

**JEE MAIN - 2014**

**MATHEMATICS**

**ANSWER KEY AND EXPLANATIONS**

**1. Sol.**

$$X = -1 \quad x = 2$$

Are maxima & minima

$$\Rightarrow \alpha \log|x| + \beta x^2 + x = f(x)$$

Taking  $x > 0$

$$F(x) = \alpha \log x + \beta x^2 + x$$

$$F'(x) = \frac{\alpha}{x} + 2\beta x + 1 = 0$$

$$2\beta x^2 + x + \alpha = 0$$

$$\text{Now } x = -1 \& 2$$

Must satisfy this as these are critical points

$$X = -1$$

$$2\beta - 1 + \alpha = 0$$

$$X = 2$$

$$2\beta + 2 + \alpha = 0$$

$$\text{Solving } \beta = -1/2$$

$$\alpha = 2$$

**2. Sol.**

Foot of perpendicular is given by :

$$\frac{h-x}{x} = \frac{k-y}{b} = -\frac{[ax+by+c]}{a^2+b^2}$$

$X, y = 0, 0$  eqn tangent:

$$\frac{x \cos \theta}{a} + \frac{y \sin \theta}{b} = 1$$

Putting values:

$$\frac{ah}{\cos \theta} = \frac{bk}{\sin \theta} = \frac{1}{\frac{\cos \theta}{a^2} + \frac{\sin^2 \theta}{b^2}}$$

$$\Rightarrow h = \frac{ab^2 \cos \theta}{b^2 \cos^2 \theta + a^2 \sin^2 \theta}$$

$$k = \frac{a^2 b \sin \theta}{b^2 \cos^2 \theta + a^2 \sin^2 \theta}$$

Now it is difficult to eliminate  $\theta$  so we check option.

$$\text{Answer} = (x^2 + y^2)^2 = 6x^2 + 2y^2$$

**3. Sol.**

$$f_4 = \frac{\sin^4 x + \cos^4 x}{4}$$

$$= 1 - \frac{2 \sin^2 x \cos^2 x}{6}$$

$$f_6 = \frac{\sin^6 x + \cos^6 x}{6}$$

$$= 1 \cdot \left( \frac{\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x}{6} \right)$$

By formula  $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$$= \left[ \frac{1 - 3 \sin^2 x \cos^2 x}{6} \right]$$

$$f_4 - f_6 = \frac{1 - 2 \sin^2 x \cos^2 x}{4} - \frac{1 - 3 \sin^2 x \cos^2 x}{6}$$

$$= 6 - \frac{12 \sin^2 \cos^2 x - 4 + 12 \sin^2 \cos^2 x}{24}$$

$$= \frac{2}{24} = \frac{1}{12} \text{ Ans.}$$

**4. Sol.**

$$X = 4n - 3n - 1$$

Rewriting:

$$X = (3+1)^n - 3n - 1$$

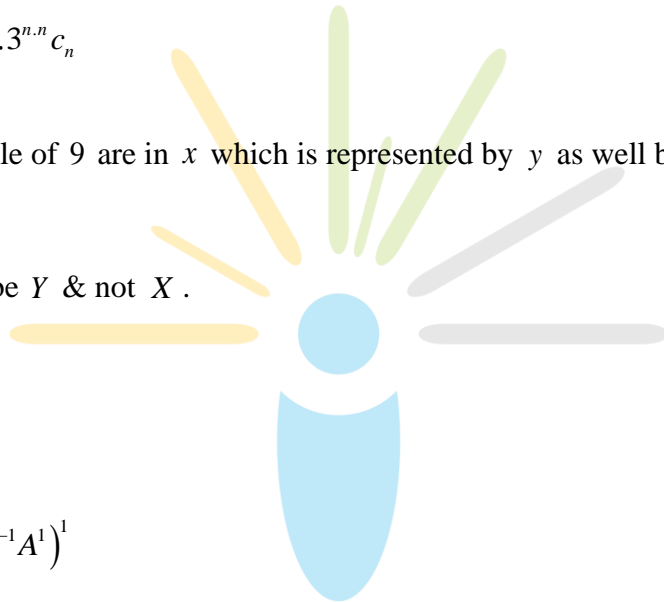
Expanding  $(1+3)^n$

$$X = \left( 1 + 3n + \frac{3.n.(n-1)}{1.2} \dots \dots 3^{n.n} c_n \right) - 3^{n-1}$$

$$= \frac{3.n.(n-1)}{1.2} \dots \dots 3^{n.n} c_n$$

$\Rightarrow$  All the multiple of 9 are in  $x$  which is represented by  $y$  as well but  $y$  will exceed  $x$  at some point.

