IIT-JEE-2010

PAPER– II

MATHEMATICS

SECTION – I (Single Correct Choice Type)

20. ***Sol. (A)***

Distance of point $(1,-2,1)$ from plane $x+2y-2z=\alpha$ is $5 \Rightarrow \alpha=10$.

Equation of $PQ \dfrac{x-1}{1}=\dfrac{y+2}{2}=\dfrac{z-1}{-2}=t$

$Q\equiv(t+1,2t-2,-2t+1)$ and $PQ=5 \Rightarrow t=\dfrac{5+\alpha}{9}=\dfrac{5}{3} \Rightarrow Q\equiv\left(\dfrac{8}{3},\dfrac{4}{3},\dfrac{-7}{3}\right)$.

21. ***Sol. (C)***

Event G = original signal is green

E_1 = A receives tlie signal correct

E_2 = B receives the signal correct

E = signal received by B is green

$P(\text{signal received by } B \text{ is green}) = P(GE_1E_2)+P(G\bar{E}_1\bar{E}_2)+P(\bar{G}E_1\bar{E}_2)+P(\bar{G}\bar{E}_1E_2)$

$P(E)=\dfrac{46}{5\times16}$

$P(G/E)=\dfrac{40/5\times16}{46/5\times16}=\dfrac{20}{23}$.

22. ***Sol. (B)***

$\overrightarrow{AD}=\overrightarrow{AB}\times\left(\overrightarrow{AB}\times\overrightarrow{AD}\right)=5\left(6l\hat{i}-10\hat{j}-2l\hat{k}\right) \Rightarrow \cos\alpha=\dfrac{\left|\overrightarrow{AD'}\cdot\overrightarrow{AD}\right|}{\left|\overrightarrow{AD'}\right|\left|\overrightarrow{AD}\right|}=\dfrac{\sqrt{17}}{9}$.

23. *Sol. (D)

$$\text{Let } y=\sum_{r=1}^{10} A_r\left(B_{10}B_r-C_{10}A_r\right)$$

$$\sum_{r=1}^{10} A_rB_r=\text{coefficient of } x^{20}\text{in}\left((1+x)^{10}(x+1)^{20}\right)-1$$

$$=C_{20}-1=C_{10}-1\text{and}\sum_{r=1}^{10}\left(A_r\right)^2=\text{coefficient of } x^{10}\text{in}\left((1+x)^{10}(x+1)^{10}\right)-1=B_{10}-1$$

$$\Rightarrow y=B_{10}\left(C_{10}-1\right)-C_{10}\left(B_{10}-1\right)=C_{10}-B_{10}.$$

24. *Sol. (B)*

$$e^{-x}f(x)=2+\int_0^x\sqrt{t^4+1}\,dt \quad \text{K } (1)$$

$$f\left(f^{-1}(x)\right)=x$$

$$\Rightarrow f'\left(f^{-1}(x)\right)\left(f^{-1}(x)\right)'=1\Rightarrow\left(f^{-1}(2)\right)'=\frac{1}{f'\left(f^{-1}(2)\right)}\Rightarrow f(0)=2\Rightarrow f^{-1}(2)=0$$

$$\left(f^{-1}(2)\right)'=\frac{1}{f'(0)}$$

$$e^{-x}\left(f'(x)-f(x)\right)=\sqrt{x^4+1}$$

$$\text{Put } x=0\Rightarrow f'(0)-2=1\Rightarrow f'(0)=3$$

$$\left(f^{-1}(2)\right)'=1/3.$$

25. *Sol. (D)

Total number of unordered pairs of disjoint subsets

$$=\frac{3^4+1}{2}=41.$$

26. *Sol. (0)

$a_k = 2a_{k-1} - a_{k-2} \Rightarrow a_1, a_2, \ldots a_{11}$ are in $A.P.$

$$\therefore \frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = \frac{11a^2 + 35 \times 11d^2 + 10ad}{11} = 90$$

$$\Rightarrow 225 + 35d^2 + 150d = 90$$

$$35d^2 + 150d + 135 = 0 \Rightarrow d = -3, -9/7$$

Given $a_2 < \frac{27}{2} \therefore d = -3$ and $d \neq -9/7 \Rightarrow \frac{a_1 + a_2 + \ldots + a_{11}}{11} = \frac{11}{2}[30 - 10 \times 3] = 0.$

27. *Sol. (1)*

$$f(x) = In\{g(x)\}$$

$$g(x) = e^{f(x)}$$

$$g'(x) = e^{f(x)} \cdot f'(x)$$

$$g'(x) = 0 \Rightarrow f'(x) = 0 \text{ as } e^{f(x)} \neq 0$$

$$\Rightarrow 2010(x-2009)(x-2010)^2(x-2011)^3(x-2012)^4 = 0$$

so there is only one point of local maxima.

