JEE MAIN-2017

MATHEMATICS

1. **Sol. (4)**

Here, $D=\begin{vmatrix}1 & 1 & 1\\ 1 & a & 1\\ a & b & 1\end{vmatrix}=0 \Rightarrow a=1$

For $a=1$, the equations become

$$x+y+z=1$$
$$x+y+z=1$$
$$x+by+z=0$$

These equations give no solution for $b=1$

$\Rightarrow S$ is singleton set

2. **Sol. (1)**

$$(p\to q)\to((\sim p\to q)\to q)$$
$$=(p\to q)\to((p\vee q)\to q)$$
$$=(p\to q)\to((\sim p\wedge \sim q)\vee q)$$
$$=(p\to q)\to((\sim p\vee q)\wedge(\sim q\vee q))$$
$$=(p\to q)\to(\sim p\vee q)$$
$$=(p\to q)\to(p\to q)$$
$$=T$$

3. **Sol. (4)**

$$5\left(\sec^2 x-1-\cos^2 x\right)=2\left(2\cos^2 x-1\right)+9$$

$$5\left(\frac{1}{t}-1-t\right)=2(2t-1)+9$$

$$5\left(1-t-t^2\right)=4t^2+7t$$

$$\Rightarrow 9t^2+12t-5=0$$

$$t=\frac{1}{3},-\frac{5}{3}$$

$$\Rightarrow \cos x^2=\frac{1}{3}$$

$$\Rightarrow \cos 2x=\frac{2}{3}-1=-\frac{1}{3}$$

$$\Rightarrow \cos 4x=-\frac{7}{9}$$

4. **Sol. (2)**

$$P(A)+P(B)-2P(A\cap B)=\frac{1}{4}$$

$$P(B)+P(C)-2P(A\cap B)=\frac{1}{4}$$

$$P(A)+P(C)-2P(A\cap C)=\frac{1}{4}$$

$$\Rightarrow \sum P(A)-\sum P(A\cap B)-\frac{3}{8}$$

$$\Rightarrow P(A\cup B\cup C)=\sum P(A)-\sum P(A\cap B)+P(A\cap B\cap C)$$

$$=\frac{3}{8}+\frac{1}{16}=\frac{7}{16}.$$

5. **Sol. (1)**

Determinant simplifies to $3k=\begin{vmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{vmatrix}$

$$=3\begin{vmatrix}1 & 1 & 1\\ 0 & \omega & \omega^2\\ 0 & \omega^2 & \omega\end{vmatrix}$$
$$=-3z$$
$$=k=-z$$

6. **Sol. (4)**

$$\Delta=\frac{1}{2}\left\|\begin{matrix}k & -3k & 1\\ 5 & k & 1\\ -k & 2 & 1\end{matrix}\right\|=28$$
$$\Rightarrow k=2(\text{since } k\in I)$$
$$\Rightarrow \text{Orthocentre is}\left(2,\frac{1}{2}\right)$$

7. **Sol. (3)**

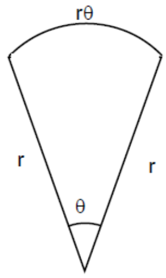
Length of wire $=r(\theta+2)=20\,\text{m}$

Area $A=\frac{\theta}{2}r^2$

$\Rightarrow A(r)=10r-r^2$

$\Rightarrow$ Area is maximum if $r=5$.

Maximum area $A=25\,\text{sq.m}$

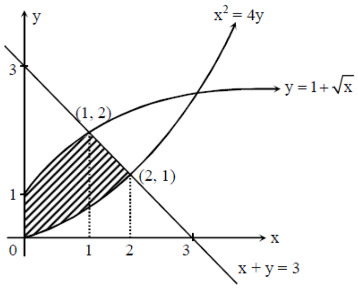


8. **Sol. (4)**

Required area

$$=\int_0^1\left(1+\sqrt{x}\right)dx+\frac{1}{2}(3\times1)-\int_0^2\frac{x^2}{4}dx$$

$$=\frac{5}{2}\text{sq.units}$$



9. **Sol. (2)**

$$\frac{x-1}{1}=\frac{y+2}{4}=\frac{z-3}{5}=\lambda$$

Let midpoint of PQ be M which lines on the plane

$$\Rightarrow M(x, y, z) = (1+\lambda, 4\lambda - 2, 5\lambda + 3)$$
$$2(1+\lambda) + 3(4\lambda - 2) - 4(5\lambda + 3) + 22 = 0$$
$$\Rightarrow -6\lambda + 6 = 0 \Rightarrow \lambda = 1$$
$$\Rightarrow M(2, 2, 8), P(1, -2, 3)$$
$$PM = \sqrt{1+16+25} = \sqrt{42}$$
$$PQ = 2\sqrt{42}.$$