So  $XUY$  has to be  $Y$  & not  $X$ .



**5. Sol.**

$$BB^1 = (A^{-1}A^1)(A^{-1}A^1)^1$$

$$(A^{-1}A^1)\{(A^1)1(A^{-1})1\}$$

$$= A^{-1}(A^1A)(A^1 - 1)$$

$$= A^{-1}(AA^1)(A^1) - 1$$

$$= A^{-1}AA^{-1}A^1$$

$$I.I = I \text{ Ans.}$$

**6. Sol.**

$$\int \left(1 + x - \frac{1}{x}\right) e^{x+\frac{1}{x}} dx$$

$$= \int \left(e^{x+\frac{1}{x}} dx + \int \left(x - \frac{1}{x}\right) e^{x+\frac{1}{x}} dx\right)$$

By parts

$$= x e^{x+\frac{1}{x}} \int e^{x+\frac{1}{x}} \left(1 - \frac{1}{x^2}\right) x dx + \int \left(1 - \frac{1}{x}\right) e^{x+\frac{1}{x}} dx$$

$$= x e^{x+\frac{1}{x}} \text{ Ans.}$$

**7. Sol.**

Req. Area = Area  $ACB$  + Area  $BCD$

$$= \int_0^1 1 - y^2 dy + \frac{\pi r^2}{2}$$

$$= 2 \left( y - \frac{y^3}{3} \right) + \frac{\pi}{2}$$

$$= \frac{\pi}{2} + \frac{4}{2} \text{ Ans.}$$

**8. Sol.**

Plane and line are parallel.

Eqn of normal to plane

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = K.$$

Point  $\rightarrow 2K + 1, 3 - K, 4 + K$

$$\Rightarrow \frac{2K+2}{2}, \frac{6-K}{2}, \frac{8+K}{2}$$

Lies on plane

$$2(K+1) - \frac{(6-k)}{2} + \frac{8+K}{2} + 3 = 0$$

$$K = -2$$

Point through which image passes  $(-3, 5, 2)$

$$\text{Hence, } \frac{x+3}{3} = \frac{Y-5}{1} = \frac{z-2}{-5}$$

**9. Sol.**

Even Natural No.

$$= 2, 4, 6, 8, \dots, 100$$

$$\text{Variance} = \sum \frac{(x - \bar{x})^2}{n}$$

$$\bar{x} = \text{mean} = 5$$

$$n = 50$$

$$x = 2, 4, 6, \dots, 100$$

$$= \frac{(2-51)^2 + (4-51)^2 + \dots + (100-51)^2}{50}$$

$$= 833 \text{ Ans.}$$



**10. Sol.**

$|z| \geq 2$  represents a circle with

Radius  $\geq 2$

$|z + \frac{1}{2}|$  represent distance

From point  $(-\frac{1}{2}, 0)$

[image]

$$|2 - \frac{1}{2}| = \frac{3}{2}$$

**11. Sol.**

Let  $GP$  be:

$$a, ar, ar^2$$

also  $a, 2ar, ar^2$  (are in  $AP$ )

