JEE MAIN – 2015

MATHEMATICS

ANSWER KEY AND EXPLANATIONS

Q61. Sol. (1)

$$n(A)=4$$
$$n(B)=2$$
$$n(A\times B)\ =n(A).n(B)$$
$$=4\times 2$$
$$=8$$

Number of subsets having atleat 3 elements $= {}^8C_3+{}^8C_4+{}^8C_5\dots{}^8C_8$

$$=2^8-\left({}^8C_0+{}^8C_1+{}^8C_2\right)$$
$$=219$$

Q62. Sol. (3)

$$|z|=1$$
$$\left|\frac{z_1-2z_2}{z-z_1z_2}\right|=1$$
$$|z_1-2z_2|^2=|2-z_1\overline{z}_2|^2$$
$$(z_1-2z_2)(\overline{z}_1-2\overline{z}_2)=(2-z_1\overline{z}_2)(2-\overline{z}_1z_2)$$
$$|z_1|^2-2z_1\overline{z}_2-2\overline{z}_1z_2+4|z_2|^2=4-2z_1z_2-2z_1\overline{z}_2+|z_1|^2|z_2|^2$$
$$|z_1|^2+4|z_2|^2=4+|z_1|^2|z_2|^2$$
$$|z_1|^2-|z_1|^2|z_2|^2=4-4|z_2|^2$$
$$|z_1|^2\left(1-|z_2|^2\right)=4\left(1-|z_2|^2\right)$$
$$\Rightarrow |z_1|^2=4$$
$$\Rightarrow |z_1|=2$$

Q63. Sol. (3)

$$x^2-6x-2=0$$

$$\frac{\left(\alpha^{10}-ß^{10}\right)-2\left(\alpha^{8}-ß^{8}\right)}{2\left(\alpha^{9}-ß^{9}\right)}$$

$$\alpha^2-6\alpha-2=0 \qquad \alpha^{10}-2\alpha^{8}=6^{\alpha}$$

$$\beta^{10}-2\beta^{8}=6\,ß^{9}$$

$$\frac{6\left(\alpha^{9}-ß^{9}\right)}{2\left(\alpha^{9}-ß^{9}\right)}=3$$

Q64. Sol. (4)

$$A=\begin{bmatrix}1&2&2\\2&1&-2\\a&2&b\end{bmatrix}$$

$$AA^T=\begin{bmatrix}1&2&2\\2&1&-2\\a&2&b\end{bmatrix}\begin{bmatrix}1&2&a\\2&1&2\\2&-2&b\end{bmatrix}$$

$$\begin{bmatrix}9&0&a+4+2b\\0&9&2a+2-2b\\a+4+2b&2a+2-2b&a^2+4+b^2\end{bmatrix}$$

$$a+4+2b=0$$

$$2a+2-2b=0$$

$$\overline{3a+6=0}$$

$$a=-2$$

$$-2+4+2b=0$$

$$2+2b=0$$

$$b=-1$$

$$(-2,-1)$$

Q65. Sol. (3)

$$(2-\lambda)x_1 - 2x_2 + x_3 = 0$$
$$2x_1 - (3+\lambda)x_2 + 2x_3$$
$$-x_1 + 2x_2 - \lambda x_3 = 0$$
$$\begin{bmatrix} 2-\lambda & -2 & 1 \\ 2 & -(3+\lambda) & 2 \\ -1 & 2 & -\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$\Delta = \left[(2-\lambda)\left[\lambda(3+\lambda)-4\right] + 2(-2\lambda+2) + 1(4-(3+\lambda)\right]$$
$$(2-\lambda)\left[3\lambda+\lambda^2-4\right] + 4 - 4\lambda + 1 - \lambda = 0$$
$$6\lambda + 2\lambda 2 - 8 - 3\lambda^2 - \lambda^3 + 4\lambda - 4\lambda + 4 + 1 - \lambda$$
$$5\lambda - \lambda^2 - 3 - \lambda^3 = 0$$
$$\lambda^3 + \lambda^2 - 5\lambda + 3 = 0$$
$$(\lambda-1)\left[\lambda^2 + 2\lambda - 3\right]$$
$$\left[\lambda^2 + 3\lambda - \lambda - 3\right]$$

for more – trivial solutions

$\Delta = 0$

Hence for 2 values

$1, -3$

Q66. Sol. (2)