10. **Sol. (1)**

Here, $y = 2\tan^{-1} 3x^{\frac{3}{2}}$

$$\frac{dy}{dx} = \frac{9x^{\frac{1}{2}}}{1+9x^3} = \sqrt{x}g(x)$$
$$\Rightarrow g(x) = \frac{9}{1+9x^3}$$

11. **Sol. (1)**

$$(2+\sin x)dy + \cos x(y+1)dx = 0$$
$$(y+1)(2+\sin x) = C$$
$$\Rightarrow (1+1)(2+0) = C = 4$$
$$(y+1)\cdot(2+\sin x) = 4$$

Put $x = \frac{\pi}{2}$

$$y = \frac{1}{3}$$

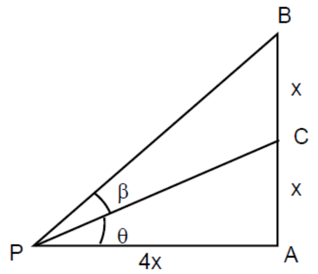
12. **Sol. (3)**

$$\tan(\theta+\beta)=\frac{1}{2} \text{ and } \tan\theta=\frac{1}{4}$$

$$\Rightarrow \frac{\frac{1}{4}+\tan\beta}{1-\frac{1}{4}\tan\beta}=\frac{1}{2}$$

$$\Rightarrow 9\tan\beta=2$$

$$\Rightarrow \tan\beta=\frac{2}{9}$$



13. **Sol. (2)**

$$A^2=\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}$$

$$=\begin{bmatrix}16 & -9\\ -12 & 13\end{bmatrix}$$

$$(3A^2+12A)=\begin{bmatrix}72 & -63\\ -84 & 51\end{bmatrix}$$

$$\text{Adj}(3A^2+12A)=\begin{bmatrix}51 & 63\\ 84 & 72\end{bmatrix}.$$

14. **Sol. (2)**

$$(15a-3b)^2+(15a-5c)^2+(3b-5c)^2=0$$
Let, $15a=3b=5c=45\lambda$
$\Rightarrow a=3\lambda; b=15\lambda; c=9\lambda$
$\Rightarrow 2c=a+b$

b, c, a are in $A.P.$

15. **Sol. (2)**

Equation of plane is $\begin{vmatrix} x-1 & y+1 & z+1 \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix}=0$

$5x+7y+3z+5=0$

Distance from $(1,3,-7)=\frac{|5+21-21+5|}{\sqrt{83}}=\frac{10}{\sqrt{83}}$

16. **Sol. (2)**

$$\begin{aligned} I_n &= \int \tan^n x dx, n>1 \\ &= \int \tan^{n-2} x(\sec^2 x-1)dx \\ &= \frac{\tan^{n-1} x}{n-1} - I_{n-2} + C \end{aligned}$$

$\Rightarrow I_n + I_{n-2} = \frac{\tan^{n-1} x}{n-1} + C$

$\Rightarrow I_6 + I_4 = \frac{\tan^5 x}{5} + C$

Given, $I_4 + I_6 = a\tan^5 x + bx^3 + C$

$\Rightarrow a=\frac{1}{5}, b=0$

17. **Sol. (2)**

Eccentricity, $e = \frac{1}{2}$

Let $2a$ be the length of major axis and $2b$ be the length of minor axis

$\Rightarrow \frac{a}{e} = 4$
$\Rightarrow a = 2$

Also, $b = \sqrt{3}$, as $e = \frac{1}{2}$

$\Rightarrow$ Equation of ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$\Rightarrow$ Equation of normal at $\left(1, \frac{3}{2}\right)$ is $4x - 2y = 1$

