JEE MAIN-2017

MATHEMATICS

General Instructions :

1. Immediately fill in the particulars on this page of the Test Booklet with ***only Black Ball Point Pen*** provided in the examination hall.

2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.

3. The test is of **3 hours** duration.

4. The Test Booklet consists of **90** questions. The maximum marks are **360**.

5. Candidates will be awarded marks as started above in instruction No. 5 for correct response of each question. ¼ (one fourth) marks of the total marks allotted to the question (i.e. 1 mark) will be deducted for indicating incorrect response of each question. No deduction from that total score will be made if no response is indicated for an item in the answer sheet.

6. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.

7. For writing particulars / marking responses on ***Side–1*** and ***Side–2*** of the Answer Sheet use ***only Black Ball Point Pen*** provided in the examination hall.

8. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination room/hall.

9. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in **four** pages (Page **20-30**) at the end of the booklet.

10. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room / Hall. ***However, the candidates are allowed to take away this Test Booklet with them***.

11. The CODE for this Booklet is **D**. Make sure that the CODE printed on **Side–2** of the Answer Sheet and also tally the serial number of the Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.

12. **Do not fold or make any stray mark on the Answer Sheet.**

1. If S is the set of distinct values of 'b' for which the following system of linear equations

$$x+y+z=1$$
$$x+ay+z=1$$
$$ax+by+z=0$$

has no solution, then S is:

(1) an empty set

(2) an infinite set

(3) a finite set containing two or more elements

(4) a singleton

2. The following statement $(p \to q)\left[(\sim p \to q) \to q\right]$ is:

(1) a tautology

(2) equivalent to $\sim p \to q$

(3) equivalent to $p \to \sim q$

(4) a fallacy

3. If $5\left(\tan^2 x - \cos^2 x\right) = 2\cos 2x + 9$, then the value of $\cos 4x$ is :

(1) $-\frac{3}{5}$

(2) $\frac{1}{3}$

(3) $\frac{2}{9}$

(4) $-\frac{7}{9}$

4. For three events A, B and C, P (Exactly one of A or B occurs) $= P$ (Exactly one of B or C occurs) $= P$ (Exactly one of C or A occurs) $= \frac{1}{4}$ and P (All the three events occur simultaneously) $= \frac{1}{16}$. Then the probability that at least one of the events occurs, is:

(1) $\frac{7}{32}$

(2) $\frac{7}{16}$

(3) $\frac{7}{64}$

(4) $\frac{3}{16}$

5. Let ω be a complex number such that $2\omega+1=z$ where $z=\sqrt{-3}$. If
$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to:

(1) $-z$

(2) z

(3) -1

(4) 1

6. Let k be an integer such that the triangle with vertices $(k,-3k),(5,k)$ and $(-k,2)$ has area $28\,\text{sq}$. units. Then the orthocentre of this triangle is at the point:

(1) $\left(2,-\frac{1}{2}\right)$

(2) $\left(1,\frac{3}{4}\right)$

(3) $\left(1,-\frac{3}{4}\right)$

(4) $\left(2,\frac{1}{2}\right)$

7. Twenty meters of wire is available for fencing off a flower–bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is:

(1) 12.5

(2) 10

(3) 25

(4) 30

8. The area (in sq. units) of the region $\left\{(x,y): x\geq 0, x+y\leq 3, x^2\leq 4y \text{ and } y\leq 1+\sqrt{x}\right\}$ is:

(1) $\frac{59}{12}$

(2) $\frac{3}{2}$

(3) $\frac{7}{3}$

(4) $\frac{5}{2}$

9. If the image of the point $P(1,-2,3)$ in the plane, $2x+3y-4z+22=0$ measured parallel to the line, $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$ is Q, then PQ is equal to:

(1) $3\sqrt{5}$

(2) $2\sqrt{42}$

(3) $\sqrt{42}$

(4) $6\sqrt{5}$

10. If for $x \in \left(0,\frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}.g(x)$, then $g(x)$ equals:

(1) $\frac{9}{1+9x^3}$

(2) $\frac{3x\sqrt{x}}{1-9x^3}$

(3) $\frac{3x}{1-9x^3}$

(4) $\frac{3}{1+9x^3}$

11. If $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$ and $y(0)=1$, then $y\left(\frac{\pi}{2}\right)$ is equal to:

(1) $\frac{1}{3}$

(2) $-\frac{2}{3}$

(3) $-\frac{1}{3}$

(4) $\frac{4}{3}$

12. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that $AP = 2AB$. If $\angle BPC = \beta$, then $\tan\beta$ is equal to:

(1) $\frac{6}{7}$

(2) $\frac{1}{4}$

(3) $\frac{2}{9}$

(4) $\frac{4}{9}$

13. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then $adj\left(3A^2 + 12A\right)$ is equal to:

(1) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

(2) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

(3) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

(3) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

14. For any three positive real numbers a, b and $c, \left(25a^2+b^2\right)+25\left(c^2-3ac\right)=15b\left(3a+c\right)$. Then:

