JEE MAIN - 2014

MATHEMATICS

1. If $x=1$ and $x=2$ are extreme points of $f(x)=a\log|x|+Bx^2+x$ then :

a. $\propto=-6,\beta=-\frac{1}{2}$

b. $\propto=-2,\beta=-\frac{1}{2}$

c. $\propto=-6,\beta=-\frac{1}{2}$

d. $\propto=-6,\beta=\frac{1}{2}$

2. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2+3y^2=6$ on any tangent to it is:

a. $\left(x^2-y^2\right)^2=6x^2-2y^2$

b. $\left(x^2+y^2\right)^2=6x^2+2y^2$

c. $\left(x^2+y^2\right)^2=6x^2-2y^2$

d. $\left(x^2-y^2\right)^2=6x^2+2y^2$

3. Let $f_k(x)=\frac{1}{k}\left(\sin^k x+\cos^k x\right)$ where $x\in R$ and $k\geq 1$. Then $f_4(x)-f_6(x)$ equals :

a. $\frac{1}{3}$

b. $\frac{1}{4}$

c. $\frac{1}{12}$

d. $\frac{1}{6}$

4. If $X=\left\{4^n-3n-1:n\in N\right\}$ and $Y=\left\{9(n-1):n\in N\right\}$, where N is the set of natural, then $X\cup Y$ is equal :

a. $Y-X$

b. X

c. Y

d. N

5. If A is a 3×3 non – singular matrix such that $AA'=A'A$ and $B=A^{-1}A'$, then BB' equals:

a. 1

b. B^{-1}

c. $\left(B^{-1}y\right.$

d. $1+B$

6. The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to :

e. $xe^{x+\frac{1}{x}}+c$

f. $(x+1)e^{x+\frac{1}{x}}+c$

g. $-xe^{x+\frac{1}{x}}+c$

h. $(x-1)e^{x+\frac{1}{x}}+c$

7. The area of the region described by $A=\{(x,y):x^2+y^2\leq 1 \text{and} \, y^2\leq 1-x\}$ is :

i. $\frac{\pi}{2}-\frac{4}{3}$

j. $\frac{\pi}{2}-\frac{2}{3}$

k. $\frac{\pi}{2}+\frac{2}{3}$

l. $\frac{\pi}{2}+\frac{4}{3}$

8. The image of the line $\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane $2x-y+z+3=0$ is the line :

a. $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$

b. $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$

c. $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$

d. $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

9. The variance of first 50 even natural numbers is :

a. 833

b. 437

c. $\frac{437}{4}$

d. $\frac{833}{4}$

10. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z+\frac{1}{2}\right|$:

11. Three positive numbers form an increasing $G.P$. If the middle term in this $G.P$. is doubled, the new number is in $A.P$. Then the common ratio of the $G.P >$ is :

12. If the coefficients of x^3 and x^4 in the expansion of $\left(1+ax+bx^2\right)(1-2x)^{18}$ in powers of x are both zero, then (a,b) is equal to :

13 Let a,b, and d be non-zero numbers. If the point of intersection of the lines $4ax+2ay+c=0$ and $5bx+2by+d=0$ lies in the fourth quadrant and is equidistant from the two axes then:

14. If $\left[\vec{a}\times\vec{b}\vec{b}\times\vec{c}\vec{c}\times\vec{a}\right]=\lambda\left[\vec{a}\vec{b}\vec{c}\right]^2$ then λ is equal to :

15. Let A and B be two events such that $P\left(\overline{A\cup B}\right)=\frac{1}{6}, P\left(A\cup B\right)=\frac{1}{4}$ and $P\left(\overline{A}\right)=\frac{1}{6}$, where $\overline{A}$ stands for the complement of the event A. Then the events A and B are :

16. Let PS be the median of the triangle with vertices $P(2,2), Q(6,-1)$ and $R(7,3)$, The equation of the line passing through $(1,-1)$ and parallel to PS is :

17. $\lim_{x\to 0} \cdot \dfrac{\sin\left(\pi \cos^2 x\right)}{x^2}$ is equal to :

18. Let $\propto$ and β be the roots of equation $px^2+qx+r=0, p\neq 0$. If p, q, r are in $A.P.$ and $\dfrac{1}{\propto}+\dfrac{1}{\beta}=4$, then the value of $|\propto -\beta|$ is:

19. A bird is sitting on the top of a vertical pole $20\,\text{m}$ high and its elevation from a point O on the ground is $45°$. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to $30°$. Then the speed (in m/s) of the bird is :

20. If $a\in R$ and the equation $-3\left(x-[x]\right)^2+2\left(x+a^2=0\right.$ (where $[x]$ denotes the greatest integer $\leq x)$ has no integral solution, then all possible values of a lie in the interval :

21. The integral $\int_0^{\pi}\sqrt{1+4\sin^2\dfrac{x}{2}-4\sin\dfrac{x}{2}}\,dx$ equals :

22. If f and g are differentiable functions in $[0,1]$ satisfying $f(0)=g(1), g(0)=0$, and $f(1)=6$, then for some $C\in\,]0,1[$:

23. If g is the inverse of a function f and $f'(x)=\dfrac{1}{1+x^5}$,then $g'(x0$ is equal to :

24. If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9 = k(10)9$, then k is equal to :

25. If $\propto, \beta \neq 0$, and $f\left(n0 = \propto^n + \beta^n\right.$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = \mathrm{K}(1-\propto)^2(1-\beta)^2(\propto-\beta)^2$$, then K is equal to

26. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is

27. The statement $\sim(p \leftrightarrow \sim q)$ is :

28. Let the population of rabbits surviving at a time t be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$ If $P(0) = 100$, than $P(t)$ equals :

29. Let C be the circle with centre at $(1,1)$ and radius $= 1$. If T is the circle centred at $(0, y)$, passing through origin and touching the circle C externally, then the radius of T is equal to :

30. The angle between the lines whose direction cosine satisfy the equations $l + m + n = 0$ and $l^2 = m^2 + n^2$ is :