IIT-JEE-2012 PAPER 2

PART-I: Mathematics

[Time allowed: 3 hours Maximum Marks: 198]

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose

INSTRUCTIONS

A. General:

1. This booklet is your Question paper. Do not break the seats of his booklet before being instructed to do so by the invigilators.

2. The question paper CODE is printed on the right hand top corner of this page and on the back page of this booklet.

3. Blank spaces and blank pages are provided in this booklet for your rough work. No additional sheets will be provided for rough work.

4. Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers, and electronic gadgets are NOT allowed inside the examination hall.

5. Answers to the questions and personal details are to be filled on a two-part carbon-less paper, which is provided separately. You should not separate these parts. The invigilator will separate them at the end of examination. The upper sheet is machine-gradable Objective Response Sheet (ORS) which will be taken back by the invigilator.

6. **Using a black ball point pen, darken the bubbles on the upper original sheet.** Apply sufficient pressure so that the impression is created on the bottom sheet.

7. DO NOT TAMPER WITH /MUTILATE THE ORS OR THE BOOKLET.

8. On breaking the seals of the booklet check that it contains 28 pages and all 60 questions and corresponding answer choices are legible. Read carefully the instructions printed at the beginning of each section.

B. Filling the Right Part of the ORS:

9. The ORS also has a **CODES** printed on its left and right parts.

10. Check that the same CODE is printed on the ORS and on this booklet. **IF IT IS NOT THEN ASK FOR A CHANGE OF THE BOOKLET.** Sign at the place provided on the ORS affirming that you have verified that all the code are same.

11. Write your Name, Registration Number and the name of examination centre and sign with pen in the boxes provided on the right part of the ORS. **Do not write any of this information anywhere else.** Darken the appropriate bubble UNDER each digit of your Registration Number in such a way that the impression is created on the bottom sheet. Also darken the paper CODE given on the right side of **ORS(R_4).**

C. Question paper format

The question paper consists of **3 parts** (Physics, Chemistry and Mathematics). Each part consists of three sections.

12. **Section I** contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

13. **Section II** contains **3 paragraphs** each describing theory, experiment, date etc. There are **6 multiple choice questions** relating to three paragraphs with **2 questions on each paragraph**. Each question has four choice (A), (B), (C) and (D) out of which ONE or MORE are correct.

14. **Section III** contains **6 multiple choice questions.** Each question has four choice (A), (B), (C) and (D) out of which ONE or MORE are correct.

D. Marking Scheme

15. For each question in **Section I and Section II,** you will be awarded **3 marks** if you darken the bubble corresponding to the correct answer **ONLY** and **zero marks** if no bubbles are darkened. In all other cases, **minus one (—1)** mark will be awarded in this sections.

16. For each question in **Section III,** you will be awarded **4 marks** if you darken **ALL** the bubble)s) corresponding to the correct answer(s) **ONLY.** In all other cases **zero (0) marks** will be awarded. **No negative marks** will be awarded for incorrect answer in this section.

Write your Name, Registration Number and sign in the space provided on the hack page of this booklet.

SECTION I : Single Correct Answer Type

This section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

41. Let $a_1, a_2, a_3, \ldots\ldots$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$

(A) 22

(B) 23

(C) 24

(D) 25

42. The equation of a plane passing through the line of intersection of the planes $x+2y+3z=2$ and $x-y+z=3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3,1,-1)$ is

(A) $5x-11y+z=17$

(B) $\sqrt{2}x+y=3\sqrt{2}-1$

(C) $x+y+z=\sqrt{3}$

(D) $x-\sqrt{2}y=1-\sqrt{2}$

43. Let PQR be a triangle of area Δ with $a=2, b=\frac{7}{2}$ and $c=\frac{5}{2}$, where $a, b,$ and c are the lengths of the sides of the tringle opposite to the angles at P, Q and R respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals

(A) $\frac{3}{4\Delta}$

(B) $\frac{45}{4\Delta}$

(C) $\left(\frac{3}{4\Delta}\right)^2$

(D) $\left(\frac{45}{4\Delta}\right)^2$

44. If $\vec{a}$ and $\vec{b}$ are vectors such $|\vec{a}+\vec{b}|=\sqrt{29}$ and $\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k})=(2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$, then a possible value of $(\vec{a}+\vec{b})\cdot(-7\hat{i}+2\hat{j}+3\hat{k})$ is

(A) 0

(B) 3

(C) 4

(D) 8

45. If P is a 3×3 matrix such that $P^T=2P+I$, where P^T is the transpose of P and I is the 3×3 identity matrix, then three exists a column matrix $X=\begin{bmatrix}x\\y\\x\end{bmatrix}\neq\begin{bmatrix}0\\0\\0\end{bmatrix}$ such that