*18. **Sol. (2)**

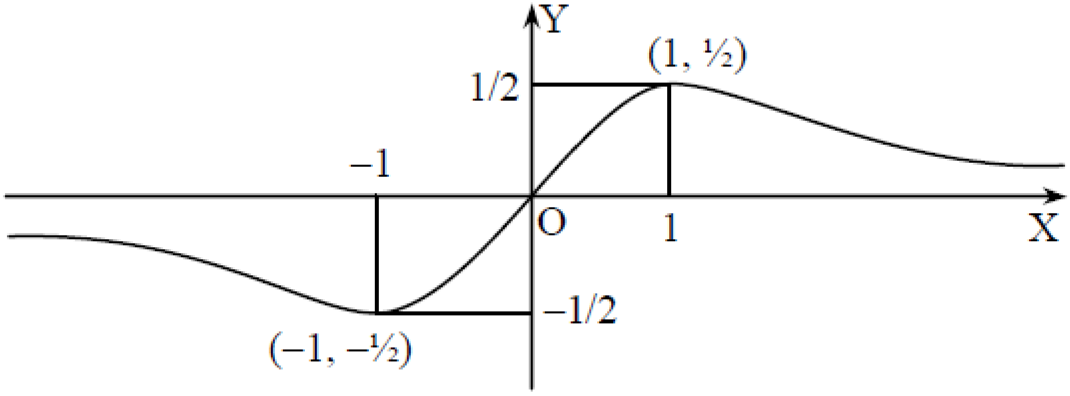
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
$\frac{x^2}{a^2} - \frac{y^2}{4-a^2} = 1$
$\Rightarrow a^2 = 8, 1, \left(a^2 \neq 8\right)$
$\Rightarrow \frac{x^2}{1} - \frac{y^2}{3} = 1.$

Hence equation of tangent at $P\left(\sqrt{2}, \sqrt{3}\right)$ is $\frac{x\sqrt{2}}{1} - \frac{y\sqrt{3}}{3} = 1$

$\Rightarrow \sqrt{6}x - y = \sqrt{3}$

19. **Sol. (3)**

For, $f(x) = \frac{x}{1+x^2}$ the curve has graph as shown



Which is onto but not one-one for, $f : R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$

20. **Sol. (2)**

$$\lim_{x\to\frac{\pi}{2}} \frac{\cos x - \cot x}{8\left(x-\frac{\pi}{2}\right)^3}$$

Put $x - \frac{\pi}{2} = t; x = t + \frac{\pi}{2}$

$$\lim_{t\to 0} \frac{1}{8}\cdot\frac{-\sin t + \tan t}{t^3}$$
$$\lim_{t\to 0} \frac{1}{8}\cdot\frac{\sin t(1-\cos t)}{t\cdot\cos t\cdot t^2}$$
$$= \frac{1}{8}\cdot 1\cdot\frac{1}{2} = \frac{1}{16}$$

21. **Sol. (2)**

$|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a}\cdot\vec{c} = 9$

$\Rightarrow |\vec{c}|^2 - 2\vec{a}\cdot\vec{c} = 0$

and $|\vec{a}\times\vec{b}|\cdot|\vec{c}|\cdot\sin 30^\circ = 3$

$\Rightarrow 3\times|\vec{c}|\times\frac{1}{2} = 3$

$\Rightarrow |\vec{c}| = 2$

$\therefore \vec{a}\cdot\vec{c} = 2$

22. **Sol. (2)**

$\frac{dy}{dx}$ at $x=0$ is 1

$\Rightarrow$ Slope of normal at $(0,1)$ is -1

$\Rightarrow$ Equation of normal is $x+y=1$

23. **Sol. (1)**

Consider two sequences: $0,4,8$ and $2,6,10$

Take both numbers from either of these sequences.

