JEE MAIN - 2014

MATHEMATICS

ANSWER KEY AND EXPLANATIONS

1. Sol.

$X = -1 \quad x = 2$

Are maxima & minima

$\Rightarrow \alpha \ \log|x| + \beta x^2 + x = f(x)$

Taking $x > 0$

$F(x) = \alpha \ \log x + \beta x^2 + x$

$F'(x) = \frac{\alpha}{\beta} + 2\beta x + 1 = 0$

$2\beta x 2 + x + \alpha = 0$

Now $\quad x = -1 \& 2$

Must satisfy this as these are critical points

$X = -1$

$2\beta - 1 + \alpha = 0$

$X = 2$

$2\beta + 2 + \alpha = 0$

Solving $\quad \beta = -1/2$

$\alpha = 2$

2. Sol.

Foot of perpendicular is given by :

$$\frac{h-x}{x} = \frac{k-y}{b} = -\frac{[ax+by+c]}{a^2+b^2}$$

$X, y = 0, 0$ eqn tangent:

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$$

Putting values:

$$\frac{ah}{\cos\theta}=\frac{bk}{\sin\theta}=\frac{1}{\frac{\cos\theta}{a^2}+\frac{\sin^2\theta}{b^2}}$$

$$\Rightarrow h=\frac{ab^2\cos\theta}{b^2\cos^2\theta+a^2\sin^2\theta}$$

$$k=\frac{a^2b\sin\theta}{b^2\cos^2\theta+a^2\sin^2\theta}$$

Now it is difficult to eliminate θ so we check option.

Answer $=\left(x^2+y^2\right)^2=6x^2+2y^2$

3. Sol.

$$f_4=\frac{\sin^4 x+\cos^4 x}{4}$$

$$=1-\frac{2\sin^2 x\cos^2 x}{6}$$

$$f_6=\frac{\sin^6 \quad x+\cos^6 x}{6}$$

$$=1.\left(\frac{\sin^4 x-\sin^2 x\cos^2 x+\cos^4 x}{6}\right)$$

By formula $a^3+b^3=(a+b)(a^2-ab+b^2$

$$=\left[\frac{1-3\sin^2 x\cos^2 x}{6}\right]$$

$$f_4-f_6=\frac{1-2\sin^2 x\cos^2 x}{4}-\frac{(1-3\sin^2 x\cos^2 x}{6}$$

$$6-\frac{12\sin^2\cos^2 x-4+12\sin^2\cos^2 x}{24}$$

$$=\frac{2}{24}=\frac{1}{12}\text{ Ans.}$$

4. Sol.

$X = 4n - 3n - 1$

Rewriting:

$X = (3+1)^n - 3n - 1$

Expanding $(1+3)^n$

$X = \left(1 + 3n + \frac{3.n.(n-1)}{1.2} \ldots \ldots 3^{n.n} c_n\right) - 3^{n-1}$

$= \frac{3.n.(n-1)}{1.2} \ldots \ldots 3^{n.n} c_n$

$\Rightarrow$ All the multiple of 9 are in x which is represented by y as well but y will exceed x at some point.

So $X \, UY$ has to be Y & not X .

5. Sol.

$BB^1 = (A^{-1}A^1)(A^{-1}A^1)^1$

$(A^{-1}A^1)\{(A^1)1(A^{-1})1\}$

$= A^{-1}(A^1A)(A^1 - 1)$

$= A^{-1}(AA^1)(A^1) - 1$

$= A^{-1}AA^{-1}A^1$

$I.I = I$ Ans.

6. Sol.

$$\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}.dx$$

$$=\int\left(e^{x+\frac{b}{x}}.dx+\int\left(x-\frac{1}{x}\right)e^{x+\frac{1}{x.dx}}\right.$$

By pasts

$$=xe^{x+\frac{1}{x-}}\int e^{x+\frac{1}{x}}\left(1-\frac{1}{x^2}\right)x.dx+\int\left(1-\frac{1}{x}\right)e^{x+\frac{1}{x}}.dx$$

$$=xe^{x+\frac{1}{x}}\text{ Ans.}$$

7. Sol.

Req. $\text{Area}=\text{Area }ACB+\text{Area }BCD$

$$=\int_0^1 1-y^2.dy+\frac{\pi r^2}{2}$$

$$=2\left(y-\frac{y^3}{3}\right)+\frac{\pi}{2}$$

$$=\frac{\pi}{2}+\frac{4}{2}\text{ Ans.}$$

8. Sol.

Plane and line are parallel.

