JEE MAIN - 2016

MATHEMATICS

ANSWER KEY AND EXPLANATIONS

Q1. Sol. (A)

$$Z=\frac{2+3i\sin\theta}{2-2i\sin\theta}$$

$$\Rightarrow Z =\frac{(2+3i\sin\theta)(1+2i\sin\theta)}{1+4\sin^2\theta}$$

$$=\frac{(2-6\sin^2\theta)+7i\sin\theta}{1+4\sin^2\theta}$$

for purely imaginary $Z, \operatorname{Re}(Z)=0$

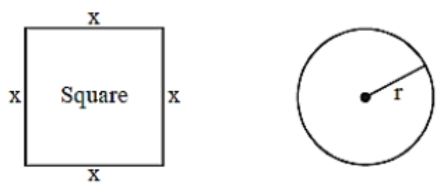
$$\Rightarrow 2-6\sin^2\theta=0\Rightarrow \sin\theta=\pm\frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta=\pm\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Q2. Sol. (A)

$$\begin{vmatrix}1 & \lambda & -1\\ \lambda & -1 & -1\\ 1 & 1 & -\lambda\end{vmatrix}=0\Rightarrow \lambda=0,1,-1$$

Q3. Sol. (D)



given that $4x+2\pi r=2$

i.e $2x+\pi r=1$

$$\therefore r=\frac{1-2x}{\pi} \quad \ldots(\text{i})$$

Area $A=x^2+\pi r^2$

$$=x^2+\frac{1}{\pi}(2x-1)^2$$

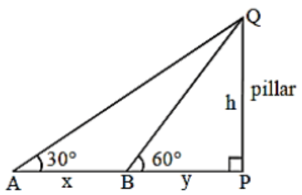
for min value of are A

$$\frac{dA}{dx}=0 \text{ gives } x=\frac{2}{\pi+4} \quad \ldots(\text{ii})$$

from (i) & (ii)

$$r=\frac{1}{\pi+4} \quad \ldots(\text{iii})$$
$$\therefore x=2r$$

Q4. Sol. (A)



$$\Delta QPA:\frac{h}{x+y}=\tan 30^\circ \Rightarrow \sqrt{3}h=x+y\ldots(\text{i})$$
$$\Delta QPB:\frac{h}{y}=\tan 60^\circ \Rightarrow h=\sqrt{3}y\ldots(\text{ii})$$

By (i) and (ii): $3y=x+y \Rightarrow y=\frac{x}{2}$

$\because$ speed is uniform

Distance x in 10 mins

$\Rightarrow$ Distance $\frac{x}{2}$ in 5 mins

Q5. Sol. (A)

$E1 \to A$ shows up 4

$E2 \to B$ shows up 2

$E3 \to$ Sum is odd (i.e. even + odd or odd + even)

$$P(E_1) = \frac{6}{6.6} = \frac{1}{6}$$
$$P(E_2) = \frac{6}{6.6} = \frac{1}{6}$$
$$P(E_3) = \frac{3\times3\times2}{6.6} = \frac{1}{2}$$

$$P(E_1 \cap E_2) = \frac{1}{6.6} = P(E_1).P(E_2)$$
$$\Rightarrow E_1 \,\&\, E_2 \text{ are independent}$$
$$P(E_1 \cap E_3) = \frac{1.3}{6.6} = P(E_1).P(E_3)$$
$$\Rightarrow E_1 \,\&\, E_3 \text{ are independent}$$
$$P(E_2 \cap E_3) = \frac{1.3}{6.6} = \frac{1}{12} = P(E_2).P(E_3)$$
$$\Rightarrow E_2 \,\&\, E_3 \text{ are independent}$$
$$P(E_1 \cap E_2 \cap E_3) = 0 \text{ ie imposible event.}$$

Q6. Sol. (C)

$$\because S.D = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$
$$\therefore \frac{49}{4} = \frac{4+9+a^2+121}{4} - \left(\frac{16+a}{4}\right)^2$$
$$\Rightarrow 3a^2 - 32a + 84 = 0$$

Q7. Sol. (C)