$$\Rightarrow 4ar = a + ar^2$$

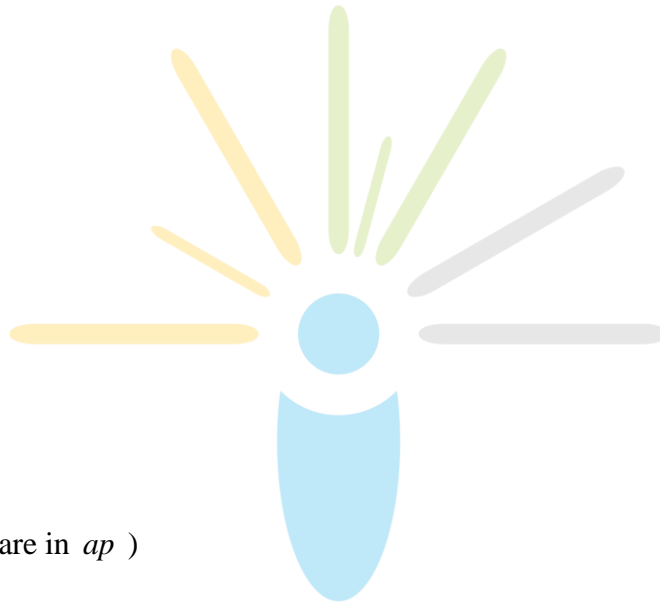
$$4r = 1 + r^2$$

$$r = \frac{4 \pm \sqrt{-12}}{2}$$

$$= 2 \pm \sqrt{3}$$

$r = 2 - \sqrt{3}$  doesn't satisfy  $AP$  condition

$$\Rightarrow r = 2 - \sqrt{3} \text{ Answer}$$



**12. Sol.**

$$(1 + ax + bx^2)(1 - 2x^{18})$$

$x^3$  terms:

$$= 18_{C_3} + (-2x)^3 + 18_{C_2}(-2x)^3 \cdot ax + 18_{C_1}(-2x) \cdot bx^2 = 0$$

$$-18_{C_3} \cdot 8 + 18_{C_2} \cdot 4a - 18_{C_1} \cdot 2b = 0 \dots (1)$$

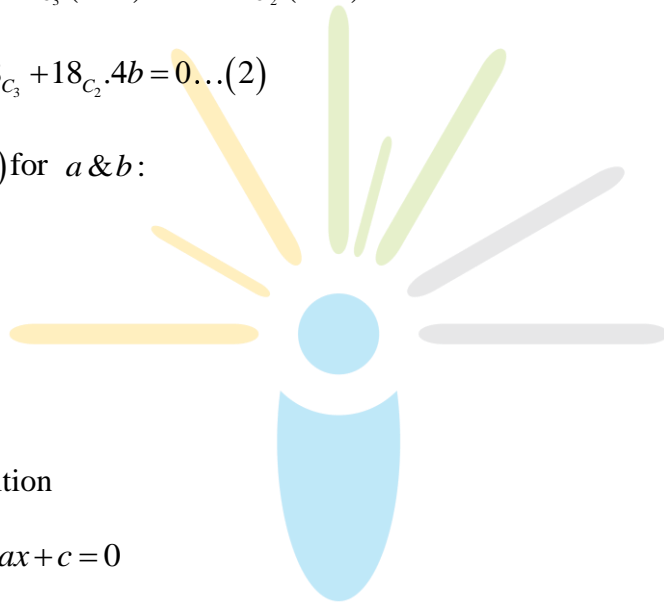
$x^4$  terms:

$$= 18_{C_4} + (-2x)^4 + 18_{C_3}(-2x)^3 \cdot ax + 18_{C_2}(-2x)^2 \cdot bx^2 = 0$$

$$-16 \cdot 18_{C_4} - 8a \cdot 18_{C_3} + 18_{C_2} \cdot 4b = 0 \dots (2)$$

Solving (1) & (2) for  $a$  &  $b$ :

$$a = 16 \quad b = \frac{272}{3}$$



**13. Sol.**

Putting the condition

$$\text{Solving: } 4ax - 2ax + c = 0$$

$$2ax + c = 0 \text{ _____ (1)}$$

$$5bx - 2bx + d = 0$$

$$3bx + d = 0 \text{ _____ (2)}$$

Putting value of  $x$

$$2a \cdot \left( \frac{-d}{3b} \right) + c = 0$$

$$3bc - 2ab = 0$$

14. Sol.

$$\begin{aligned} [\vec{a} \times \vec{b} \vec{b} \times \vec{c} \vec{c} \times \vec{a}] &= (\vec{a} \times \vec{b}) \cdot [(\vec{b} \times \vec{c})(\vec{c} \times \vec{a})]^2 \\ &= [(\vec{a} \times \vec{b}) \cdot (\vec{p}(\vec{c} \times \vec{a}))] \end{aligned}$$

$$\begin{aligned} \text{Let } \vec{p} &= (\vec{b} \times \vec{c}) \\ &= (\vec{a} \times \vec{b}) \cdot ((p.a)\vec{c} - (p.a)\vec{c}) \\ &= (\vec{a} \times \vec{b}) \cdot (((\vec{b} \times \vec{c}) \cdot \vec{a})\vec{c} - (((\vec{b} \times \vec{c}) \cdot \vec{c})\vec{a})) \\ &= (a \times v)[(bca)\vec{c}] - 0 \\ \because (\vec{b} \times \vec{c}) \cdot \vec{c} &= 0 \\ \Rightarrow (\vec{a} \times \vec{b}) \cdot \vec{c} &= [\vec{a}\vec{b}\vec{c}] \\ &= [\vec{a}\vec{b}\vec{c}] = 1 \end{aligned}$$