Hence, probability $= \frac{{}^3C_2 + {}^3C_2}{{}^{11}C_2} = \frac{6}{55}$

*24. **Sol. (1)**

$X : 4L, 3M ; Y : 3L, 4M$

Possible combinations

	(1)	(2)	(3)	(4)
X	$3L$	$2L, 1M$	$1L, 2M$	$3M$
Y	$3M$	$1L, 2M$	$2L, 1M$	$3L$

$\therefore$ Number of ways $= {}^4C_3 \cdot {}^4C_3 + {}^4C_2 \cdot {}^3C_1 \cdot {}^3C_1 \cdot {}^4C_2 + {}^4C_1 \cdot {}^3C_2 \cdot {}^3C_2 \cdot {}^4C_1 + {}^3C_3 \cdot {}^3C_3$

$= 485$

*25. **Sol. (4)**

Let $S = \left({}^{21}C_1 - {}^{10}C_1\right) + \left({}^{21}C_2 - {}^{10}C_2\right) + \ldots + \left({}^{21}C_{10} - {}^{10}C_{10}\right)$

$\Rightarrow S = \left({}^{21}C_0 + {}^{21}C_1 + \ldots + {}^{21}C_{10}\right) - \left({}^{10}C_0 + {}^{10}C_1 + \ldots + {}^{10}C_{10}\right)$

$\Rightarrow S = 2^{20} - 2^{10}.$

26. **Sol. (1)**

$p = \frac{15}{25}, q = \frac{10}{25}, n = 10$

$\sigma^2 = npq = 10 \cdot \frac{3}{5} \cdot \frac{2}{5} = \frac{12}{5}.$

27. **Sol. (1)**

Partially differentiating, we get

$f'(x) - x = \text{constant} = \lambda$

$\Rightarrow f(x) = \frac{x^2}{2} + \lambda x + k$

$f(0) = 0 \Rightarrow k = 0$

$\frac{1}{2} + \lambda = 3 \Rightarrow \lambda = \frac{5}{2}$

$\sum_{n=1}^{10} f(n) = a\sum_{n=1}^{10} n^2 + b\sum_{n=1}^{10} n$

$= \frac{1}{2}\frac{n(n+1)(2n+1)}{6} + \frac{5}{2} \times \frac{n(n+1)}{2}$

$= \frac{1}{2} \cdot \frac{10 \times 11 \times 21}{6} + \frac{5}{2} \times \frac{10 \times 11}{2} = 330.$

28. **Sol. (3)**

Let $P\left(\frac{t}{2},4-\frac{t^2}{4}\right)$ be point where circle touches the parabola $y=4-x^2$

$\Rightarrow$ Normal at $P: ty-x+\frac{t^3}{4}-\frac{7t}{2}=0$ to the parabola passes through centre (c) of the circle $(0,\beta)$.

$\Rightarrow t^3-14t+4\beta t=0 \quad \ldots(1)$

Also, radius $r=\frac{|\beta|}{\sqrt{2}}$

$\Rightarrow t^4+4t^2+8\beta t^2-32t^2-128\beta+256+16\beta^2=16r^2$

$\Rightarrow t^4+(8\beta-28)t^2-128\beta+256+8\beta^2=0 \quad \ldots(2)$

From equation (1) and (2), we get

Either $\beta=8\pm4\sqrt{2}$ for $t=0$

or $\beta=\frac{-\sqrt{2}\pm\sqrt{17}}{\sqrt{2}}$ for $t^2=14-4\beta$

As, $r=\frac{|\beta|}{2}\Rightarrow r=4\sqrt{2}\pm4,\frac{\sqrt{17}-\sqrt{2}}{2}$

$\Rightarrow$ Minimum possible radius, $r=\frac{\sqrt{17}-\sqrt{2}}{2}$

[But of the given options $r=4\left(\sqrt{2}-1\right)$ **is minimum]**

29. **Sol. (4)**

$$x^2 + nx + \frac{n^2 - 31}{3} = 0$$

Let I and $I+1$ be the roots of the equation

$$2I + 1 = -n \quad \ldots(1)$$

$$I(I+1) = \frac{n^2 - 31}{3} \quad \ldots(2)$$

Eliminating I from (1) and (2), we get

$$\frac{n^2 - 1}{4} = \frac{n^2 - 31}{3}$$

$$\Rightarrow n^2 = 121$$
$$\Rightarrow n = 11.$$

30. **Sol. (2)**

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left(\operatorname{cosec}^2 x - \operatorname{cosec} x \cdot \cot x\right) dx$$
$$= 2$$