Solutions to JEE (MAIN) - 2013

MATHEMATICS

31. Sol. **(2)**

$(x-3)^2 + y^2 + \lambda y = 0$

The circle passes through $(1,-2)$

$\Rightarrow 4+4-2\lambda = 0 \Rightarrow \lambda = 4$

$(x-3)^2 + y^2 + 4y = 0 \Rightarrow$ Clearly $(5,-2)$ satisfies.

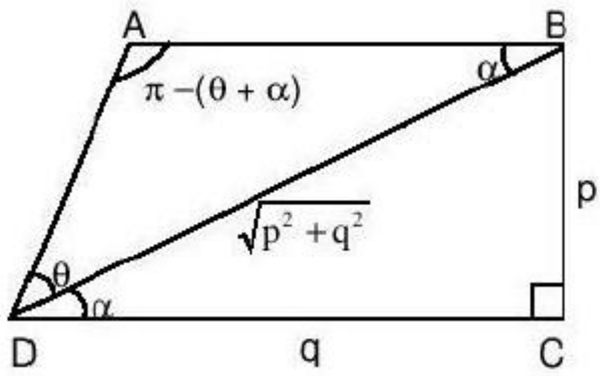
32. Sol. **(4)**

Using sine rule in triangle ABD

$$\frac{AB}{\sin\theta} = \frac{BD}{\sin(\theta+\alpha)}$$

$$\Rightarrow AB = \frac{\sqrt{p^2+q^2}\sin\theta}{\sin\theta\cos\alpha+\cos\theta\sin\alpha} = \frac{\sqrt{p^2+q^2}\sin\theta}{\frac{\sin\theta\cdot q}{\sqrt{p^2+q^2}}+\frac{\cos\theta\cdot p}{\sqrt{p^2+q^2}}}.$$

$$\Rightarrow AB = \frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)}$$



33. Sol. **(1)**

Let the tangent to the parabola be $y = mx + \frac{\sqrt{5}}{m}(m \neq 0)$.

Now, its distance from the center of the circle must be equal to the radius of the circle.

So, $\left|\frac{\sqrt{5}}{m}\right| = \frac{\sqrt{5}}{\sqrt{2}}\sqrt{1+m^2} \Rightarrow (1+m^2)m^2 = 2 \Rightarrow m^4 + m^2 - 2 = 0$.

$\Rightarrow (m^2-1)(m^2+2) = 0 \Rightarrow m = \pm 1$

So, the common tangents are $y = x + \sqrt{5}$ and $y = -x - \sqrt{5}$.

34. Sol. **(1)**

Slope of the incident ray is $-\frac{1}{\sqrt{3}}$.

So, the slope of the reflected ray must be $\frac{1}{\sqrt{3}}$.

The point of incidence is $(\sqrt{3}, 0)$. So, the equation of reflected ray is $y = \frac{1}{\sqrt{3}}(x - \sqrt{3})$.

35. Sot. **(3)**

Variance is not changed by the change of origin.

Alternate Solution:

$$\sigma = \sqrt{\frac{\sum |x-\bar{x}|^2}{n}} \text{ for } y = x + 10 \Rightarrow \bar{y} = \bar{x} + 10$$

$$\sigma_1 = \sqrt{\frac{\sum |y+10-\bar{y}-10|^2}{n}} = \sqrt{\frac{\sum |y-\bar{y}|^2}{n}} = \sigma.$$

36. Sol. **(4)**

If x, y, z are in $A.P.$

$2y = x + z$

and $\tan^{-1} x, \tan^{-1} y, \tan^{-1} z$ are in $A.P.$

$2\tan^{-1} y = \tan^{-1} x + \tan^{-1} z \Rightarrow x = y = z$.

Note: If $y = 0$, then none of the options is appropriate.

37. Sol. **(2)**

$\int f(x)dx = \psi(x)$

Let $x^3 = t$

$3x^2 dx = dt$

then $\int x^5 f(x^3)\, dx = \frac{1}{3}\int tf(t)dt$

$= \frac{1}{3}\left[t\int f(t)\,dt - \int\left\{1\cdot\int f(t)\,dt\right\}dt\right] = \frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)\,dx + C$.

38. Sol. **(4)**

$foci \equiv (\pm ae, 0)$

We have $a^2e^2 = a^2 - b^2 = 7$

Equation of circle $(x-0)^2 + (y-3)^2 = (\sqrt{7}-0)^2 + (0-3)^2$.

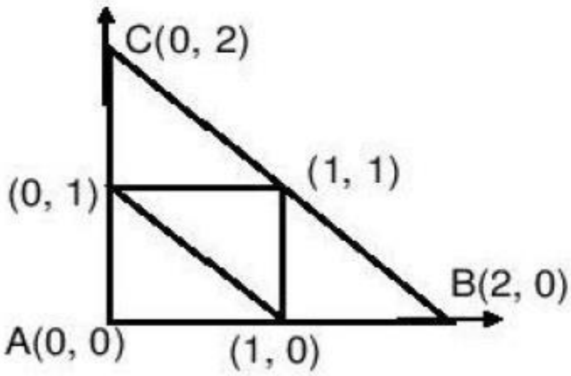
$\Rightarrow x^2 + y^2 - 6y - 7 = 0$.