$6,000$

$\Rightarrow 3 \times 4 \times 3 \times 2 = 72$

$\underline{\ldots\ldots\ldots\ldots\ldots 5! = 120}$

$= 192$

Q67. Sol. (1)

$$\left(1-2\sqrt{x}\right)^{50} = {}^{50}C_0\left(1\right)^0\left(-2\sqrt{x}\right)^{50} - {}^{50}C_1\left(1\right)^1\left(2\sqrt{x}\right)^{49}$$

$$\left(1+2\sqrt{x}\right)^{50} = {}^{50}C_0\left(1\right)^0\left(2\sqrt{x}\right)^{50} + \ldots\ldots\ldots$$

$$\left(1+2\sqrt{x}\right)^{50} + \left(1-2\sqrt{x}\right)^{50} = \left[{}^{50}C_0 + \ldots\ldots\right]$$

$$3^{50} + \left(-1\right)^{50} = 2\times \text{Re}\,quried.$$

$$\frac{3^{50}+1}{2} = \text{Re}\,quried.$$

Q68. Sol. (2)

$$m = \frac{l+n}{2}$$

$$lG_1G_2G_3n$$

$$G_1 = rl$$

$$G_2 = r^2 l$$

$$G_3 = r^3 l$$

$$n = r^4 l \quad \Rightarrow r = \left(\frac{n}{l}\right)^{1/4}$$

$$\begin{aligned}
\text{Requrired} \quad &= G_1^4 + 2G_2^4 + G_3^4 \\
&= r^4 l^4 + 2\left(r^2 l\right)^4 + \left(r^3 l\right)^4 \\
&= l^4\left\{r^4 + 2\left(r^4\right) + \left(r^4\right)^3\right\} \\
&= l^4\left\{\frac{n}{l} + \frac{2n^2}{l^2} + \frac{n^3}{l^3}\right\} \\
&= nl^3 + 2n^2l^2 + n^3 l. \\
&= nl\left\{l^2 + 2nl + n^2\right\} \\
&= nl\left\{l+n\right\}^2 \\
&= nl\left(2m\right)^2 = 4lm^2 n
\end{aligned}$$

Q69. Sol. (2)

$$T_r=\frac{1^3+2^3+3^3+\ldots\ldots r^3}{1+3+5+\ldots\ldots(2r-1)}$$

$$\frac{\left(\frac{r(r+1)}{2}\right)^2}{r^2}=\frac{(r+1)^2}{4}$$

$$\sum T_t=\frac{1}{4}\sum_{r=1}^{9}(r+1)^2$$

$$=\frac{1}{4}\{2^2+3^2+\ldots\ldots 10^2\}$$

$$=\frac{1}{4}\left\{\frac{10}{6}(11)(21)-1\right\}$$

$$=96$$

Q70. Sol. (3)

$$\lim_{x\to\infty}\frac{(1-\cos 2x)(3+\cos x)}{x\left(\frac{\tan 4x}{4x}\right)\times 4x}$$

$$=\lim_{x\to\infty}\frac{2\sin^2 x(3+\cos x)}{4x^2}$$

$$\Rightarrow \frac{1}{2}\times 4=2$$

Q71. Sol. (1)

$$g(x)=\begin{cases}k\sqrt{x+1} & 0\le x\le 3\\ mx+2 & 3<x\le 5\end{cases}$$

$$g'(x)=\begin{cases}\frac{k}{2\sqrt{x+1}} & 0\le x\le 3\\ m & 3<x\le 5\end{cases}$$

For differentiability at 3,

$$\frac{k}{2\sqrt{3+1}}=m$$

$k = 4m \quad \ldots(1)$

Also $g\left(3^{-}\right) = g\left(3^{+}\right)$

$k\sqrt{3+1} = m(3)+2$

$2k = 3m+2 \quad \ldots(2)$

From (1) and (2) $m = \frac{2}{5} k = \frac{8}{5}$

$k + m = 2$

Q72. Sol. (4)