In the neighbourhood of $x = 0,\ \ f(x) = \log 2 - \sin x$

$\therefore g(x) = f(f(x)) = \log 2 - \sin(f(x))$

$= \log 2 - \sin(\log 2 - \sin x)$

It is differentiable at $x = 0$, So

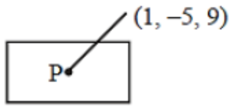
$\therefore ''(x) = -\cos(\log 2 - \sin x)(-\cos x)$

$\therefore g'(0) = \cos(\log 2)$

Q8. Sol. (C)

Equation of the line parallel to $x = y = z$ through $(1, -5, 9)$ is $\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1}\lambda$

If $P(\lambda+1, \lambda-5, \lambda+9)$ be point of intersection of line and plane.



$\Rightarrow \lambda + 1 - \lambda + 5 + \lambda + 9 = 5$

$\Rightarrow \lambda = -10$

$\Rightarrow$ Coordinates point are $(-9, -15, -1)$

$\Rightarrow$ Required distance $= 10\sqrt{3}$

Q9. Sol. (D)

Given

$\frac{2b^2}{a}=8 \quad \ldots(1)$

$2b=ae \quad \ldots(2)$

We know

$b^2=a^2\left(e^2-1\right) \quad \ldots(3)$

substitute $\frac{b}{a}=\frac{e}{2}$ from (2) in (3)

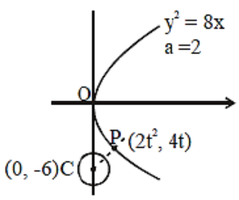
$\Rightarrow \frac{e^2}{4}=e^2-1$

$\Rightarrow 4=3e^2$

$\Rightarrow e=\frac{2}{\sqrt{3}}$

Q10. Sol. (B)

Circle and parabola are as shown:



Minimum distance occurs along common normal.

Let normal to parabola be $y+tx=2.2.t+2t^3$ pass through $(0,-6)$:

$-6=4t+2t^3 \Rightarrow t^3+2t+3=0$

$\Rightarrow t=-1$ (only real value)

$\therefore P(2,-4)$

$\therefore CP=\sqrt{4+4}=2\sqrt{2}$

$\therefore$ equitation of circle

$(x-2)^2+(y+4)^2=(2\sqrt{2})^2$

$\Rightarrow x^2+y^2-4x+8y+12=0$

Q11. Sol. (C)

$A=\begin{bmatrix}5a & -b\\ 3 & 2\end{bmatrix}$ and $A^T=\begin{bmatrix}5a & 3\\ -b & 2\end{bmatrix}$

$AA^T=\begin{bmatrix}25a^2+b^2 & 15a-2b\\ 15a-2b & 13\end{bmatrix}$

Now, $A\,\text{adj}\,A=|A|I_2=\begin{bmatrix}10a+3b & 0\\ 0 & 10a+3b\end{bmatrix}$

Given $AA^T=A.\text{adj}\,A$

$15a-2b=0 \quad \ldots(1)$

$10a+3b=13 \quad \ldots(2)$

Solving we get

$5a=2$ and $b=3$

$\therefore 5a+b=5$

Q12. Sol. (C)

$$f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right) \text{where } x \in \left(0, \frac{\pi}{2}\right)$$

$$= \tan^{-1}\left(\sqrt{\frac{(1+\sin x)^2}{1-\sin^2 x}}\right)$$

$$= \tan^{-1}\left(\frac{1+\sin x}{|\cos x|}\right)$$

$$= \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)\left(\text{as } x \in \left(0, \frac{\pi}{2}\right)\right)$$

$$= \tan^{-1}\left(\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$$

$$f(x) = \frac{\pi}{4}+\frac{\pi}{2} \text{as } x \in \left(0, \frac{\pi}{2}\right) \Rightarrow f'\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$\therefore$ Equitation of normal

$$\left(y-\frac{\pi}{3}\right) = -2\left(x-\frac{\pi}{6}\right)$$

which passes through $\left(0, \frac{2\pi}{3}\right)$

Q13. Sol. (D)