15. Sol.

$$\begin{aligned} P(\overline{A \cup B}) &= \frac{1}{6} \\ P(A \cap B) &= \frac{5}{6} \\ P(\overline{A}) &= \frac{1}{4} \\ P(A) &= \frac{3}{4} \\ P(B) &= P(A \cup B) - P(A) + P(A \cap B) \\ &= \frac{5}{6} - \frac{3}{4} + \frac{1}{4} \\ P(B) &= \frac{5}{6} - \frac{1}{2} = \frac{1}{3} \\ \text{Also } P(A) \cdot P(B) &= (A \cap B) \\ &= \text{Independent events} \\ P(A) &\neq P(B) \text{ Unlikely} \end{aligned}$$





**16. Sol.**

Coordinate of  $S = \left(\frac{13}{2}, 1\right)$  by mid point formula

$$\text{Slope } PS = \frac{2-1}{2-\frac{13}{2}} = \frac{-2}{9}$$

$$y = \frac{-2}{9}x + C$$

Putting  $(1, -1)$

$$-1 = \frac{-2}{9}C$$

$$C = \frac{-7}{9}$$

$$2x + 9y - 7 = 0$$

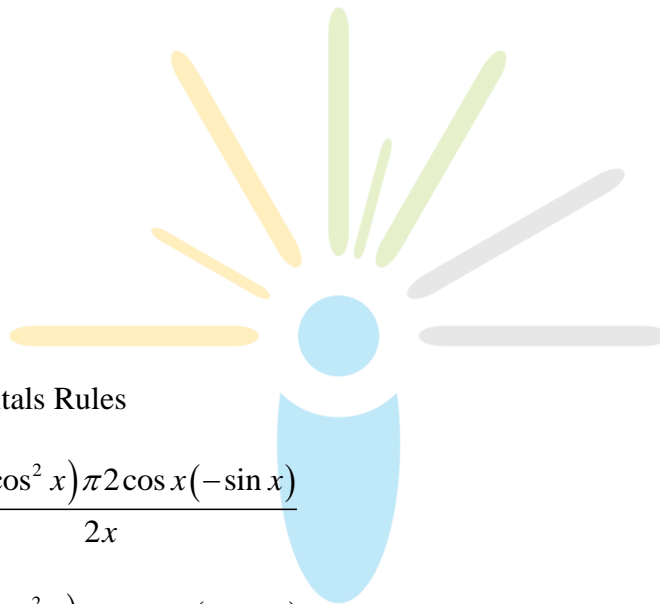
**17. Sol.**

Applying Hospitals Rules

$$= \lim_{x \rightarrow 0} \frac{\cos(\pi \cos^2 x) \pi 2 \cos x (-\sin x)}{2x}$$

$$= \lim_{x \rightarrow 0} \frac{\cos(\pi \cos^2 x)}{2} \pi 2 \cos x \frac{(-\sin x)}{x}$$