Eqn of normal to plane

$$\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-4}{1}=K.$$

Point $\rightarrow 2K+1,3-K,\ 4+K$

$\Rightarrow \frac{2K+2}{2}, \frac{6-K}{2}, \frac{8+K}{2}$

Lies on plane

$2(K+1) - \frac{(6-k)}{2} + \frac{8+K}{2} + 3 = 0$
$K = -2$

Point through which image passes $(-3, 5, 2)$

Hence, $\frac{x+3}{3} = \frac{Y-5}{1} = \frac{z-2}{-5}$

9. Sol.

Even Natural No.

$= 2, 4, 6, 8 \ldots\ldots\ldots 100$

Variance $= \sum \frac{(x-\bar{x})^2}{n} \backslash$

$\bar{x} = \text{mean} = 5$

$n = 50$

$x = 2, 4, 6, \ldots\ldots\ldots 100$

$= \frac{(2-51)^2 + (4-51)^2 \ldots\ldots\ldots\ldots\ldots (100-51)^2}{50}$
$= 833$ Ans.

10. Sol.

$|z| \succeq 2$ represents a circle with

Radius $\succeq 2$

$\left|z + \frac{1}{2}\right|$ represent distance

From point $\text{From point})-1/2,0)$

[image]

$\left|2 - \frac{1}{2}\right| = 3/2$

11. Sol.

Let GP be:

a, ar, ar^2

also $a \ 2ar \ ar^2$ (are in ap)

$\Rightarrow 4ar = a + ar^2$

$4r = 1 + r^2$

$r = \dfrac{4 \pm \sqrt{-12}}{2}$

$= 2 \pm \sqrt{3}$

$r = 2 - \sqrt{3}$ doesn't satisfy $A.P$ condition

$\Rightarrow r = 2 - \sqrt{3}$ Answer

12. Sol.

$\left(1+ax+bx^2\right)\left(1-2x^{18}\right)$

x^3 terms:

$=18_{C_3}+(-2x)^3+18_{C2}(-2x)^3.ax+18_{C_1}(-2x).bx^2=0$

$-18_{C_3}.8+18_{C_2}.4a-18_{C_1}.2b=0\ldots(1)$

x^4 terms:

$=18_{C_4}+(-2x)^4+18_{C_3}(-2x)^3.ax+18_{C_2}(-2x)^2.bx^2=0$

$-16.18_{C_4}-8a.18_{C_3}+18_{C_2}.4b=0\ldots(2)$

Solving (1) & (2) for a & b:

$a=16 \quad b=\dfrac{272}{3}$

13. Sol.

Putting the condition

Solving: $4ax-2ax+c=0$

$2ax+c=0$ ________ (1)

$5bx-2bx+d=0$

$3bx+d=0$ ________ (2)

Putting value of x

$2a.\left(\dfrac{-d}{3b}\right)+c=0$

$3bc-2ab=0$

14. Sol.

$$\left[\vec{a}\times\vec{b}\vec{b}\times\vec{c}\vec{c}\times\vec{a}\right]=\left(\vec{a}\times\vec{b}\right).\left[\left(\vec{b}\times\vec{c}\right)\left(\vec{c}\times\vec{a}\right)\right]^{2}$$

$$=\left[\left(\vec{a}\times\vec{b}\right).\left(\vec{p}\left(\vec{c}\times\vec{a}\right)\right]$$

$$Let\ \vec{p}=\left(\vec{b}\times\vec{c}\right)$$

$$=\left(\vec{a}\times\vec{b}\right).\left(\left(p.a\right)\vec{c}-\left(p.a\right)\vec{c}\right.$$

$$=\left(\vec{a}\times\vec{b}\right)\left(\left(\left(\vec{b}\times\vec{c}\right).\vec{a}\vec{c}-\left(\left(\left(\vec{b}\times\vec{c}\right).\vec{c}\right)\vec{a}\right)\right.\right.$$

$$=\left(a\times v\right)\left[\left(bca\right)\vec{c}\right)-0$$

$$\because\left(\vec{b}\times\vec{c}\right).\vec{c}=0$$

$$\Rightarrow\left(\vec{o}\times\vec{b}\right).\vec{c}\left[\vec{b}\vec{c}\vec{a}\right]$$