28. *Sol. (5)*

$|A| = (2k+1)^3, |B| = 0$ (Since B is a skew-symmetric matrix of order 3)

$$\Rightarrow \det(adj\, A) = |A|^{n-1} = \left((2k+1)^3\right)^2 = 106 \Rightarrow 2k+1 = 10 \Rightarrow 2k = 9$$

$$[k] = 4.$$

29. *Sol. (2)

$$2\cos\frac{\pi}{2k}+2\cos\frac{\pi}{k}=\sqrt{3}+1$$

$$\cos\frac{\pi}{2k}+\cos\frac{\pi}{k}=\frac{\sqrt{3}+1}{2}$$

$$\text{Let }\frac{\pi}{k}=0,\cos\theta+\cos\frac{\theta}{2}=\frac{\sqrt{3}+1}{2}\Rightarrow 2\cos^2\frac{\theta}{2}-1+\cos\frac{\theta}{2}=\frac{\sqrt{3}+1}{2}$$

$$\cos\frac{\theta}{2}=t\quad 2t^2+t-\frac{\sqrt{3}+3}{2}=0$$

$$t=\frac{-1\pm\sqrt{1+4\left(3+\sqrt{3}\right)}}{4}=\frac{-1\pm2\sqrt{3}+1}{4}=\frac{-2-2\sqrt{3}}{4},\frac{\sqrt{3}}{2}\text{Q } t\in[-1,1],\cos\frac{\theta}{2}=\frac{\sqrt{3}}{2}$$

$$\frac{\theta}{2}=\frac{\pi}{6}\Rightarrow k=3.$$

30. *Sol. (3)*

$$\Delta=\frac{1}{2}ab\sin C\Rightarrow\sin C=\frac{2\Delta}{ab}=\frac{2\times15\sqrt{3}}{6\times10}=\frac{\sqrt{3}}{2}\Rightarrow C=120^\circ$$

$$\Rightarrow c=\sqrt{a^2+b^2-2ab\cos C}$$

$$=\sqrt{6^2+10^2-2\times6\times10\times\cos120^\circ}=144$$

$$\therefore r=\frac{\Delta}{s}\Rightarrow r^2=\frac{225\times3}{\left(\frac{6+10+14}{2}\right)^2}=3.$$

31. *Sol. (C)*

Since, $f\left(-\frac{1}{2}\right)\cdot f\left(-\frac{3}{4}\right)<0\Rightarrow s$ lie in $\left(-\frac{3}{4},-\frac{1}{2}\right)$.

32. *Sol. (A)*

$$-\frac{3}{4} < s < -\frac{1}{2}$$
$$\frac{1}{2} < t \frac{3}{4}$$
$$\int_0^{1/2} \left(4x^3 + 3x^2 + 2x + 1\right) dx < area < \int_0^{3/4} \left(4x^3 + 3x^2 + 2x + 1\right) dx$$
$$\left[x^4 + x^3 + x^2 + x\right]_0^{1/2} < area < \left[x^4 + x^3 + x^2 + x\right]_0^{5/4}$$
$$\frac{1}{16} + \frac{1}{8} + \frac{1}{4} + \frac{1}{2} < area < \frac{81}{256} + \frac{27}{64} + \frac{9}{16} + \frac{3}{4}$$
$$\frac{15}{16} < area < \frac{525}{256}.$$

33. *Sol. (B)*

$$f'(x) = 2[12x + 3] = 0 \Rightarrow x = -1/4.$$

Paragraph for questions 34 to 36.

Tangents are drawn from the point $P(3,4)$ to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at points A and B.

34. *Sol. (D)

$$y = mx + \sqrt{9m^2 + 4}$$
$$4 - 3m = \sqrt{9m^2 + 4}$$
$$16 + 9m^2 - 24m = 9m^2 + 4 \Rightarrow m = \frac{12}{24} = \frac{1}{2}$$

Equation is $y - 4 = \frac{1}{2}(x - 3)$

$$2y - 8 = x - 3 \Rightarrow x - 2y + 5 = 0$$

Let $B = (\alpha, \beta) \Rightarrow \frac{x\alpha}{9} + \frac{y\beta}{4} - 1 = 0 \Rightarrow \frac{\alpha/9}{1} = \frac{\beta/4}{-2} = \frac{-1}{5} \Rightarrow \alpha = -\frac{9}{5}, \beta = \frac{8}{5}$

$$B = \left(-\frac{9}{5}, \frac{8}{5}\right).$$

35. *Sol. (C)

Slope of BD must be 0

$$\Rightarrow y-\frac{8}{5}=0\left(x+\frac{9}{5}\right)\Rightarrow y=\frac{8}{5}$$

Hence y coordinate of D is $8/5$.

36. *Sol. (A)

Locus is parabola

Equation of AB Is $\frac{3x}{9}+\frac{4y}{4}=1\Rightarrow\frac{x}{3}+y=1\Rightarrow x+3y-3=0$

$$(x-3)^2+(y-4)^2=\frac{(x+3y-3)^2}{10}$$

$$10x^2+90-60x+10y^2+160-80y=x^2+9y^2+9+6xy-x-18y$$

$$\Rightarrow 9x^2+y^2-6xy-54x-62y+241=0.$$