JEE MAIN-2010

MATHEMATICS

A. General Instructions :

1. This Question Paper contains 32 pages having 84 questions.

2. The question paper CODE is printed on the right hand top corner of this sheet and also on the back page (page no. 32) of this booklet

3. No additional sheets will be provided for rough work.

4. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets in any form are not allowed.

5. The answer sheet, a machine-gradable Objective Response Sheet (ORS), is provided separately.

6. Do not Tamper / mutilate the ORS or this booklet.

7. Do not break the seals of the question - paper booklet before instructed to do so by the invigilators.

B. Filling the bottom-half of the ORS:

8. The ORS has CODE printed on its lower and upper Parts.

9. Make sure the CODE on the ORS is die same as that on this booklet. If the Codes do not match, ask for a change of the Booklet.

10. Write your Registration No., Name and Name of centre and sign with pen in appropriate boxes. Do not write these any where else.

11. Darken the appropriate bubbles below your registration number with HB Pencil.

C. Question paper format and Marking scheme:

12. The question paper consists of 3 parts (Mathematics). Each part consists of four Sections.

13 For each question in Section I, you will be awarded 3 marks if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. In all other cases, minus one (-1) mark will be awarded.

14. For each question in Section II, you will be awarded 3 marks if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. Partial marks will be awarded for partially coned answers. No negative marks will be awarded in this Section.

15. For each question in Section III, you will be awarded 3 marks if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. In all other cases, minus one (—1) mark will be awarded.

16. For each question in Section IV, you will be awarded 3 marks if you darken the bubble corresponding to the correct answer and zero mark if no bubble is darkened. No negative marks will be awarded for in this Section

Useful Data

Atomic Numbers: Be 4; C 6; N 7; O 8; A1 13; Si 14; Cr24; Fe 26; Zn 30; Br 35.

1 amu = 1.66×10^{-27} kg	R = 0.082 L-atm K^{-1} mol^{-1}
h = 6.626×10^{-34} J s	N_A = 6.022×10^{23}
m_e = 9.1×10^{-31} kg	e = 1.6×10^{-19} C
c = 3.0×10^{8} m $^{s-1}$	F = 96500 C $mo1^{-1}$
R_H = 2.18×10^{-18} J	$4\pi \epsilon_0$ = 1.11×10^{-10} J^{-1} C^2 m^{-1}

SECTION I

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of **which ONLY ONE** is correct.

29. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If $r_{1,} r_2$ and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is

(A) $\frac{1}{18}$

(B) $\frac{1}{9}$

(C) $\frac{2}{9}$

(D) $\frac{1}{36}$

30. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i} - \hat{j}, 4\hat{i}, +3\hat{i} + 3\hat{j}$ and $-3\hat{i} + 2\hat{j}$ respectively. The quadrilateral $PQRS$ must be a

(A) parallelogram, which is neither a rhombus nor a rectangle

(B) square

(C) rectangle, but not a square

(D) rhombus, but not a square

31. The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$ has exactly two distinct solutions, is

(A) 0

(B) 2^9-1

(C) 168

(D) 2

32. The value of $\lim_{x\to0}\frac{1}{x^3}\int_0^x\frac{t\ln(1+t)}{t^4+4}dt$ is

(A) 0

(B) $\frac{1}{12}$

(C) $\frac{1}{24}$

(D) $\frac{1}{64}$

33. Let p and q be real numbers such that $p\neq0, p^3\neq q$ and $p^3\neq-q$. If α and β are nonzero complex numbers satisfying $\alpha+\beta=-p$ and $\alpha^3+\beta^3=q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

(A) $(p^3+q)x^2-(p^3+2q)x+(p^3+q)=0$

(B) $(p^3+q)x^2-(p^3-2q)x+(p^3+q)=0$

(C) $\left(p^3-q\right)x^2-\left(5p^3-2q\right)x+\left(p^3-q\right)=0$

(D) $\left(p^3-q\right)x^2-\left(5p^3+2q\right)x+\left(p^3-q\right)=0$

34. Let f, g and h be real-valued functions defined on the interval $[0,1]$ by $f(x)=e^{x^2}+e^{-x^2}, g(x)=xe^{x^2}+e^{-x^2}$ and $h(x)=x^2e^{x^2}+e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum off, f, g and h on $[0,1]$, then

(A) $a=b$ and $c\neq b$

(B) $a=c$ and $a\neq b$

(C) $a\neq b$ and $c\neq b$

(D) $a=b=c$

35. If the angles A, B and C of a triangle are in an arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C+\frac{c}{a}\sin 2A$ is

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) 1

(D) $\sqrt{3}$

36. Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is

(A) $x+2y-2z=0$

(B) $3x+2y-2z=0$

(C) $x-2y+z=0$

(D) $5x+2y-4z=0$

SECTION II

This section contains **5 multiple choice questions**. Each question has four choices A), B), Q and D) out of which **ONE OR MORE** may be correct

37. Let z_1 and z_2 be two distinct complex numbers and let $z=(1-t)z_1+tz_2$ for some real number t with $0<t<1$. If $\text{Arg}(w)$ denotes the principal argument of a non-zero complex number w, then

(A) $|z-z_1|+|z-z_2|=|z_1-z_2|$

(B) $\text{Arg}(z-z_1)=\text{Arg}(z-z_2)$

(C) $\begin{vmatrix} z-z_1 & \bar{z}-\bar{z}_1 \\ z_2-z_1 & \bar{z}_2-\bar{z}_1 \end{vmatrix}=0$