Equation of angle bisector of the lines $x-y+1=0$ and $7x-y-5=0$ is given by

$$\frac{x-y+1}{\sqrt{2}} = \pm\frac{7x-y-5}{5\sqrt{2}}$$

$$\Rightarrow 5(x-y+1) = 7x-y-5$$

and

$5(x-y+1)=-7x+y+5$
$\therefore 2x+4y-10=0 \Rightarrow x+2y-5=0$

and

$12x-6y=0 \Rightarrow 2x-y=0$

Now equation of diagonal are

$(x+1)+2(y+2)=0 \Rightarrow x+2y+5=0 \quad \ldots(1)$

and

$2(x+1)-(y+2)=0 \Rightarrow 2x-y=0 \quad \ldots(2)$

Clearly $\left(\frac{1}{3},-\frac{8}{3}\right)$ lies on (1)

Q14. Sol. (A)

Given differential equitation

$$ydx+xy^2dx=xdy$$
$$\Rightarrow \quad \frac{xdy-ydx}{y^2}=xdx$$
$$\Rightarrow \quad -d\left(\frac{x}{y}\right)=d\left(\frac{x^2}{2}\right)$$

Integrating we get

$$-\frac{x}{y}=\frac{x^2}{2}+C$$

$\because$ It passes through $(1,-1)$

$$\therefore 1=\frac{1}{2}+C \Rightarrow C=\frac{1}{2}$$
$$\therefore x^2+1+\frac{2x}{y}=0 \Rightarrow y=\frac{-2x}{x^2+1}$$
$$\therefore f\left(-\frac{1}{2}\right)=\frac{4}{5}$$

Q15. Sol. (A)

Total number of words which can be formed using all the letters of the word 'SMALL'
$=\frac{5!}{2!}=60$

Now, 60^{th} word is $\rightarrow$ SMLLA
59^{th} word is $\rightarrow$ SMLAL
58^{th} word is $\rightarrow$ SMALL

Q16. Sol. (C)

Let 'a' be the first term and d be the common difference 2^{nd} term $=a+d$,
5^{th} term $=a+4d$, 9^{th} term $=4+8d$

$\therefore$ common ratio $=\frac{a+4d}{a+d}=\frac{a+8d}{a+4d}=\frac{4d}{3d}=\frac{4}{3}$

Q.17 Sol. (A)

Number of terms in the expansion of

$\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$ is ${}^{n+2}C_2$ (considering $\frac{1}{x}$ and $\frac{1}{x^2}$ Distinct).

$\therefore {}^{n+2}C_2=28 \Rightarrow n=6$

$\therefore$ Sum of coefficents $=(1-2+4)^6=729$

But number of dissimilar terms actually will be $2n+1$ (as $1/x$ and $1/x^2$ are functions as same variable)

Hence it contains error, so a bonus can be expected.

Q18. Sol. (C)

Given series is

$$S = \frac{8^2}{5^2}+\frac{12^2}{5^2}+\frac{16^2}{5^2}+\ldots 10\text{ terms}$$
$$= \frac{4^2}{5^2}\left(2^2+3^2+4^2+\ldots 10\text{ terms}\right)$$
$$= \frac{16}{25}\left(\frac{11.12.23}{6}-1\right)=\frac{16}{25}\times 505$$
$$\therefore \quad m = 101$$

Q19. Sol. (A)

Given line

$$\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$$

and Given plane is $\ell x + my - z = 9$

Now, it is given that line on plane

$$\therefore 2\ell - m - 3 = 0 \Rightarrow 2\ell - m = 3 \quad \ldots(1)$$

Also, $(3,-2,-4)$ lies on plane

$$3\ell - 2m = 5 \quad \ldots(2)$$

Solving (1) and (2), we get

$$\ell = 1, m = -1$$
$$\therefore \quad \ell^2 + m^2 = 2$$

Q20. Sol. (D)

Given Boolean expression is

$$(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$$
$$(p \wedge \sim q) \vee q = (p \vee q) \wedge (\sim q \vee q) = (p \vee q) \wedge t = (p \vee q)$$