$$=\left[\vec{a}\vec{b}\vec{c}\right]=1$$

15. **Sol.**

$$P\left(\overline{A\cup B}\right)=\frac{1}{6}$$

$$P\left(A\cap B\right)=\frac{5}{6}$$

$$P\left(\bar{A}\right)=\frac{1}{4}$$

$$P\left(A\right)=\frac{3}{4}$$

$$P\left(B\right)=P\left(A\cup B\right)-P\left(A\right)+P\left(A\cap B\right)$$

$$=\frac{5}{6}-\frac{3}{4}+\frac{1}{4}$$

$$P\left(B\right)=\frac{5}{6}-\frac{1}{2}=\frac{1}{3}$$

Also $P(A).\left(PB\right)=\left(A\cap B\right)$

$=$ Independent events

$P\left(A\right)\neq P\left(B\right)$ Unlikely

16. Sol.

Coordinate of $S=\left(\frac{13}{2},1\right)$ by mid point formula

Slope $PS=\frac{2-1}{2-\frac{13}{2}}=\frac{-2}{9}$

$y=\frac{-2}{9}x+C$

Putting $(1,-1)$

$-1=\frac{-2}{9}C$

$C=\frac{-7}{9}$

$2x+9y-7=0$

17. Sol.

Applying Hospitals Rules

$=\lim_{x\to 0}\frac{\cos\left(\pi\cos^2 x\right)\pi 2\cos x(-\sin x)}{2x}$

$=\lim_{x\to 0}\frac{\cos\left(\pi\cos^2 x\right)}{2}\pi 2\cos x\frac{(-\sin x)}{x}$

$=\pi\lim_{x\to 0}\frac{\sin x}{x}=1$

18. Sol.

$$A-\beta=\sqrt{(\alpha+\beta^2-4\alpha\beta}$$

$$=\sqrt{\frac{q^2}{p}-4.\frac{r}{p}}$$

$$=\frac{\alpha+\beta}{\alpha\beta}4$$

$$=-\frac{\frac{q}{p}}{\frac{r}{p}}=\frac{-q}{r}=4\ldots(1)$$

$$2q=r+p$$

$$2=\frac{r}{q}+\frac{p}{q}$$

$$2=\frac{-1}{4}+\frac{p}{q}$$

$$=\frac{p}{q}-\frac{9}{4}\ldots(2)$$

Form $(1)\&(2)$

$$=\frac{r}{p}=\frac{-1}{9}\ldots(3)$$

putting the values from $(2)\&(3)$

$$(\alpha-\beta)=\sqrt{\frac{4^2}{9}\quad+4.\frac{1}{9}}$$

$$=\sqrt{\frac{52}{81}}=\frac{2\sqrt{13}}{9}\text{ Ans.}$$

19. Sol.

In ΔAOB

$$= \frac{AB}{OB} = \frac{20}{OB} = \tan 45 = 1$$
$$OB = 20$$

Similarly in ΔA^1OB^1

$$OB^1 = 20\sqrt{3}$$

Distance moved :

$$OB^1 - OB = 20\left(\sqrt{3}-1\right)$$

$$\text{Velocity} = \frac{20\left(\sqrt{3}-1\right)}{2}$$
$$= 20\left(\sqrt{3}-1\right)$$

20. Sol.

$$-3\left(x-[x]\right)^2 + 2\left(x-[x]\right) + a^2$$
$$\left(x-[x]\right) = \{x\}$$
$$R = [0,1]$$
$$\text{for } \{x\} = 0$$
$$a^2 = 0$$
$$\text{for } \{x\} = 1$$
$$-1 + a^2 = 0$$

for the eqn. not to hold

$$a * (0,1) u (-1,0)$$
$$(-1,0) u \ (0,1)$$

21. Sol.

$$\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}dx}$$
$$=\int_0^{\pi}\sqrt{\left(1-2\sin\frac{x}{2}\right)^2}.dx$$
$$=\int_0^{\pi}\left(\left(1-2\sin\frac{x}{2}\right).dx\right.$$
$$x+2\cos\frac{x}{2}.2$$
$$=(\pi-4)$$

22. Sol.

$$\begin{array}{lll} x= & 0 & 1 \\ f(x) & 2 & 6 \\ g(x) & 2 & 6 \end{array}$$

By Rolles theorem:

$$f^1(x)=\frac{6-2}{1}=4$$
$$g^1(x)=\frac{2-0}{1}=2$$
$$f^1(x)=2g^1(x)$$

23. Sol.

$$f^1(x)=\frac{1}{1+x^5}$$
$$f(x)=\int_0^x\frac{1}{1+x^5}.dx$$
$\rightarrow$ Inverse fun:
$$X=\int_0^{g(x)}\frac{1}{1+xg^{5(x)}}.d\left(d_{(x)}\right)$$
Differentiating:
$$1=\frac{1}{1+xg^{5(x)}}.g(x)$$
$$G(x)=1+g^5x$$