$x^2 + 2xy = 3y^2 = 0$

$2x + 2xy' + 2y - 6y\,y' = 0$

$(2x-6y)\,y' = -(2x+2y)$

$y' = -\frac{(2x+2y)}{2x-6y}$

$y'\Big|_{(1,1)} = \frac{-4}{-4} = 1$

$y'\Big|_{(1,1)}(\text{normal}) = -1$

Equation of normal

$(y-1) = -1(x-1)$

$y-1 = -x+1$

$x+y=2 \quad x = 2-y$

Solve it with curve

$(2-4)^2 + 2(2-y)y - 3y^2 = 0$

$4 + y^2 - 4y + 4y - 2y^2 - 3y^2 = 0$

$4y^2 = 4$

$y = \pm 11$

$x = 1, 3$

$(1,1)\,\text{and}\,(3-1)$

Hence meets the curve again in 4^{th} quadrant

Q73. Sol. (3)

$$\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]$$

Let $f(x)=ax^4+bx^3+cx^2+dx+e$

$$\lim_{x\to 0}\frac{f(x)}{x^2}=2$$

$$\frac{ax^4}{x^2}+\frac{bx^3}{x^2}+\frac{cx^2}{x^2}+\frac{dx}{x^2}+\frac{e}{x^2}=2$$

For limit to exist

d and $e=0$

So $c=2$

$f(x)$ becomes $ax^4+bx^3+2x^2$

$f'(x)=4ax^3+3bx^2+4x$

$f'(1)=4a+3b+h=0 \quad \ldots\ldots 1$

$f'(z)=32a+12b+8=0 \quad \ldots\ldots 2$

$8x(1)-(2)$

$12b+24 \quad =0$

$b=-2$

$a=½$

$f(x)\frac{x^4}{2}-2x^3+2x^2 \Rightarrow f(z) \quad =8-16+8$

$=0$

Q74. Sol. (5)

$$\int \frac{dx}{x^2\left(x^4+1\right)^{3/4}}$$

$$\int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$$

$$1/x^4 = t$$

$$dt = -4\frac{1}{x^5}dx$$

$$\int \frac{-x^5}{4x^5(1+t)^{3/4}}dt$$

$$-\int \frac{1}{4(1+t)^{3/4}}dt$$

$$-\frac{(1+t)}{4\times\frac{1}{4}}+c = -\left(1+\frac{1}{x^4}\right)^{1/4}+c$$

Q75. Sol. (3)

$$I - \int_2^4 \frac{\log x^2}{\log x^2+\log(x-6)^2} \quad \ldots(1)$$

Using property $\int_a^b +(x)\,da = \int_a^b +(a+b-x)$

$$I = \int_2^4 \frac{\log(6-x)^2}{\log(6-x)^2+\log(6-x-6)^2}$$

$$= \int_2^4 \frac{\log(6-x)^2}{\log(6-x)+\log x^2} \quad \ldots(2)$$

Form (1) and (2)

$$21 = \int_2^4 1\,dx$$

$$21 = x\Big|_2^4 = 2$$

$$I = 1$$

Q76. Sol. (4)

$x=\frac{1}{2} \quad x=\frac{1}{8}$

$y^2=2x$

when $x=\frac{1}{8} \quad y^2=2\left(\frac{1}{8}\right)_4$

$y=\frac{1}{2}$

$y=4x-1$

when $x=\frac{1}{8} y=4\left(\frac{1}{8}\right)-1$

$y=\frac{1}{2}-1$

$y=-\frac{1}{2}$

$$\int_{-1/2}^{+1}\left(\frac{y+1}{4}-\frac{y^2}{2}\right)dx$$

$$\frac{1}{4}\left(\frac{y^2}{2}+y\right)_{-1/2}^{+1} -\frac{1}{2}\left(\frac{y^3}{3}\right)_{-1/2}^{+1}$$

$$\frac{1}{4}\left[\left(\frac{1}{2}+1\right)-\left(\frac{1}{8}-\frac{1}{2}\right)\right]-\frac{1}{2}\left\{\frac{1}{3}+\frac{1}{24}\right\}$$

$$\frac{1}{4}\left\{\frac{3}{2}+\frac{3}{8}\right\}-\frac{1}{2}\left\{\frac{9}{24}\right\}$$

$$\Rightarrow \frac{3}{4}\left(\frac{5}{8}\right)-\frac{9}{48}=\frac{15}{-32}-\frac{9}{48}=\frac{9}{32}$$

