IIT-JEE-2010

PAPER– II

MATHEMATICS

General Instruction:

A. General:

1. The **question paper** CODE is prited on the right hand top corner of this sheet and also on the back page of this booklet.

2. No additional sheets will be provided for rough work.

3. Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers, and electronic gadgets in any form are not allowed.

4. The answer sheet, a machine-gradable Objective Response Sheet (ORS), is provided separately

5. DO NOT TAMPER WITH /MUTILATE THE ORS OR THE BOOKLET.

6. Do not break the seals of the question – paper booklet before instructed to do so by the invigilators.

B. Filling the bottom-half of the ORS:

7. The ORS has **CODE** printed on its lower and upper parts.

8. Make sure the CODE on the **ORS** is the same as that on this booklet. **IF THE CODES DO NOT MATCH, ASK FOR A CHANGE OF THE BOOKLET.**

9. Write your Name, Registration Number and the name of examination centre and sign with pen in the boxes. **Do not write these anywhere else.**

10. Darken the appropriate bubbles below your Registration Number with HB Pencil.

C. Question paper format and Marking scheme:

11. For each question in **Section I,** you will be **awarded 5 marks** if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. In all other cases, **minus two (-2) mark** will be awarded.

12. For each question in **Section II,** you will be **awarded 3 marks** if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. No negative marks will be awarded in this Section.

13. For each question in **Section III,** you will be **awarded 3 marks** if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. In all other cases, **minus one (-1) mark** will be awarded.

14. For each question in **Section IV,** you will be **awarded 2 marks** if you darken only the bubble corresponding to the correct answer and zero mark if no bubbles are darkened. No negative marks will be awarded for in this Section.

Write your Name, Registration Number and sign in the space provided on the hack page of this booklet.

SECTION – I (Single Correct Choice Type)

20. If the distance of the point $P(1,-2,1)$ from the plane $x+2y-2z=\alpha$, where $\alpha>0$, is 5, then the foot of the perpendicular from P to the plane is

A) $\left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$

B) $\left(\frac{4}{3},-\frac{4}{3},\frac{1}{3}\right)$

C) $\left(\frac{1}{3},\frac{2}{3},\frac{10}{3}\right)$

D) $\left(\frac{2}{3},-\frac{1}{3},\frac{5}{2}\right)$

21. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and that transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is

A) $\frac{3}{5}$

B) $\frac{6}{7}$

C) $\frac{20}{23}$

D) $\frac{9}{20}$

22. Two adjacent sides of a parallelogram $ABCD$ are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by

A) $\frac{8}{9}$

B) $\frac{\sqrt{17}}{9}$

C) $\frac{1}{9}$

D) $\frac{4\sqrt{5}}{9}$

*23. For $r = 0, 1, \ldots 10$, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of $(1+x)^{10}, (1+x)^{20}$ and $(1+x)^{30}$. Then $\sum_{r=1}^{10} A_r\left(B_{10}B_r - C_{10}A_r\right)$ is equal to

A) $B_{10} - C_{10}$

B) $A_{10}\left(B_{10}^2 - C_{10}A_{10}\right)$

C) 0

D) $C_{10} - B_{10}$

24. Let f be a real-valued function defined on the interval $(-1,1)$ such that

$e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4+1}\,dt$, for all $x \in (-1,1)$ and let f^{-1} be the inverse function of f.

Then $\left(f^{1}\right)'(2)$ is equal to

A)1

B) $1/3$

C) $1/2$

D) $1/e$

*25. Let $S = \{1,2,3,4\}$. The total number of unordered pairs of disjoint subsets of S is equal to

A) 25

B) 34

C) 42

D) 41

*26. Let $a_1, a_2, a_3 K, a_{11}$ be real numbers satisfying $a_1 = 15, 27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, K, 11$. If $\frac{a_1^2 + a_2^2 + K + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + K + a_{11}}{11}$ is equal to

27. Let f be a function defined on R (the set of all real numbers) such that $f'(x)=2010(x-2009)(x-2010)^2(x-2011)^3(x-2012)^4$, for all $x\in R$. If g is a function defined on R with values in the interval $(0,\infty)$ such that $f(x)=In(g(x))$, for all $x\in R$, then the number of points in R at which g has a local maximum is

28. Let k be a positive real number and let $A=\begin{vmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{vmatrix}$ and

$B=\begin{vmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{vmatrix}$. If $\det(adj\,A)+\det(adj\,B)=10^6$, then $[k]$ is equal to

[Note: $adj\,M$ denotes the adjoint of a square matrix M and $[k]$ denotes the largest integer less than or equal to k].

*29. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the centre, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k>0$, then the value of $[k]$ is

[Note: $[k]$ denotes the largest integer less than or equal to k].

30. Consider a triangle ABC and let a a,b and c denote the lengths of the sides opposite to vertices A,B and C respectively. Suppose $a=6, b=10$ and the area of the triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and if r denotes the radius of the incircle of the triangle, then r^2 is equal to

Paragraph for questions 31 to 33.

Consider the polynomial $f(x) = 1 + 2x + 3x^2 + 4x^3$. Let s be the sum of all distinct real roots of $f(x)$ and let $t = |s|$.

31. The real number s lies in the interval

(A) $\left(-\frac{1}{4}, 0\right)$

(B) $\left(-11, -\frac{3}{4}\right)$

(C) $\left(-\frac{3}{4}, -\frac{1}{2}\right)$

(D) $\left(0, \frac{1}{4}\right)$

32. The area bounded by the curve $y = f(x)$ and the lines $x = 0, y = 0$ and $x = t$, lies in the interval

(A) $\left(\frac{3}{4}, 3\right)$

(B) $\left(\frac{21}{64}, \frac{11}{16}\right)$

(C) $(9, 10)$

(D) $\left(0, \frac{21}{64}\right)$

33. The function $f'(x)$ is

(A) increasing in $\left(-t,-\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4},t\right)$

(B) decreasing in $\left(-t,-\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4},t\right)$

(C) increasing in $(-t,t)$

(D) decreasing in $(-t,t)$

*34. The coordinates of A and B are

(A) $(3,0)$ and $(0,2)$

(B) $\left(-\frac{8}{5},\frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5},\frac{8}{5}\right)$

(C) $\left(-\frac{8}{5},\frac{2\sqrt{161}}{15}\right)$ and $(0,2)$

(D) $(3,0)$ and $\left(-\frac{9}{5},\frac{8}{5}\right)$

*35. The orthocentre of the triangle PAB is

(A) $\left(5, \frac{8}{7}\right)$

(B) $\left(\frac{7}{5}, \frac{25}{8}\right)$

(C) $\left(\frac{11}{5}, \frac{8}{5}\right)$

(D) $\left(\frac{8}{25}, \frac{7}{5}\right)$

*36. The equation of the locus of the point whose distances from the point P and the line AB are equal, is

(A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

(B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

(C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$

(D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$