$$= \pi \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$



18. Sol.

$$A - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$$

$$= \sqrt{\frac{q^2}{p} - 4 \cdot \frac{r}{p}}$$

$$= \frac{\alpha + \beta}{\alpha\beta} 4$$

$$= -\frac{\frac{q}{r}}{\frac{p}{r}} = -\frac{q}{r} = 4 \dots (1)$$

$$2q = r + p$$

$$2 = \frac{r}{q} + \frac{p}{q}$$

$$2 = \frac{-1}{4} + \frac{p}{q}$$

$$= \frac{p}{q} - \frac{9}{4} \dots (2)$$

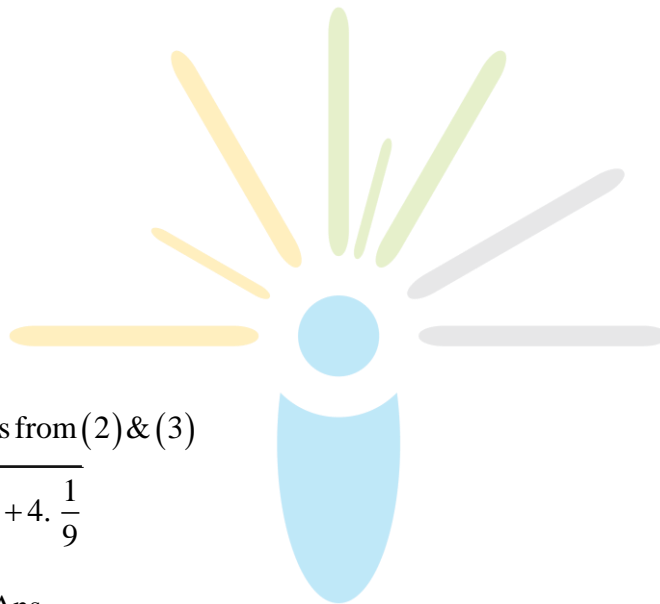
Form (1) & (2)

$$= \frac{r}{p} = \frac{-1}{9} \dots (3)$$

putting the values from (2) & (3)

$$(\alpha - \beta) = \sqrt{\frac{4^2}{9} + 4 \cdot \frac{1}{9}}$$

$$= \sqrt{\frac{52}{81}} = \frac{2\sqrt{13}}{9} \text{ Ans.}$$



**19. Sol.**

In  $\triangle AOB$

$$= \frac{AB}{OB} = \frac{20}{OB} = \tan 45 = 1$$

$$OB = 20$$

Similarly in  $\triangle A^1OB^1$

$$OB^1 = 20\sqrt{3}$$

Distance moved :

$$OB^1 - OB = 20(\sqrt{3} - 1)$$

$$\text{Velocity} = \frac{20(\sqrt{3} - 1)}{2}$$

$$= 20(\sqrt{3} - 1)$$

**20. Sol.**

$$-3(x - [x])^2 + 2(x - [x]) + a^2$$

$$(x - [x]) = \{x\}$$

$$R = [0, 1]$$

$$\text{for } \{x\} = 0$$

$$a^2 = 0$$

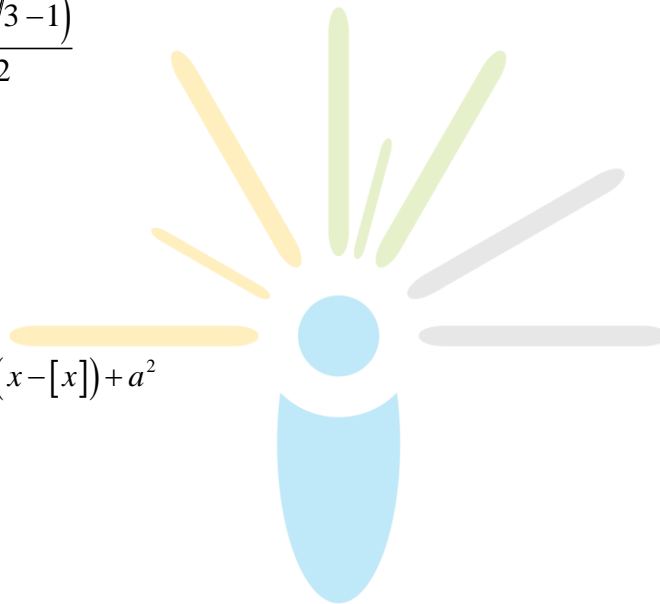
$$\text{for } \{x\} = 1$$

$$-1 + a^2 = 0$$

for the eqn. not to hold

$$a * (0, 1) \cup (-1, 0)$$

$$(-1, 0) \cup (0, 1)$$



**21. Sol.**

$$\begin{aligned} & \int_0^{\pi} \sqrt{1 + 4 \sin^2 \frac{x}{2} - 4 \sin \frac{x}{2}} dx \\ &= \int_0^{\pi} \sqrt{\left(1 - 2 \sin \frac{x}{2}\right)^2} dx \\ &= \int_0^{\pi} \left(1 - 2 \sin \frac{x}{2}\right) dx \\ & x + 2 \cos \frac{x}{2} \cdot 2 \\ &= (\pi - 4) \end{aligned}$$