Q77. Sol. (2)

$$(x\log x)\frac{dy}{dx}+y=2x\log x$$

$$\frac{dy}{dx}+\left(\frac{1}{x\log x}\right)y=2$$

$$IF \quad = e^{\int pdx}$$

$$= e^{\int \frac{1}{x\log x}dx}$$

$$= e\log\log x$$

$$= \log x$$

$$y\times IF = \int IF \times Qdx$$

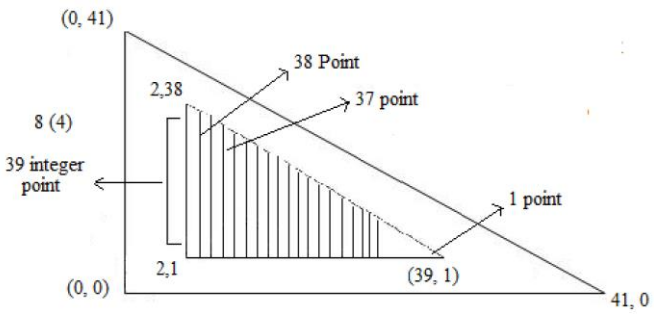
$$y\log x = \int 2\log x\, dx$$

$$y\log x = 2x(\log x - 1) + c$$

$$y(c) = \frac{2e(\log e - 1) + c}{\log e}$$

$$y(e) = \frac{c}{\log e} = c\,(c \text{ is not given so assume } c \text{ to be } 0)$$

Q78. Sol. (4)



Numbers of points interior of triangle $= 39 + 38 + 37 = H$

$$= \frac{39\times 40}{2}$$

$$= 780$$

Q79. Sol. (3)

$$(2x-3y+4)+k(a-2y+3)=0$$

This is an eqn. of formally of straight lines.

So, fixed point $\left.\begin{matrix}2x-3y+4=0\\ x-2y+3=0\end{matrix}\right\}$ solving

So fixed point $\dfrac{y=+2 \text{and} \, x=+1}{(1,2)}$

Locus will be circle

$$\text{With radius } =\sqrt{(2-1)^2+(3-2)^2}$$
$$=\sqrt{1^2+1^2}$$
$$=\sqrt{2}$$

Q80. Sol. (3)

Given circles are

$$(x-2)^2+(y-3)^2=5^2$$
$$(x+3)^2+(y+9)^2=8^2$$

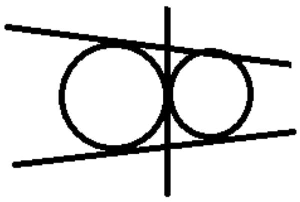
$$\text{Distance between centers } =\sqrt{(3+2)^2+(9+3)^2}$$
$$=\sqrt{25+144}$$
$$=\sqrt{169}=13$$

Sum of radius $=8+5=13$

which means circles are touching externally

Hence 3 common tangents.

Q81. Sol. (4)



Tangent at $\left(2, \frac{5}{3}\right)$

$\frac{2x}{9} + \frac{5 \times y}{3 \times} = 1$

$\frac{2x}{9} + \frac{y}{3} = 1$

$6x + y = 9$

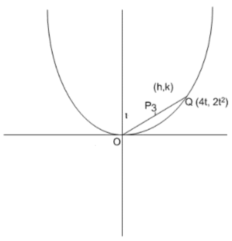
$\frac{x}{9/2} + \frac{4}{9} = 0 \rightarrow l$

Area of Δ formed $= \frac{27}{4}$

by 1 with axis

Now due to symmetry area of quad will be 4 times i.e 27 sq units.