24. Sol.

$$(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9 = k(10)9$$

$$1 = 2\left(\frac{11}{10}\right) + 3\left(\frac{11}{10}\right)^2 \ldots 10.\frac{11^9}{10^9} = K$$

Subtracting:

$$1 + \frac{11}{10} + \left(\frac{11}{10}\right)^2 \ldots \left(\frac{11}{10}\right)^9 - 10.\left(\frac{11}{10}\right)^{10} = \frac{-K}{10}$$

$$= \frac{\left(\frac{11}{10}\right)^{10-1}}{\frac{1}{10}} 10.\left(\frac{11}{10}\right)^{10} = \frac{K}{10}$$

$$10.\left(\frac{11}{10}\right)^{10} - 10\ 10.\left(\frac{11}{10}\right)^{10} = \frac{-K}{10}$$

$K = 100$ Ans.

25. Sol.

$$\begin{vmatrix} 3 & 1+\alpha+\beta) & 1+\alpha(2)+\beta(2) \\ 1+\alpha+\beta) & 1+\alpha(2)+\beta(2) & 1+\alpha(3)+\beta(3) \\ 1+\alpha(2)+\beta(2) & 1+\alpha(3)+\beta(3) & 1+\alpha(4)+\beta(3) \end{vmatrix}$$

$$\begin{matrix} 3 & 2 & 6 \\ 2 & 6 & 8 \\ 6 & 8 & 18 \end{matrix}$$

Solving we get $K = 1$

26. Sol.

$y^2 = 4x$

Tangent $y = mx + \frac{1}{m}$

Touches $x^2 = -32y$

$= x^2 = -32y\left(mx + \frac{1}{m}\right)$

$= x^2 + 32mx + \frac{32}{m} = 0$

$D = 0 \rightarrow (32mx)^2 - 4.\frac{32}{m} = 0$

$= m^3 = \frac{1}{8}$

$m = \frac{1}{2}$ Ans.

27. Sol.

P	Q	$\sim P$	$\sim q$	$P \leftrightarrow q$	$P \leftrightarrow \sim q$	$\sim P \leftrightarrow q$	$\sim(P \leftrightarrow \sim q)$
T	F	F	T	F	T	T	F
F	T	T	F	F	T	T	F
T	T	F	F	T	F	F	T
F	F	T	T	T	F	F	T

28. Sol.

$$\frac{dp(t)}{dt}=\frac{1}{2}p(t)-200$$

$$=\int_{100}^{p}\frac{dp(t)}{\frac{1}{2}p(t)-200}=\int_{0}^{t}dt$$

$$2\left[\log\left(\frac{1}{2}p(t)-200\right)-\log(-150)\right]$$

$$\text{Log}\frac{\frac{1}{2\,p(t)-200}}{-150}=\frac{t}{2}$$

$$=\frac{1}{2}p(t)-200=-150e^{\frac{t}{2}}$$

$$P(t)=400-300e^{\frac{t}{2}} \quad \text{Ans.}$$

29. Sol.

(Image)

$$=r_1+r_2=\sqrt{(1-0)^2}\,(1-y)^2$$

$$1+\sqrt{(y-0)^2+(0-0)^2}=\sqrt{1+(1-y)^2}$$

$$=(1+y)^2=2+y^2-2y$$

$$1+y^2+2y=2+y^2-2p$$

$$4y=1$$

$$Y=\frac{1}{4} \quad \text{Ans.}$$

30. Sol.

$$(l+\text{n})=-m$$

$$l^2=(l+\text{n})^2+n^2$$

$$l^2=l^2+n^2 2ln+n^2$$

$$2n^2+2ln=0$$

$$2n(n+l)=0$$

$$N=0$$

$$N=-l$$

$$L=-m$$

$$M=o$$

$$dr\text{'}sl,-l,o$$

$$l,0,-l.$$

$$\cos\theta=\frac{1}{\sqrt{2\sqrt{2}}}=\frac{1}{2}$$

$$\theta=\frac{\pi}{3}\text{ Ans.}$$