JEE MAIN-2011

MATHEMATICS

41. Let $P(6,3)$ be a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ If the normal at the point P intersects the x-axis at (9.0) . then the eccentricity of the hyperbola is -

(A) $\sqrt{\frac{5}{2}}$

(B) $\sqrt{\frac{3}{2}}$

(C) $\sqrt{2}$

(D) $\sqrt{3}$

42. Let (x,y) be any point on the parabola $y^2=4x$. Let P be the point that divides the line segment from $(0,0)$ to (x,y) in the ratio $1:3$ Than the locus of P is-

(A) $x^2=y$

(B) $y^2=2x$

(C) $y^2=x$

(D) $x^2=2y$

43. Let $f(x)=x^2$ and $g(x)=\sin x$ for all $x\in R$. Then the set of all x satisfying $(f\,0\,g\,0\,g\,0\,f)(x)=(g\,0\,g\,0\,f)(x)$, where $(f\,0\,g)(x)=f(g(x))$, is-

(A) $\pm\sqrt{n\pi}, n\in\{0,1,2,\ldots\}$

(B) $\pm\sqrt{n\pi}, n\in\{1,2,\ldots\}$

(C) $\frac{\pi}{2}+2n\pi, n\in\{\ldots,-2,-1,0,1,2\ldots\}$

(D) $2n\pi, n\in\{\ldots,-2,-1,0,1,2\ldots\}$

44. Let $f:[-1,2]\to[0,\infty]$ be a continuous function such that $f(x)=f(1-x)$ for all $x\in[-1,2]$. Let $R_1=\int_{-1} xf(x)\,dx$, and R_2 be the area of the region bounded by $y=f(x), x=-1, x=2$, and the x-axis. Then-

(A) $R_1=2R_2$

(B) $R_1=3R_2$

(C) $2R_1=R_2$

(D) $3R_1=R_2$

45. If $\lim_{x\to 0}\left[1+x\ell n\left(1+b^2\right)\right]^{\frac{1}{x}}=2b\sin^2\theta, b>0$ and $\theta\in(-\pi,\pi)$, then the value of θ is-

(A) $\pm\frac{\pi}{4}$

(B) $\pm\frac{\pi}{3}$

(C) $\pm\frac{\pi}{6}$

(D) $\pm\frac{\pi}{2}$

46. The circle passing through the point $-1,0$ and touching the y-axis at (0.2) also passes through the point -

(A) $\left(-\frac{3}{2},0\right)$

(B) $\left(-\frac{5}{2},0\right)$

(C) $\left(-\frac{3}{2},\frac{5}{2}\right)$

(D) $(-4,0)$

47. Let $\omega\neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of a,b and c is either ω or ω^2. Then the number of distinct matrices in the set S is-

(A) 2

(B) 6

(C) 4

(D) 8

48. A value of b for which the equations

$x^2 + bx - 1 = 0$
$x^2 + x + b = 0,$

have one root in common is -

(A) $-\sqrt{2}$

(B) $-i\sqrt{3}$

(C) $i\sqrt{5}$

(D) $\sqrt{2}$

49. If $f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \le -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2} < x \le 0 \\ x - 1, & 0 < x \le 1 \\ \ell n x, & x > 1 \end{cases}$ then –

(A) $f(x)$ is continuous at $x = \frac{\pi}{2}$

(B) $f(x)$ is not differentiable at $x = 0$

(C) $f(x)$ is differentiable at $x = 1$

(D) $f(x)$ is differentiable at $x = -\frac{3}{2}$

50. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point $(9,6)$, then L is given by-

(A) $y - x + 3 = 0$

(B) $y + 3x - 33 = 0$

(C) $y + x - 15 = 0$

(D) $y - 2x + 12 = 0$

51. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If $P(T)$ denotes the probability of occurrence of the event T, then -

