JEE (MAIN) - 2013

MATHEMATICS

Read the instructions carefully:

Important Instructions:

1	Immediately fill in the particulars on this page of the Test Booklet with *Blue/Black Ball Point Pen. Use of pencil is strictly prohibited*.
2.	The test is of **3 hours duration.**
3.	This paper consists of **30** questions. The maximum marks are **120**.
4.	There are 30 questions in this paper which are equal weightage. Each question is allotted **4 (four)** marks for correct response.
5.	*Candidates will be awarded marks as stated above in instruction No. 4 for correct response of each question. (1/4) (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
6.	There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 5 above.

31. The circle passing through $(1,-2)$ and touching the axis of x at $(3,0)$ also passes through the point

(1) $(2,-5)$

(2) $(5,-2)$

(3) $(-2,5)$

(4) $(-5,2)$

32. $ABCD$ is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta, BC = p$ and $CD = q$, then AB is equal to

(1) $\dfrac{p^2 + q^2\cos\theta}{p\cos\theta + q\sin\theta}$

(2) $\dfrac{p^2 + q^2}{p^2\cos\theta + q^2\sin\theta}$

(3) $\dfrac{(p^2 + q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$

(4) $\dfrac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$

33. Given: A circle, $2x^2 + 2y^2 = 5$ and a parabola. $y^2 = 4\sqrt{5}x$.

Statement — I: An equation of a common tangent to these curves is.

Statement — II: If the line, $y = mx + \dfrac{\sqrt{5}}{m}(m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$.

(1) Statement -1 is True; Statement -II is true; Statement-II is **not** a correct explanation for Statement-I

(2) Statement -I is True; Statement -II is False.

(3) Statement -I is False; Statement -II is True

(4) Statement -I is True; Statement -II is True; Statement-II is a **correct** explanation for Statement-I

34. A ray of light along $x+\sqrt{3}y=\sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected rays is

(1) $\sqrt{3}y=x-\sqrt{3}$

(2) $y=\sqrt{3}x-\sqrt{3}$

(3) $\sqrt{3}y=x-1$

(4) $y=x+\sqrt{3}$

35. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ?

(1) median

(2) mode

(3) variance

(4) mean

36. If x, y, z are in $A.P.$ and $\tan^{-1}x, \tan^{-1}y$ and $\tan^{-1}z$ are also in $A.P.$., then

(1) $2x=3y=6z$

(2) $6x=3y=2z$

(3) $6x=4y=3z$

(4) $x = y = z$

37. If $\int f(x)dx = \Psi(x)$, then $\int x^5 f(x^3)dx$ is equal to

(1) $\frac{1}{3}x^3\Psi(x^3) - 3\int x^3\Psi(x^3)dx + C$

(2) $\frac{1}{3}x^3\Psi(x^3) - \int x^2\Psi(x^3)dx + C$

(3) $\frac{1}{3}\left[x^3\Psi(x^3) - \int x^3\Psi(x^3)dx\right] + C$

(4) $\frac{1}{3}\left[x^3\Psi(x^3) - \int x^2\Psi(x^3)dx\right] + C$

38. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having centre at $(0,3)$ is

(1) $x^2 + y^2 - 6y + 7 = 0$

(2) $x^2 + y^2 - 6y - 5 = 0$

(3) $x^2 + y^2 - 6y + 5 = 0$

(4) $x^2 + y^2 - 6y - 7 = 0$

39. The x- coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as $(0,1)(1,1)$ and $(1,0)$ is

(1) $2 - \sqrt{2}$

(2) $1 + \sqrt{2}$

(3) $1 - \sqrt{2}$

(4) $2+\sqrt{2}$

40. The intercepts on x-axis made by tangents to the curve, $y=\int_0^x |t|\,dt, x\in R$, which are parallel to the line $y=2x$, are equal to

(1) ± 2

(2) ± 3

(3) ± 4

(4) ± 1

41. The sum of first 20 terms of the sequence $0.7, 0.77, 0.777, \ldots,$ is

(1) $\frac{7}{9}\left(99-10^{-20}\right)$

(2) $\frac{7}{81}\left(179+10^{-20}\right)$

(3) $\frac{7}{9}\left(99+10^{-20}\right)$

(4) $\frac{7}{81}\left(179-10^{-20}\right)$

42. Consider:

Statement — I: $(p\wedge\sim q)\wedge(\sim p\wedge q)$ is a fallacy. **Statement — II:** $(p\to q)\leftrightarrow(\sim q\to\sim p)$ is a tautology.