(D) $\text{Arg}(z-z_1)=\text{Arg}(z_2-z_1)$

38. The value(s) of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$ is (are)

(A) $\frac{22}{7}-\pi$

(B) $\frac{2}{105}$

(C) 0

(D) $\frac{71}{15}-\frac{3\pi}{2}$

39. Let ABC be a triangle such that $\angle ACB=\frac{\pi}{6}$ and let a,b and c denote the lengths of the sides opposite to A,B and C respectively. The value(s) of x for which $a=x^2+x+1, b=x^2-1$ and $c=2x+1$ is (are)

(A) $-2\left(2+\sqrt{3}\right)$

(B) $1+\sqrt{3}$

(C) $2+\sqrt{3}$

(D) $4\sqrt{3}$

40. Let A and B be two distinct points on the parabola $y^2=4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be

(A) $-\frac{1}{r}$

(B) $\frac{1}{r}$

(C) $\frac{2}{r}$

(D) $-\frac{2}{r}$

41. Let f be a real-valued function defined on the interval $(0,\infty)$ by $f(x)=\ln x+\int_0^x \sqrt{1+\sin t}\,dt$. Then which of the following statement(s) is (are) true?

(A) $f''(x)$ exists for all $x\in(0,\infty)$

(B) $f'(x)$ exists for all $x\in(0,\infty)$ and f' is continuous on $(0,\infty)$, but not differentiable on $(0,\infty)$

(C) there exists $\alpha>1$ such that $|f'(x)|<|f(x)|$ for all $x\in(\alpha,\infty)$

(D) there exists $\beta>0$ such that $|f(x)|+|f'(x)|\le\beta$ for all $x\in(0,\infty)$

SECTION III

This section contains 2 paragraphs. Based upon the first paragraph 2 multiple choice questions and based upon tine second paragraph 3 multiple choice questions have to be answered Each of these questions has four choices A), B), C) and D) out of WHICH ONLY ONE CORRECT.

Paragraph for Questions 42 to 43

The circle $x^2+y^2-8x=0$ and hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ intersect at the points A and B.

42. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

(A) $2x-\sqrt{5}y-20=0$

(B) $2x-\sqrt{5}y+4=0$

(C) $3x-4y+8=0$

(D) $4x-3y+4=0$

43. Equation of the circle with AB as its diameter is

(A) $x^2+y^2-12x+24=0$

(B) $x^2+y^2+12x+24=0$

(C) $x^2+y^2+24x-12=0$

(D) $x^2+y^2-24x-12=0$

Paragraph for Questions 44 to 46

Let p be an odd prime number and T_p be the following set of 2×2 matrices:

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, \ldots, p-1\} \right\}$$

44. The number of A in T_p such that A is either symmetric or skew-symmetric or both, and $\det(A)$ divisible by p is

(A) $(p-1)^2$

(B) $2(p-1)$

(C) $(p-1)^2 + 1$

(D) $2p-1$

45. The number of A in T_p such that the trace of A is not divisible by p but $\det(A)$ is divisible by p is [Note: The trace of a matrix is the sum of its diagonal entries.]

(A) $(p-1)(p^2-p+1)$

(B) $p^3-(p-1)^2$

(C) $(p-1)^2$

(D) $(p-1)(p^2-2)$

46. The number of A in T_p such that $\det(A)$ is not divisible by p is

(A) $2p^2$

(B) $p^3 - 5p$

(C) $p^3 - 3p$

(D) $p^3 - p^2$

SECTION IV

This section contains TEN questions. The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled

47. Let $S_k, k = 1, 2 \ldots, 100$, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!} + \sum_{k=1}^{100} \left| \left(k^2 - 3k + 1\right) S_k \right|$ is

48. The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations

$$(y+z)\cos 3\theta = (xyz)\sin 3\theta$$

$$x\sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$$

$$(xyz)\sin 3\theta = (y+2z)\cos 3\theta + y\sin 3\theta$$

have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is

49. Let f be a real-valued differentiable function on R (the set of all real numbers) such that $f(1)=1$. If the y-intercept of the tangent at any point $P(x,y)$ on the curve $y=f(x)$ is equal to the cube of the abscissa of P, then the value of $f(-3)$ is equal to

50. The number of values of θ in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n=0,\pm1,\pm2$ and $\tan\theta=\cot 5\theta$ as well as $\sin 2\theta=\cos 4\theta$ is

51. The maximum value of the expression $\frac{1}{\sin^2\theta+3\sin\theta\cos\theta+5\cos^2\theta}$ is

52. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a}=\frac{\hat{i}-2\hat{j}}{\sqrt{5}}$ and $\vec{b}=\frac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$, then the value of $\left(2\vec{a}+\vec{b}\right)\cdot\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}-2\vec{b}\right)\right]$ is

53. The line $2x+y=1$ is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

54. If the distance between the plane $Ax-2y+z=d$ and the plane containing the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is $\sqrt{6}$, then $|d|$ is

55. For any real number x, let $|x|$ denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval $[-10,10]$ by

$$f(x)=\begin{cases} x-[x] & \text{if } [x] \text{ is odd} \\ 1+[x]-x & \text{if } [x] \text{ is even} \end{cases}$$

Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x\, dx$ is

56. Let ω be the complex number $\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$. Then the number of distinct complex numbers z satisfying $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=0$ is equal to