Q82. Sol. (4)



$\frac{OP}{PQ} = 1:3$

$h = \frac{4t}{4} t = h$

$k = \frac{2t}{4} t^2 = 2k$

$\therefore h^2 = 2k$

$x^2 = 2y$

Q83. Sol. (4)

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}=k$$
$$x=3k+2$$
$$y=4k+1$$
$$z=12k+2$$
$$\therefore x+z-y=16$$
$$3k+2+12k+2-4k+1=16$$
$$11k=11$$
$$k=1$$
$$\therefore (x,y,z)=(5,3,14)$$

Distance

$$=\sqrt{16+9+144}$$
$$=\sqrt{144+25}$$
$$\sqrt{169}=13$$

Q84. Sol. (3)

plane parallel to $x+3y+6z=1$

will be $x+3y+6z=k$

and as the lines lieon the plane

so it must pass through common point

i e $(1,0,1)$

$$\therefore 1+0+6=k$$
$$k=7$$
$$\therefore x+3y+6z=7$$

Q85. Sol. (1)

$$\left(\vec{a}\times\vec{b}\right)\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$$

$$\left(\vec{b}\cdot\vec{c}\right)\vec{a}=\left(\vec{a}\cdot\vec{c}\right)\vec{b}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$$

$$\left(\vec{b}\cdot\vec{c}-\frac{1}{3}|\vec{b}||\vec{c}|\right)\vec{a}=\left(\vec{a}\cdot\vec{c}\right)\vec{b}$$

$$\left(|\vec{b}||\vec{c}|\cos\theta-\frac{1}{3}|\vec{b}||\vec{c}|\right)\vec{a}=\left(\vec{a}\cdot\vec{c}\right)\vec{b}$$

Since $\vec{a}$ and $\vec{b}$ non collinear so $\vec{a}\neq\lambda\vec{b}$ non zero

i.e. $|\vec{b}||\vec{c}|\cos\theta-\frac{1}{3}|\vec{b}||\vec{c}|=0$ (one of the possible values)

$$\cos\theta=\frac{1}{3}$$

$$\sin\theta=\frac{2\sqrt{2}}{3}$$

Q86. Sol

Q87. Sol. (4)

Sum of 16 observations $=16\times16$

$=256$

Sum of remaining 15 observations $=240$

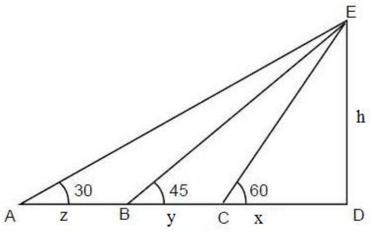
New sum of 18 observations $=240+3+4+5$

$=252$

Mean of 18 observations $=\frac{252}{18}$

$=14$

Q88. Sol. (1)



$d \in$ is tower

$$\tan 60 = \frac{h}{x}$$

$$x = \frac{h}{\sqrt{3}} \quad \dots(1)$$

$$\tan 45 = \frac{h}{x+y}$$

$$x + y = h \quad \dots(2)$$

$$\tan 30 = \frac{h}{x+y+z}$$

$$x + y + z = \sqrt{3h} \quad \dots(3)$$

$$\text{To find } \frac{AB}{BC} = \frac{z}{Y}$$

From eq. (2) − (1)

$$y = h\left(1 - \frac{1}{\sqrt{3}}\right) \quad \dots(4)$$

From eq. (3) − (2)

$$z = \cdot h\left(\sqrt{3} - 1\right) \quad \dots(5)$$

From eq. (5) ÷ (4)

$$\frac{z}{y} = \frac{h\left(\sqrt{3}-1\right)}{h\left(1 - \frac{1}{\sqrt{3}}\right)} = \sqrt{3}$$

Q89. Sol. (1)

$$|x| < \frac{1}{\sqrt{3}}$$

$$\frac{-1}{\sqrt{3}} \le x \le \frac{1}{\sqrt{3}}$$

$$\therefore -\frac{\pi}{3} \le \tan^{-1} x \le \frac{\pi}{3}$$

$$\text{and } 2\tan^{-1} x = \tan^{-1}\left\{\frac{2x}{1-x^2}\right\}$$

$$= \tan^{-1} x + 2\tan^{-1} x$$

$$\tan^{-1} x = 3\tan^{-1} x$$

$$y = \tan\left[3\tan^{-1} x\right]$$

$$\therefore y = \frac{3x - x^3}{1-3x^2}$$

Q90. Sol. (4)

r	s	$\sim r$	$\sim r \wedge s$	$\sim s$	$\sim s \vee (\sim r \wedge s)$	Negation
T	T	F	F	F	F	T
T	F	F	T	T	T	F
F	T	T	T	F	T	F
F	F	T	F	T	T	F