(1) Statement -1 is True; Statement -II is true; Statement-II is **not** a correct explanation for Statement-I

(2) Statement -I is True; Statement -II is False.

(3) Statement -I is False; Statement -II is True

(4) Statement -I is True; Statement -II is True; Statement-II is a **correct** explanation for Statement-I

43. The area (in square units) bounded by the curves $y=\sqrt{x}, 2y-x+3=0, x$-axis , and lying in the first quadrant is

(1) 36

(2) 18

(3) $\frac{27}{4}$

(4) 9

44. The expression $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$ can be written as

(1) $\sec A\operatorname{cosec} A+1$

(2) $\tan A+\cot A$

(3) $\sec A+\operatorname{cosec} A$

(4) $\sin A\cos A+1$

45. The real number k for which the equation, $2x^3+3x+k=0$ has two distinct real roots in $[0,1]$

(1) lies between 2 and 3

(2) lies between -1 and 0

(3) does not exist

(4) lies between 1 and 2

46. $\lim\limits_{x\to 0}\dfrac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to

(1) $\dfrac{1}{2}$

(2) 1

(3) 2

(4) $-\dfrac{1}{4}$

47. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1}-T_n=10$, then the value of n is

(1) 5

(2) 10

(3) 8

(4) 7

48. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\dfrac{dP}{dx}=100-12\sqrt{x}$. If the firm employs 25 more workers, then dx the new level of production of items is

(1) 3000

(2) 3500

(3) 4500

(4) 2500

49. **Statement - I:** The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ equal to $\frac{\pi}{6}$.

Statement-II: $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$.

(1) Statement -1 is True; Statement -II is true; Statement-II is **not** a correct explanation for Statement-I

(2) Statement -I is True; Statement -II is False.

(3) Statement -I is False; Statement -II is True

(4) Statement -I is True; Statement -II is True; Statement-II is a **correct** explanation for Statement-I

50. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to

(1) 11

(2) 5

(3) 0

(4) 4

51. The number of values of k, for which the system of equations

$(k+1)x + 8y = 4k$

$kx + (k+3)y = 3k - 1$

has no solution, is

(1) 1

(2) 2

(3) 3

(4) infinite

52. If $y = \sec\left(\tan^{-1} x\right)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to

(1) $\frac{1}{2}$

(2) 1

(3) $\sqrt{2}$

(4) $\frac{1}{\sqrt{2}}$

53. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then k can have

(1) exactly one value

(2) exactly two values

(3) exactly three values

(4) any value

54. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is

(1) 220

(2) 219

(3) 211

(4) 256

55. If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is

(1) $\sqrt{72}$

(2) $\sqrt{33}$

(3) $\sqrt{45}$

(4) $\sqrt{18}$

56. A multiple-choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is

(1) $\frac{13}{3^5}$

(2) $\frac{11}{3^5}$

(3) $\frac{10}{3^5}$

(4) $\frac{17}{3^5}$

57. If z is a complex number of unit modulus and argument θ, then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals

(1) $\frac{\pi}{2} - \theta$

(2) θ

(3) $\pi - \theta$

(4) $-\theta$

58. If the equations $x^2+2x+3=0$ and $ax^2+bx+c=0, a,b,c\in R$, have a common root, then $a:b:c$ is

(1) $3:2:1$

(2) $1:3:2$

(3) $3:1:2$

(4) $1:2:3$

59. Distance between two parallel planes $2x+y+2z=8$ and $4x+2y+4z+5=0$ is

(1) $\frac{5}{2}$

(2) $\frac{7}{2}$

(3) $\frac{9}{2}$

(4) $\frac{3}{2}$

60. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is