JEE MAIN - 2016

MATHEMATICS

General Instructions :

1. Immediately fill in the particulars on this page of the test booklet with blue/black ball point pen.

2. This Test Booklet consists of three parts - **Part I, Part II** and **Part III. Part I** has **30** objective type questions of Mathematics Test consisting of **FOUR(4)** marks for each correct response.**Part II** Aptitude Test has **50** objective type questions consisting of **FOUR(4)** marks for each correct response. Mark your answers for these questions in the appropriate space against the number corresponding to the question in the Answer Sheet placed inside this Test Booklet. Use Blue/Black Ball Point Pen only for writing particulars/marking responses of **Side-1** and **Side-2** of the Answer Sheet. **Part III** consists of 2 questions carrying **70** marks which are to be attempted on a separate Drawing Sheet which is also placed inside the Test Booklet. Marks allotted to each question are written against each question. Use colour **pencils or crayons** only on the Drawing Sheet. Do not use water colours. For each incorrect response in **Part I** and **Part II**, **one-fourth (¼)** of the total marks allotted to the question from the total score. **No deduction** from the total score, however, will be made if no response is indicated for an item in the Answer Sheet.

3. There is only one correct response for each question in **Part I** and **Part II**. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 2 above.

4. The test is of 3 hours duration. The maximum marks are 390.

5. On completion of the test, the candidates must hand over the Answer Sheet of Mathematics and **Aptitude Test Part- I & II** and the Drawing Sheet of Aptitude **Test-Part III** along with Test Booklet for Part III to the Invigilator in the Room/Hall. Candidates are allowed to take away with them the Test Booklet of **Aptitude Test -Part I & II**

6. The CODE for this Booklet is S. Make sure that the **CODE** printed on **Side – 2** of the Answer Sheet and on the Drawing Sheet (Part III) is the same as that on this booklet. Also tally the Serial Number of the Test Booklet, Answer Sheet and Drawing Sheet and en sure that they are same. In case of discrepancy in Code or Serial Number, the candidate should immediately report the matter to the Invigilator for replacement of the Test Booklet, Answer Sheet and the Drawing Sheet.

Q1. A value θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is:

(A) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

Q2. The system of linear equation $x+\lambda y-z=0\,\lambda x-y-z=0\,x+y-\lambda z=0$ has a non-trivial solution for :

(A) Exactly three values of λ.

(B) Infinitely many values of λ.

(C) Exactly one value of λ.

(D) Exactly two values of λ.

Q3. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side $=x$ units and a circle of radius $=r$ units. If the sum of the areas of the square and the circle so formed is minimum, then:

(A) $2x=r$

(B) $2x=(\pi+4)r$

(C) $(4-\pi)x=\pi r$

(D) $x=2r$

Q4. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is $30°$. After walking for 10 minutes from A in the same direction, at a point B, he serves that the angle of elevation of the top of the pillar is $60°$. Then the time taken (in minutes) by him, form B to reach the pillar, is:

(A) 5

(B) 6

(C) 10

(D) 20

Q5. Let two fair six-faced dice A and B be thrown simultaneously. If $E1$ is the event that die A shows up four, $E2$ is the event that die B shows up two and $E3$ is the event that the sum of numbers on both dice is odd, then which of the following statements is **NOT true** ?

(A) E_1, E_2 and E_3 are independent.

(B) E_1 and E_2 are independent.

(C) E_2 and E_3 are independent.

(D) E_1 and E_3 are independent.

Q6. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true?

(A) $3a_2 - 23a + 44 = 0$

(B) $3a_2 - 26a + 55 = 0$

(C) $3a_2 - 32a + 84 = 0$

(D) $3a_2 - 34a + 91 = 0$

Q7. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then:

(A) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

(B) g is not differentiable at $x = 0$

(C) $g'(0) = \cos(\log 2)$

(D) $g'(0) = -\cos(\log 2)$

Q8. The distance of the point $(1, -5, 9)$ from the plane $x - y + z = 5$ measured along the line $x = y = z$ is :

(A) $\frac{20}{3}$

(B) $3\sqrt{10}$

(C) $10\sqrt{3}$

(D) $\frac{10}{\sqrt{3}}$

Q9. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is :

(A) $\sqrt{3}$

(B) $\frac{4}{3}$

(C) $\frac{4}{\sqrt{3}}$

(D) $\frac{2}{\sqrt{3}}$

Q10. Let P be the point on the parabola, $y2=8x$ which is at a minimum distance from the center C of the circle, $x2+(y+6)2=1$. Then the equation of the circle, passing through C and having its centre at P is:

(A) $x_2+y_2-4x+9y+18=0$

(B) $x_2+y_2-4x+8y+12=0$

(C) $x_2+y_2-x+4y-12=0$

(D) $x_2+y_2-x4+2y-24=0$

Q11. If $A=\begin{bmatrix}5a & -b\\ 3 & 2\end{bmatrix}$ and $A\text{adj}\,A=AA^T$, then $5a+b$ is equal to:

(A) 13

(B) -1

(C) 5

(D) 4

Q12. Consider $f(x)=\tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x\in\left(0,\frac{\pi}{2}\right)$. A normal to $y=f(x)$ at $x=\frac{\pi}{6}$ also through the point:

(A) $\left(\frac{\pi}{4},0\right)$

(B) $(0,0)$

(C) $0,\frac{2\pi}{3}$

(D) $\left(\frac{\pi}{6},0\right)$

Q13. Two sides of a rhombus are along the lines, $x-y+1=0$ and $7x-y-5=0$. If its diagonals intersect at $(-1,-2)$, then which one of the following is a vertex of this rhombus?