Now,

$$(p \vee q) \vee (\sim p \wedge q) = p \vee q$$

Q21. Sol. (C)

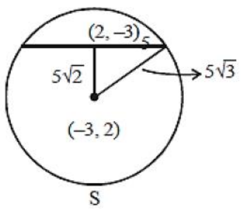
$\div$ by x^{15} in N^r & D^r

$$\int \frac{\left(\frac{2}{x^3} + \frac{5}{x^6}\right)dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3}$$

Let $1 + \frac{1}{x^2} + \frac{1}{X^5} = t \Rightarrow dt = -\left(\frac{2}{x^3} + \frac{5}{x^6}\right)dx$

$$\int \frac{-dt}{t^3} = \frac{1}{2t^2} + c$$

Q22. Sol. (C)



Q23. Sol. (C)

$$e^{\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\ell n\left(1+\frac{r}{n}\right)} = e^{\int_0^2 \ln(1+x)dx}$$

$$\Rightarrow e^{\left((x+1)\{\ell n(x+1)-1\}\right)_0^2} = e^{3\ell n 3-2} = \frac{27}{e^2}$$

Q24. Sol. (A)

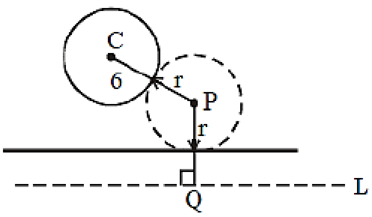
Consider line L at a dist. Of 6 unit below x axis

$\Rightarrow PC = PQ$

$\Rightarrow P$ lies on a parabola

for which C is focus

and L directrix



Q25. Sol. (A)

$$\left(\vec{a}.\vec{c}-\frac{\sqrt{3}}{2}\right)\vec{b}-\left(\vec{a}.\vec{b}+\frac{\sqrt{3}}{2}\right)\vec{c}=0$$

$$\Rightarrow \vec{a}.\vec{b}=\cos\theta=-\sqrt{3}/2 \Rightarrow \theta = 5\pi/6$$

Q26. Sol. (D)

$$p = e^{\lim_{x\to 0^+} \frac{1}{2}\left(\frac{\tan\sqrt{x}}{\sqrt{x}}\right)^2} = \sqrt{e}$$

$$\log p = \frac{1}{2}$$

Q27. Sol. (D)

$$2\cos 2x\cos x + 2\cos 3x\cos x = 0$$

$$\Rightarrow 2\cos x\left(\cos 2x + \cos 3x\right) = 0$$

$$2\cos x 2\cos 5x/2\cos x/2 = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \pi, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$$

7 Solutions

Q28. Sol. (B)

$$x^2 - 5x + 5 = 1 \Rightarrow x = 1, 4$$

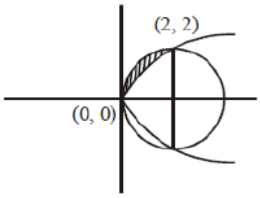
$$x^2 - 5x + 5 = -1 \Rightarrow x = 2, 3$$

but 3 is rejected

$$x^2 + 4x - 60 = 0 \Rightarrow x = -10, 6$$

$$\text{Sum} = 3$$

Q29. Sol. (C)



$$= \frac{\pi(2)^2}{4} - \sqrt{2}\int_0^2 \sqrt{x}dx$$

$$= \pi - \sqrt{2}\cdot\frac{2}{3}2\sqrt{2}$$

$$= \pi - 8/3$$

Q30. Sol. (D)

$$f(x) + 2f(1/x) = 3x \quad \ldots(1)$$

$$x \to \frac{1}{x} \Rightarrow f(1/x) + 2f(x) = 3/x \quad \ldots(2)$$

$$f(x) + 2\left(\frac{3}{x} - 2f(x)\right) = 3x$$

$$\Rightarrow \quad 3f(x) = \frac{6}{x} - 3x$$

$$\Rightarrow \quad f(x) = \frac{2}{x} - x$$

For S $\quad f(x) = f(-x) \Rightarrow \frac{2}{x} - x = 0$

$$\Rightarrow x = \pm\sqrt{2}$$