(A) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$

(B) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$

(C) $P(E) = \frac{2}{5}, P(F) = \frac{1}{5}$

(D) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$

52. Let $f:(0,1) \to R$ be defined by $f(x) = \frac{b - x}{1 - bx}$, where b is a constant such that $0 < b < 1$. Then

(A) f is not invertible on $(0,1)$

(B) $f \neq f^{-1}$ on $(0,1)$ and $f'(b) = \frac{1}{f'(0)}$

(C) $f = f^{-1}$ on $(0,1)$ and $f'(b) = \dfrac{1}{f'(0)}$

(D) f^{-1} is differentiable on $(0,1)$

53. Let $y'(x) + y(x)g'(x) = g(x)g'(x), y(0) = 0, x \in R$. where $f'(x)$ denotes $\dfrac{df(x)}{dx}$ and $g(x)$ is a given non-constant differentia ale function on R with $g(0) = g(2) = 0$. Then the value of $y(2)$ is

54. Let $\vec{a} = -\vec{i} - \vec{k}, \vec{b} = -\vec{i} + \vec{j}$ and $\vec{c} = \vec{i} + 2\vec{j} + 3\vec{k}$ be three given vectors. If $\vec{r}$ is a vector, such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$, then the value of $\vec{r}.\vec{b}$ is

55. Let $\omega = e^{i\pi/3}$, and a, b, c, x, y, z be non-zero complex numbers such that

$$a + b + c = x$$
$$a + b\omega + c\omega^2 = y$$
$$a + b\omega^2 + c\omega = z.$$

Then the value of $\dfrac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ is

56. Let M be 3×3 matrix satisfying

$$M\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix}, M\begin{bmatrix}1\\-1\\-1\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix} \text{and } M=\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}$$

Then the sum of the diagonal entries of M is

57. The number of distinct real roots of $x^4-4x^3+12x^2+x-1=0$ is

58. The straight line $2x-3y=1$ divides the circular region $x^2+y^2\le 6$ into two parts. If $S=\left\{\left(2,\frac{3}{4}\right),\left(\frac{5}{2},\frac{3}{4}\right),\left(\frac{1}{4},-\frac{1}{4}\right),\left(\frac{1}{8},\frac{1}{4}\right)\right\}$, then die number of point(s) in S lying inside die smaller part is

59. Match the statements given in Column I with the values given in Column II

Column I	Column II
(A) If $\vec{a}=\hat{j}+\sqrt{3}\hat{k}.\vec{b}=-\hat{j}+\sqrt{3}\hat{k}$ and $\vec{c}=2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is	(p) $\frac{\pi}{6}$
(B) If $\int_a^b (f(x)-3x)dx=a^2-b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is	(q) $\frac{2\pi}{3}$
(C) The value of $\frac{\pi^2}{\ell n3}\int_{7/6}^{5/6}\sec(\pi x)dx$ is	(r) $\frac{\pi}{3}$
(D) The maximum value of $\left\|Arg\left(\frac{1}{1-z}\right)\right\|$ for $\|z\|=1, z\neq 1$ is given by	(s) π
	(t) $\frac{\pi}{2}$

60. Match the statements given in Column I with the intervals/union of intervals given in Column II

Column I	Column II
(A) The set $\left\{ \text{Re}\left(\frac{2iz}{1-z^2}\right) : z \text{ is a complex number}, \lvert z\rvert = 1, z \neq \pm 1 \right\}$ is	(p) $(-\infty,-1)\cup(1,\infty)$
(B) The domain of the function $f(x) - \sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is	(q) $(-\infty,0)\cup(0,\infty)$
(C) If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set	(r) $[2,\infty)$
$\left\{ f(\theta) : 0 \le \theta < \frac{\pi}{2} \right\}$ is	(s) $(-\infty,-1]\cup[0,\infty)$
(D) If $f(x) = x^{3/2}(3x-10), x \ge 0$, then $f(x)$ is increasing in	(t) $(-\infty,0]\cup[2,\infty)$