(A) $\left(-\frac{10}{3},-\frac{7}{3}\right)$

(B) $(-3,-9)$

(C) $(-3,-8)$

(D) $\left(\frac{1}{3},\frac{8}{3}\right)$

Q14. If a curve $y=f(x)$ passes through the point $(1,-1)$ and satisfies the differential equation, $y(1+xy)dx=x\,dy$, then $f\left(-\frac{1}{2}\right)$ is equal to :

(A) $\frac{4}{5}$

(B) $-\frac{2}{5}$

(C) $-\frac{4}{5}$

(D) $\frac{2}{5}$

Q15. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is:

(A) 58^{th}

(B) 46^{th}

(C) 59^{th}

(D) 52^{nd}

Q16. If the 2^{nd}, 5^{th} and 9^{th} terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is:-

(A) $\frac{7}{4}$

(B) $\frac{8}{5}$

(C) $\frac{4}{3}$

(D) 1

Q.17. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients off all the terms in this expansion is:-

(A) 729

(B) 64

(C) 2187

(D) 243

Q18. If the sum of the first ten terms of the series

$$\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+\ldots, \text{ is } \frac{16}{5}m,$$

Then m is equals to:-

(A) 99

(B) 102

(C) 101

(D) 100

Q19. If the line, $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane, $1x+my-z=9$, then l^2+m^2 is equals to:-

(A) 2

(B) 26

(C) 18

(D) 5

Q20. The Boolean Expression $(p\wedge\sim q)\vee q\vee(\sim p\wedge q)$ is equivalent to:-

(A) (1) $p\vee\sim q$

(B) (2) $\sim p\wedge q$

(C) (3) $p\wedge q$

(D) (4) $p\vee q$

Q21. The integral $\int\frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3}dx$ is equals to:-

(A) $\frac{-x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

(B) $\frac{-x^5}{\left(x^5+x^3+1\right)^2}+C$

(C) $\frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

(D) $\frac{x^5}{2\left(x^5+x^3+1\right)^2}+C$

Where C is an arbitrary constant.

Q22. If one of the diameters of the circle, given by the equation, $x^2+y^2-4x+6y-12=0$, is a chord of a circle S, whose centre is at $(-3,\ 2)$, then the radius of S is:-

(A) 10

(B) $5\sqrt{2}$

(C) $5\sqrt{3}$

(D) 5

Q23. $\lim\limits_{n\to\infty}\left(\dfrac{(n+1)(n+2)\ldots 3n}{n^{2n}}\right)^{1/n}$ is equal to:-

(A) $3\log 3-2$

(B) $\dfrac{18}{e^4}$

(C) $\dfrac{27}{e^2}$

(D) $\dfrac{9}{e^2}$

Q24. The centres of those circles which touch the circle, $x^2+y^2-8x-8y-4=0$, externally and also touch the x-axis, lie on:-

(A) A parabola

(B) A circle

(C) An ellipse which is not a circle

(D) A hyperbola

Q25. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be there unit vectors such that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\frac{\sqrt{3}}{2}\left(\vec{b}+\vec{c}\right)$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is:-

(A) $\frac{5\pi}{6}$

(B) $\frac{3\pi}{4}$

(C) $\frac{\pi}{2}$

(D) $\frac{2\pi}{3}$

Q26. Let $p=\lim\limits_{x\to 0+}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then $\log p$ is equals to:-

(A) $\frac{1}{4}$

(B) 2

(C) 1

(D) $\frac{1}{2}$

Q27. If $0\le x<2\pi$, then the number of real values of x, which satisfy the equation $\cos x+\cos 2x+\cos 3x+\cos 4x=0$, is:-

(A) 9

(B) 3

(C) 5

(D) 7

Q28. The sum of all real values of x satisfying the equation $(x^2-5x+5)x^2+4x-60=1$ is:-

(A) 5

(B) 3

(C) –4

(D) 6

Q29. The area (in sq. units) of the region $\{(x,y): y^2 \geq 2x \text{ and } x^2+y^2 \leq 4x, x \geq 0, y \geq 0\}$ is:

(A) $\frac{\pi}{2}-\frac{2\sqrt{2}}{3}$

(B) $\pi-\frac{4}{3}$

(C) $\pi-\frac{8}{3}$

(D) $\pi-\frac{4\sqrt{2}}{3}$

Q30. If $f(x)+2f\left(\frac{1}{x}\right)=3x, x\neq 0,$ and $S=\{x\in R: f(x)=f(-x)\}$: then S:

(A) Contains more than two elements

(B) Is an empty set

(C) Contains exactly one element

(D) Contains exactly two elements