39. Sol. **(1)**

$$x\text{-coordinate} = \frac{ax_1 + bx_2 + cx_3}{a+b+c}$$

$$= \frac{2\times 2 + 2\sqrt{2}\times 0 + 2\times 0}{2+2+2\sqrt{2}}.$$

$$= \frac{4}{4+2\sqrt{2}} = \frac{2}{2+\sqrt{2}} = 2-\sqrt{2}$$



Alternate Solution:

$$x\text{-coordinate} = r = (s-a)\tan A/2$$

$$= \left(\frac{4+2\sqrt{2}}{2} - 2\sqrt{2}\right)\tan\frac{\pi}{4} = 2-\sqrt{2}.$$

40. Sol. **(4)**

$$\frac{dy}{dx} = |x| = 2 \Rightarrow x = \pm 2 \Rightarrow y = \int_0^0 |t|\,dt = 2 \text{ for } x = 2$$

$$\text{and } y = 2\int_0^{-2} |t|\,dt = -2 \text{ for } x = -2$$

$$\therefore \text{tangents are } y-2 = 2(x-2) \Rightarrow y = 2x-2 \quad .$$

$$\text{and } y+2 = 2(x+2) \Rightarrow y = 2x+2$$

Putting $y = 0$, we get $x = 1$ and -1

41. Sol. **(2)**

$t_r = 0.777\ldots r\,times$

$= 7\left(10^{-1} + 10^{-2} + 10^{-3} + \ldots + 10^{-r}\right)$

$= \frac{7}{9}\left(1 - 10^{-r}\right)$

$S_{20} = \sum_{r=1}^{20} t_r = \frac{7}{9}\left(20 - \sum_{r=1}^{20} 10^{-r}\right) = \frac{7}{9}\left(20 - \frac{1}{9}\left(1 - 10^{-20}\right)\right) = \frac{7}{81}\left(179 + 10^{-20}\right)$

42. Sol. **(1)**

$S1:$

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \wedge q$	$(p \wedge \sim q) \wedge (\sim p \wedge q)$
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	F	F
						Fallacy

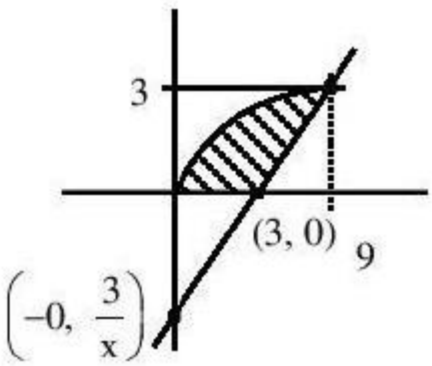
$S2:$

p	q	$\sim p$	$\sim q$	$p \Rightarrow q$	$\sim q \Rightarrow \sim p$	$(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T
						Tautology

S_2 is not an explanation of S_1

43. Sol. **(4)**

$$2\sqrt{x} = x - 3$$
$$4x = x^2 - 6x + 9$$
$$x^2 - 10x + 9$$
$$x = 9, x = 1$$
$$\int_0^3 \left(2y+3\right) - y^2 \Big] dy$$
$$\left[y^2 + 3y - \frac{y^3}{3}\right]_0^3 = 9 + 9 - 9 = 9$$



44. Sol. **(1)**

$$\frac{1}{\cot A\left(1-\cot A\right)} - \frac{\cot^2 A}{\left(1-\cot A\right)} = \frac{1-\cot^3 A}{\cot A\left(1-\cot A\right)} = \frac{\operatorname{cosec}^2 A + \cot A}{\cot A} = 1 + \sec A \operatorname{cosec} A$$

45. Sol. **(3)**

If $2x^3 + 3x + k = 0$ has 2 distinct real roots in $[0,1]$, then $f'(x)$ will change sign but $f'(x) = 6x^2 + 3 > 0$

So no value of k exists.