**22. Sol.**

$$\begin{array}{l} x = \quad 0 \quad 1 \\ f(x) \quad 2 \quad 6 \\ g(x) \quad 2 \quad 6 \end{array}$$

By Rolles theorem:

$$\begin{aligned} f^1(x) &= \frac{6-2}{1} = 4 \\ g^1(x) &= \frac{2-0}{1} = 2 \\ f^1(x) &= 2g^1(x) \end{aligned}$$



**23. Sol.**

$$\begin{aligned} f^1(x) &= \frac{1}{1+x^5} \\ f(x) &= \int_0^x \frac{1}{1+x^5} dx \\ &\rightarrow \text{Inverse fun:} \\ X &= \int_0^{g(x)} \frac{1}{1+xg^{5(x)}} \cdot d(d_{(x)}) \\ \text{Differentiating:} \\ 1 &= \frac{1}{1+xg^{5(x)}} \cdot g(x) \\ G(x) &= 1 + g^5 x \end{aligned}$$

**24. Sol.**

$$(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \dots + 10(11)^9 = k(10)^9$$

$$1 = 2\left(\frac{11}{10}\right) + 3\left(\frac{11}{10}\right)^2 \dots 10 \cdot \frac{11^9}{10^9} = K$$

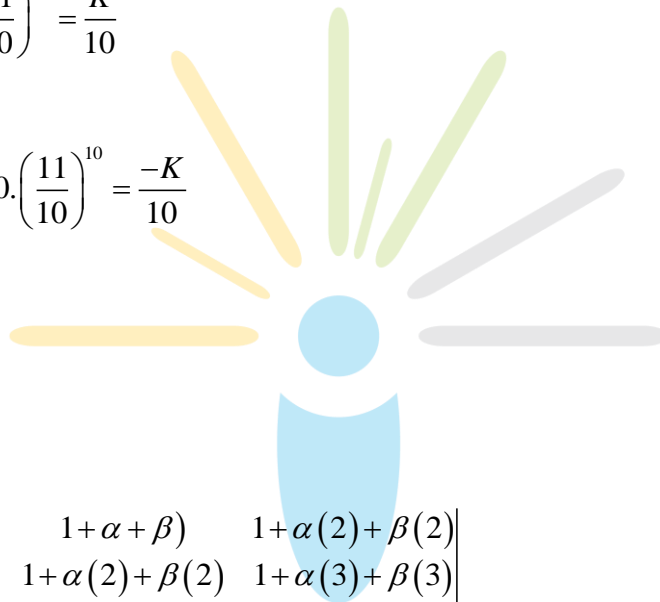
Subtracting:

$$1 + \frac{11}{10} + \left(\frac{11}{10}\right)^2 \dots \left(\frac{11}{10}\right)^9 - 10 \cdot \left(\frac{11}{10}\right)^{10} = \frac{-K}{10}$$

$$= \frac{\left(\frac{11}{10}\right)^{10-1}}{\frac{1}{10}} - 10 \cdot \left(\frac{11}{10}\right)^{10} = \frac{K}{10}$$

$$10 \cdot \left(\frac{11}{10}\right)^{10} - 10 \cdot 10 \cdot \left(\frac{11}{10}\right)^{10} = \frac{-K}{10}$$

$K = 100$  Ans.



**25. Sol.**

$$\begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha(2)+\beta(2) \\ 1+\alpha+\beta & 1+\alpha(2)+\beta(2) & 1+\alpha(3)+\beta(3) \\ 1+\alpha(2)+\beta(2) & 1+\alpha(3)+\beta(3) & 1+\alpha(4)+\beta(3) \end{vmatrix}$$

$$\begin{vmatrix} 3 & 2 & 6 \\ 2 & 6 & 8 \\ 6 & 8 & 18 \end{vmatrix}$$

Solving we get  $K = 1$

**26. Sol.**

$$y^2 = 4x$$

Tangent  $y = mx + \frac{1}{m}$

Touches  $x^2 = -32y$

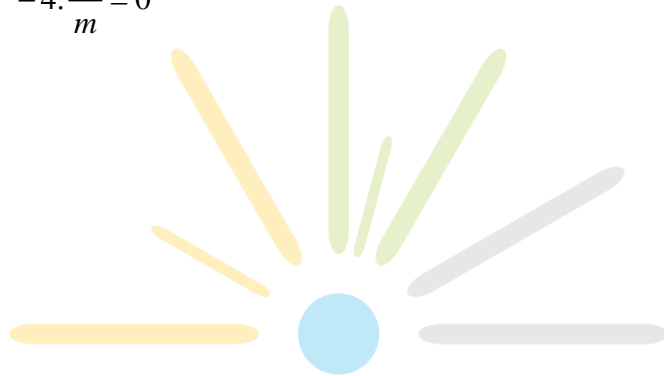
$$= x^2 = -32y \left( mx + \frac{1}{m} \right)$$

