+z=0, 4x+ky+2z=0$ and $2x+2y+z=0$ intersect in a straight line	(q) 2
(C) Value(s) of k for which $\lvert x-1\rvert+\lvert x-2\rvert+\lvert x+1\rvert+\lvert x+2\rvert=4k$ has integer solution(s)	(r) 3
(D)If $y'=y+1$and $y(0)=1$ then value(s) of y (ln2)	(s) 4
	(t) 6

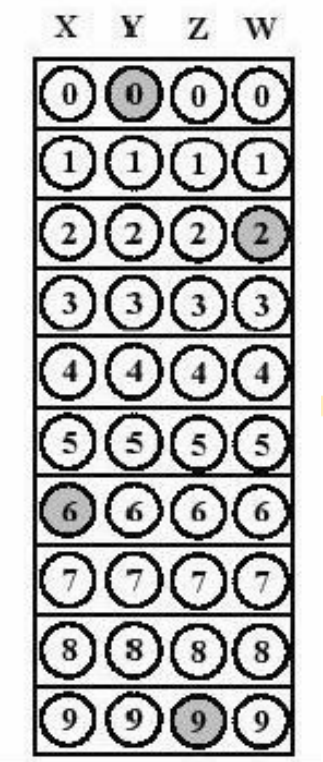
30. Match the statements/expressions in **Column I** with the values given in **Column II.**

Column-I	**Column-II**
(A) Root(s) of the expression $2\sin^2\theta+\sin^2 2\theta=2$	(p) $\frac{\pi}{6}$
(B) Points of discontinuity of the function $f(x)=\left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$, where $[y]$ denotes the largest integer less than or equal to y	(q) $\frac{\pi}{4}$
(C) Volume of the parallelopiped with its edges represented by the vectors $\hat{i}+\hat{j},\hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+\pi\hat{k}$	(r) $\frac{\pi}{3}$
(D) Angle between vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}+\vec{b}+\sqrt{3}\vec{c}=\vec{0}$	(s) $\frac{\pi}{2}$
	(t) π

SECTION-IV

Integer Answer Type

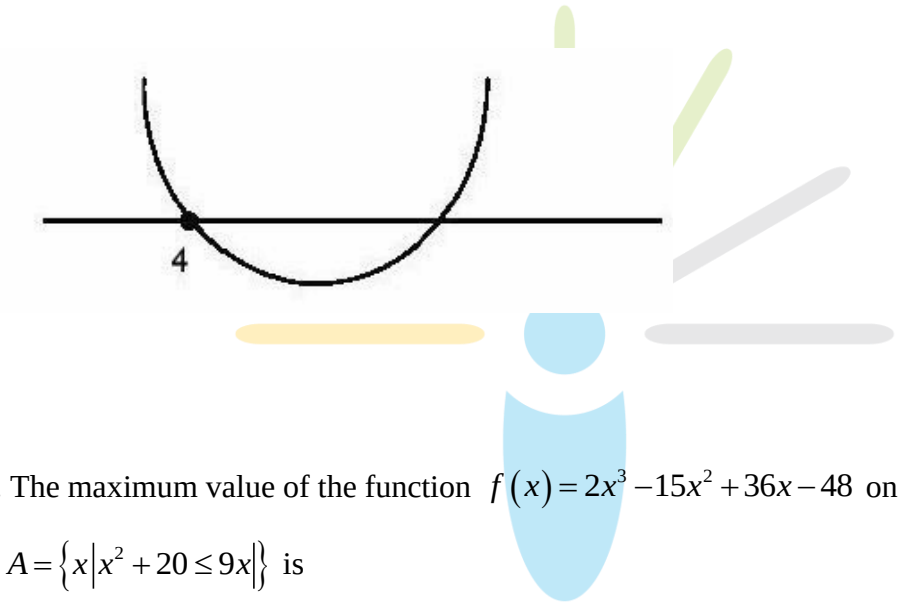
This section contains 8 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the **correct** darkening of bubbles will look like the following:



31. Let $f:R\to R$ be a continuous function which satisfies $f(x)=\int_0^x f(t)\,dt$. Then the value of $f(\ln 5)$ is

32. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to Q and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is

33. The smallest value of k, for which both the roots of the equation $x^2-8kx+16\left(k^2-k+1\right)=0$ are real, distinct and have values at least 4, is



34. The maximum value of the function $f\left(x\right)=2x^3-15x^2+36x-48$ on the set $A=\left\{x\middle|x^2+20\le 9x\right\}$ is

35. Let ABC and ABC' be two non-congruent triangles with sides $AB=4,\ AC=AC'=2\sqrt{2}$ and angle $B=30°$. The absolute value of the difference between the areas of these triangles is

36. If the function $f\left(x\right)=x^3+e^{x/2}$ and $g\left(x\right)=f^{-1}\left(x\right)$ then the value of $g'\left(1\right)$ is

37. Let $p(x)$ be a polynomial of degree 4 having extremum at $x=1,2$ and $\lim_{x\to 0}\left(1+\frac{p(x)}{x^2}\right)=2$. Then the value of $p(2)$ is

38. Let (x,y,z) be points with integer coordinates satisfying the system of homogeneous equations:

$$\begin{aligned} 3x-y-z&=0\\ -3x+z&=0\\ -3x+2y+z&=0 \end{aligned}$$

Then the number of such points for which $x^2+y^2+z^2\le 100$ is