(A) $PX=\begin{bmatrix}0\\0\\0\end{bmatrix}$

(B) $PX=X$

(C) $PX=2X$

(D) $PX=-X$

46. Let $\alpha(a)$ and $\beta(a)$ be the root of the equitation $\left(\sqrt[3]{1+a}-1\right)x^2+\left(\sqrt{1+a}-1\right)x+\left(\sqrt[6]{1+a}-1\right)=0$ where $a>-1$. Then $\alpha(a)$ and $\lim\limits_{a\to o^+}\alpha(a)$ and $\lim\limits_{a\to o^+}\beta(a)$ are

(A) $-\frac{5}{2}$ and 1

(B) $-\frac{1}{2}$ and -1

(C) $-\frac{7}{2}$ and 2

(D) $-\frac{9}{2}$ and 3

47. Four fair dies $D_1, D_2, D_3,$ and D_4, each having six faces numbered $1, 2, 3, 4, 5,$ and 6 are rolled simultaneously. The probability that D_4 shows a number appearing on one of $D_1, D_2,$ and D_3 is

(A) $\frac{91}{216}$

(B) $\frac{108}{216}$

(C) $\frac{125}{216}$

(D) $\frac{127}{216}$

48. The value of the integral $\int_{-\pi/2}^{\pi/2}\left(x^2+\ln\frac{\pi+x}{\pi-x}\right)\cos x\,dx$ is

(A) 0

(B) $\frac{\pi^2}{2}-4$

(C) $\frac{\pi^2}{2}+4$

(D) $\frac{\pi^2}{2}$

49. A possible equation or L is

(A) $x-\sqrt{3}y=1$

(B) $x+\sqrt{3}y=1$

(C) $x-\sqrt{3}y=-1$

(D) $x+\sqrt{3}y=5$

50. A common tangent of the line two circle is

(A) $x=4$

(B) $x=2$

(C) $x+\sqrt{3}y=4$

(D) $x+2\sqrt{2}y=6$

51. Consider the statement:

P: There exists some $x\in$ such that $f(x)+2x=2(1+x^2)$

Q: There exist some $x\in$ such that $2f(x)+1=2x(1+x)$

Then

(A) Both P and Q are true

(B) P is true and Q is false

(C) P is false and Q is true

(D) both P and Q are false

52. Which of the following is true?

(A) g is increasing on $(1,\infty)$

(B) g is decreasing $(1,\infty)$

(C) g is increasing on $(1,2)$ and decreasing on $(2,\infty)$

(D) g is decreasing on $(1,2)$ and increasing on $(2,\infty)$

53. The value of b_6 is

(A) 7

(B) 8

(C) 9

(D) 11

54. which of the following is correct?

(A) $a_{17} = a_{16} + a_{15}$

(B) $c_{17} \neq c_{16} + c_{15}$

(C) $b_{17} \neq b_{16} + b_{16}$

(D) $a_{17} = c_{17} + b_{16}$

55. For every integer n, let a_n and b_n be real number. Let function $f : IR \to IR$ be given by

$f(x) = \begin{cases} a_n + \sin \pi x, \ for\ x \in [2n, 2n+1] \\ b_n + \cos \pi x, \ for\ x \in (2n-1, 2n) \end{cases}$, for all integer n .If f is continuous, then which of the following hold (s) for all n ?

(A) $a_{n-1} - b_{n-1} = 0$

(B) $a_n - b_n = 1$

(C) $a_n - b_{n+1} = 1$

(D) $a_{n-1} - b_n = -1$

56. If the straight line $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$ are coplanar, then plane(S) containing these two lines is (are)

(A) $y + 2z = -1$

(B) $y + z = -1$

(C) $y - z = -1$

(D) $y-2z=-1$

57. If the adjoint of a 3×3 matrix P is $\begin{vmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{vmatrix}$, then the possible value(s) of the determinate of P is are

(A) -2

(B) -1

(C) 1

(D) 2

58. Let $f:(-1,1)\to IR$ be such that $f(\cos 4\theta)=\frac{2}{2-\sec^2\theta}$ for $\theta\in\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{4},\frac{\pi}{2}\right)$. Then the values(s) of $f\left(\frac{1}{3}\right)$ is (are)

(A) $1-\sqrt{\frac{3}{2}}$

(B) $1+\sqrt{\frac{3}{2}}$

(C) $1-\sqrt{\frac{2}{3}}$

(D) $1+\sqrt{\frac{2}{3}}$

59. Let X and Y be two events such that $P(X\mid Y)=\frac{1}{2}, P(Y\mid X)=\frac{1}{3}$ and $P(X\cap Y)=\frac{1}{6}$. which of the following is(are) correct ?

(A) $P(X\cup Y)=\frac{2}{3}$

(B) X and Y are independent

(C) X and Y are not independent

(D) $P\left(X^{C} \cap Y\right)=\frac{1}{3}$

60. If $f(x)=\int_{0}^{x} e^{t^{2}}(t-2)(t-3)\,dt$ for all $x \in(0, \infty)$, then

(A) f has local maximum at $x=2$

(B) f is decreasing on $(2,3)$

(C) there exists some $c \in(0, \infty)$ such that $f''(c)=0$

(D) f has a local maximum at $x=3$