$$= x^2 + 32mx + \frac{32}{m} = 0$$

$$D = 0 \rightarrow (32mx)^2 - 4 \cdot \frac{32}{m} = 0$$

$$= m^3 = \frac{1}{8}$$

$$m = \frac{1}{2} \text{ Ans.}$$



**27. Sol.**

$P$	$Q$	$\sim P$	$\sim q$	$P \leftrightarrow q$	$P \leftrightarrow \sim q$	$\sim P \leftrightarrow q$	$\sim (P \leftrightarrow \sim q)$
$T$	$F$	$F$	$T$	$F$	$T$	$T$	$F$
$F$	$T$	$T$	$F$	$F$	$T$	$T$	$F$
$T$	$T$	$F$	$F$	$T$	$F$	$F$	$T$
$F$	$F$	$T$	$T$	$T$	$F$	$F$	$T$

28. Sol.

$$\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$$

$$= \int_{100}^p \frac{dp(t)}{\frac{1}{2}p(t) - 200} = \int_0^t dt$$

$$2 \left[ \log \left( \frac{1}{2}p(t) - 200 \right) - \log(-150) \right]$$

$$\text{Log} \frac{\frac{1}{2}p(t) - 200}{-150} = \frac{t}{2}$$

$$= \frac{1}{2}p(t) - 200 = -150e^{\frac{t}{2}}$$

$$P(t) = 400 - 300e^{\frac{t}{2}} \text{ Ans.}$$

29. Sol.

(Image)

$$= r_1 + r_2 = \sqrt{(1-0)^2 + (1-y)^2}$$

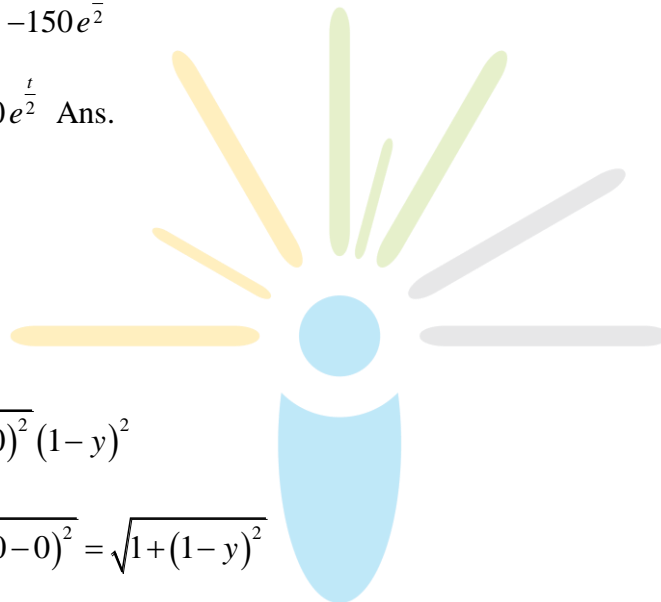
$$1 + \sqrt{(y-0)^2 + (0-0)^2} = \sqrt{1+(1-y)^2}$$

$$= (1+y)^2 = 2 + y^2 - 2y$$

$$1 + y^2 + 2y = 2 + y^2 - 2y$$

$$4y = 1$$

$$Y = \frac{1}{4} \text{ Ans.}$$



30. Sol.

$$(l+n) = -m$$

$$l^2 = (l+n)^2 + n^2$$

$$l^2 = l^2 + n^2 + 2ln + n^2$$

$$2n^2 + 2ln = 0$$

$$2n(n+l) = 0$$

$$N = 0$$

$$N = -l$$

$$L = -m$$

$$M = 0$$

$$dr'sl, -l, 0$$

$$l, 0, -l.$$

$$\cos \theta = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$$

$$\theta = \frac{\pi}{3} \text{ Ans.}$$