(1) b, c and a are in $G.P.$

(2) b, c and a are in $A.P.$

(3) a, b and c are in $A.P.$

(4) a, b and c are in $G.P.$

15. The distance of the point $(1,3,-7)$ from the plane passing through the point $(1,-1,-1)$, having normal perpendicular to both the lines $\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-4}{3}$ and $\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z+7}{-1}$,. is:

(1) $\frac{20}{\sqrt{74}}$

(2) $\frac{10}{\sqrt{83}}$

(3) $\frac{5}{\sqrt{83}}$

(4) $\frac{10}{\sqrt{74}}$

16. Let $I_n = \int \tan^n x\,dx, (n>1)$. If $I_4 + I_6 = a\tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a,b) is equal to:

(1) $\left(-\frac{1}{5},1\right)$

(2) $\left(\frac{1}{5},0\right)$

(3) $\left(\frac{1}{5},-1\right)$

(4) $\left(-\frac{1}{5},0\right)$

17. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is $x=-4$, then the equation of the normal to it at $\left(1,\frac{3}{2}\right)$ is:

(1) $2y-x=2$

(2) $4x-2y=1$

(3) $4x+2y=7$

(4) $x+2y=4$

18. A hyperbola passes through the point $P\left(\sqrt{2},\sqrt{3}\right)$ and has foci at $\left(\pm 2,0\right)$. Then the tangent to this hyperbola at P also passes through the point:

(1) $\left(3\sqrt{2},2\sqrt{3}\right)$

(2) $\left(2\sqrt{2},3\sqrt{3}\right)$

(3) $\left(\sqrt{3},\sqrt{2}\right)$

(4) $\left(-\sqrt{2},-\sqrt{3}\right)$

19. The function $f: R \to \left[-\frac{1}{2},\frac{1}{2}\right]$ defined as $f\left(x\right)=\frac{x}{1+x^2}$, is:

(1) invertible.

(2) injective but not surjective.

(3) surjective but not injective.

(4) neither injective nor surjective.

20. $\lim\limits_{x\to\frac{\pi}{2}} \frac{\cot x-\cos x}{\left(\pi-2x\right)^3}$ equals:

(1) $\frac{1}{24}$

(2) $\frac{1}{16}$

(3) $\frac{1}{8}$

(4) $\frac{1}{4}$

21. Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c}-\vec{a}|=3, \left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right|=3$ and the angle between $\vec{c}$ and $\vec{a}\times\vec{b}$ be $30°$. Then $\vec{a}\cdot\vec{c}$ is equal to:

(1) $\frac{25}{8}$

(2) 2

(3) 5

(4) $\frac{1}{8}$

22. The normal to the curve $y(x-2)(x-3)=x+6$ at the point where the curve intersects the y-axis passes through the point:

(1) $\left(-\frac{1}{2},-\frac{1}{2}\right)$

(2) $\left(\frac{1}{2},\frac{1}{2}\right)$

(3) $\left(\frac{1}{2},-\frac{1}{3}\right)$

(4) $\left(\frac{1}{2},\frac{1}{3}\right)$

23. If two different numbers are taken from the set $\{0,1,2,3,\ldots,10\}$;then the probability that their sum as well as absolute difference are both multiple of 4, is:

(1) $\frac{6}{55}$

(2) $\frac{12}{55}$

(3) $\frac{14}{45}$

(4) $\frac{7}{55}$

24. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is:

(1) 485

(2) 468

(3) 469

(4) 484

25. The value of
$\left({}^{21}C_1 - {}^{10}C_1\right) + \left({}^{21}C_2 - {}^{10}C_2\right) + \left({}^{21}C_3 - {}^{10}C_3\right) + \left({}^{21}C_4 - {}^{10}C_4\right) + \ldots + \left({}^{21}C_{10} - {}^{10}C_{10}\right)$ is:

(1) $2^{21} - 2^{11}$

(2) $2^{21} - 2^{10}$

(3) $2^{20} - 2^{9}$

(4) $2^{20} - 2^{10}$

26. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is:

(1) $\frac{12}{5}$

(2) 6

(3) 4

(4) $\frac{6}{25}$

27. Let $a, b, c \in R$. If $f(x) = ax^2 + bx + c$ is such that $a + b + c = 3$ and $f(x+y) = f(x) + f(x) + f(y) + xy, \forall x, y \in R$, then $\sum_{n=1}^{10} f(n)$ is equal to:

(1) 330

(2) 165

(3) 190

(4) 255

28. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is:

(1) $2\left(\sqrt{2}+1\right)$

(2) $2\left(\sqrt{2}-1\right)$

(3) $4\left(\sqrt{2}-1\right)$

(4) $4\left(\sqrt{2}+1\right)$

29. If, for a positive integer n, the quadratic equation, $x(x+1)+(x+1)(x+2)+\ldots+(x+\overline{n-1})(x+n)=10n$ has two consecutive integral solutions, then n is equal to:

(1) 12

(2) 9

(3) 10

(4) 11

30. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to:

(1) -2

(2) 2

(3) 4

(4) -1