46. Sol. (3)

$$\lim_{x\to 0}\frac{(1-\cos 2x)}{x(\tan 4x)}(3+\cos x)$$

$$\lim_{x\to 0}2\left(\frac{\sin x}{x}\right)^2\cdot\frac{1}{4}\left(\frac{4x}{\tan 4x}\right)(3+\cos x)=2\times 1\times\frac{1}{4}\times 1\times(3+1)=2.$$

47. Sol. (1)

$${}^{n+1}C_3-{}^{n}C_3=10\Rightarrow {}^{n}C_2=10\Rightarrow n=5.$$

48. Sol. (2)

$$\int_{2000}^{P}dP=\int_{0}^{25}\left(100-12\sqrt{x}\right)dx$$

$$(P-2000)=25\times 100-\frac{12\times 2}{3}(25)^{3/2}$$

$$P=3500.$$

49. Sol. (3)

$$I=\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$$

$$I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}dx$$

$$2I=\frac{\pi}{6}$$

$$I=\frac{\pi}{12}.$$

50. Sol. (1)

$$P=\begin{bmatrix}1 & \alpha & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$$

$$|Adj\ A|=|A|^2$$
$$|Adj\ A|=16$$
$$1(12-12)-\alpha(4-6)+3(4-6)=16.$$
$$2\alpha-6=16.$$
$$2\alpha=22.$$
$$\alpha=11.$$

51. Sol. **(1)**

For no solution

$$\frac{k+1}{k}=\frac{8}{k+3}\neq\frac{4k}{3k-1}\quad\ldots(1)$$
$$\Rightarrow(k+1)(k+3)-8k=0$$
$$\text{or } k^2-4k+3=0\Rightarrow k=1,3$$

But for $k=1$, equation (1) is not satisfied

Hence $k=3$.

52. Sol. **(4)**

$$y=\sec\left(\tan^{-1}x\right)$$
$$\frac{dy}{dx}=\sec\left(\tan^{-1}x\right)\tan\left(\tan^{-1}x\right)\cdot\frac{1}{1+x^2}$$
$$\frac{dy}{dx}\Big|_{x=1}=\sqrt{2}\times1\times\frac{1}{2}=\frac{1}{\sqrt{2}}.$$

53. Sol. **(2)**

$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$$1(1+2k)+1(1+k^2)-1(2-k)=0$$

$$k^2+1+2k+1-2+k=0$$

$$k^2+3k=0$$

$$(k)(k+3)=0$$

2 values of k.

54. Sol. **(2)**

$A \times B$ will have 8 elements.

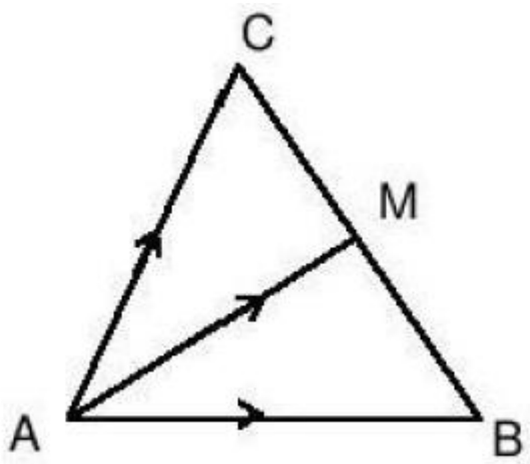
$$2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2 = 256-1-8-28=219.$$

55. Sol. **(2)**

$$\overrightarrow{AM} = \frac{\overrightarrow{AB}+\overrightarrow{AC}}{2}$$

$$\overrightarrow{AM} = 4\hat{i} - \hat{j} + 4\hat{k}$$

$$\left|\overrightarrow{AM}\right| = \sqrt{16+16+1} = \sqrt{33}$$



56. Sol. **(2)**

$$P(\text{correct answer}) = 1/3$$

$${}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1 + {}^5C_5\left(\frac{1}{3}\right)^5$$

$$\frac{5\times 2}{(3)^5} + \frac{1}{(3)^5} = \frac{11}{3^5}.$$

57. Sol. **(2)**

$$|z| = 1 \Rightarrow z\bar{z} = 1$$

$$\frac{1+z}{1+\bar{z}} = \frac{1+z}{1+\frac{1}{z}} = z.$$

58. Sol. **(4)**

For equation

$$x^2 + 2x + 3 = 0$$

both roots are imaginary.

Since $a, b, c \in R.$

If one root is common, then both roots are common

Hence, $\frac{a}{1} = \frac{b}{2} = \frac{c}{3}$

$$a : b : c = 1 : 2 : 3.$$

59. Sol. **(2)**

$$4x+2y+4z=16$$
$$4x+2y+4z=-5$$
$$d_{\min}=\frac{21}{\sqrt{36}}\frac{21}{6}=\frac{7}{2}.$$

60. Sol. **(2)**

$$\left(\frac{\left(x^{1/3}+1\right)\left(x^{2/3}-x^{1/3}+1\right)}{x^{2/3}-x^{1/3}+1}-\frac{1}{\sqrt{x}}\cdot\frac{\left(\sqrt{x}+1\right)\left(\sqrt{x-1}\right)}{\left(\sqrt{x}-1\right)}\right)^{10}=\left(x^{1/3}-x^{-1/2}\right)^{10}$$

$$T_{r+1}=\left(-1\right)^{r}\,{}^{10}C_{r}x^{\frac{20-5r}{6}}\Rightarrow r=4$$
$${}^{10}C_{4}=210.$$