JEE MAIN-2018

PART-I MATHEMATICS (B.Arch.)

1. Ans. (2)

Sol.

$f(x)+2f(1-x)=x^2-1 \quad \ldots(\text{i})$

$x \to 1-x;$

$f(1-x)+2f(x)=(1-x)^2+1 \quad \ldots(\text{ii})$

Solving (i) & (ii)

$f(x)+2f(1-x)=x^2+1$

$2f(x)+f(1-x)=(1-x)^2+1)\times 2$

$-3f(x)=(x^2+1)-2\left[(1-x)^2+1\right]$

$=(x^2+1)-2(1+x^2-2x+1)$

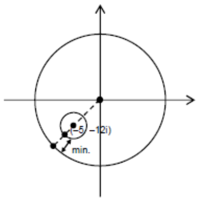
$-3f(x)=-x^2+4x-3$

$f(x)=-x^2+4x-3$

$f(x)=1/3(x^2-4x+3) \quad \therefore \quad f(x)\in[-1/3,\infty)$

2. Ans. (3)

Sol.



$\therefore \quad |Z - w|_{\min} = 8$

3. Ans. (2)

Sol.

$x^2 - 5kx + 2e^{2\log_e |k|} - 1 = 0$

$x^2 - 5kx + (2k^2 - 1) = 0 \rightarrow (\alpha, \beta)$

$\alpha + \beta = 5k$

$\alpha\beta = 2k^2 - 1 = 49$

$\therefore \quad k = \pm 5$

$\therefore \quad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (25)^2 - 98$

$= 527$

4. Ans. (4)

Sol.

$$|\text{adj}(2A)| = |2A|^{n-1} = 2^{n(n-1)}.|A|^{n-1}$$

$$|A| = 8$$

$$\therefore \quad |\text{adj}(2A)| = 2^6.8^2 = 2^{12} = 4096$$

5. Ans. (4)

Sol.

$$\lambda x + y + z = 5\lambda$$

$$2\lambda x + 2y - z = 1$$
$$3y + z = 9$$

These is no such value of 'λ' for which

$$\Delta = \Delta x = \Delta y = \Delta z = 0$$

Hence, S is an empty set.

6. Ans. (3)

Sol.

Number of ways in which he can fail

$$^9C_5 + {}^9C_6 + {}^9C_7 + {}^9C_8 + {}^9C_9$$
$$= 126 + 84 + 39 + 9 + 1$$
$$= 256$$

7. Ans. (4)

Sol.

$T_r = {}^{50}C_{r-1}.a^{51-r}$

$T_{25} + T_{27} = \left(\frac{125}{52}\right).T_{26}$

$\Rightarrow {}^{50}C_{24}.a^{26} + {}^{50}C_{26}.a^{24} = \left(\frac{125}{52}\right){}^{50}C_{25}.a^{25}$

$\Rightarrow a^2 + 1 = \left(\frac{125}{52}\right)\left(\frac{{}^{50}C_{25}}{{}^{50}C_{24}}\right).a$

$\Rightarrow a^2 + 1 = \frac{5a}{2}$

$\Rightarrow 2a^2 - 5a + 2 = 0 \rightarrow (a_1, a_2) \quad \therefore \quad a_1 + a_2 = \frac{5}{2}$

8. Ans. (2)

Sol.

Let $a, \underset{\substack{\downarrow \\ A.P.}}{\frac{16}{a}}, \frac{28}{a}, \frac{49}{a} \rightarrow$ four numbers

$\therefore \quad \frac{32}{a} = a + \frac{28}{a} \quad \Rightarrow \quad a^2 + 28 = 32 \quad \Rightarrow \quad a^2 = 4$

$\therefore$ product of 2^{nd} and 3^{rd} $= \left(\frac{16}{a}\right)\left(\frac{28}{a}\right) = \frac{16 \times 28}{4} = 112$

9. Ans. (1)

Sol.

$$e^{\left(\sin^2 x+\sin^4 x+\sin^6 x+\ldots\ldots\infty\right).\log_e 2}$$

$$=e^{\left(\frac{\sin^2 x}{1-\sin^2 x}\right)}.\log_e 2=e^{\tan^2 x.\log_e 2} \quad =2^{\tan^2 x}$$

$$y^2-5y+4=0$$

$$(y-1)(y-4)=0$$

$$y=1,4$$

$$\therefore 2^{\tan^2 x}=2^0,2^2$$

$$\tan^2 x=0,2$$

$$\therefore \tan^2 x=\sqrt{2} \quad \left[\because x\in(0,\pi/2)\right]$$

$$\frac{\sin x}{\cos x-\sin x}=\frac{\tan x}{1-\tan x}=\frac{\sqrt{2}}{1-\sqrt{2}}=-\left(2+\sqrt{2}\right)$$

10. Ans. (2)

Sol.

$$f(x)=x\left[\frac{1}{x}\right]=x\left(\frac{1}{x}-\left\{\frac{1}{x}\right\}\right)=1-x\left\{\frac{1}{x}\right\}$$

$$=\lim_{x\to 0^+} f(x)=1, \lim_{x\to 1/3^+} f(x)=1, \lim_{x\to \frac{1}{2}^-} f(x)=\frac{1}{2}, \lim_{x\to 2^-} f(x)=0$$

11. Ans. (3)

Sol.

$$f(x)=\begin{cases}\dfrac{(9^x-1)(8^x-1)}{\sqrt{2}-\sqrt{1+\cos x}}; x\neq 0\\ k\sqrt{2}\log_e 2\log_e 3, x=0\end{cases}$$

$$=\lim_{x\to 0}\frac{(9^x-1)(8^x-1)}{\sqrt{2}-\sqrt{1+\cos x}}=\lim_{x\to 0}\frac{\left(\frac{(9^x-1)}{x}\right)\left(\frac{(8^x-1)}{x}\right)}{\left(\frac{1-\cos x}{x^2}\right)}.\left(\sqrt{2}+\sqrt{1+\cos x}\right)$$

$$\Rightarrow k\sqrt{2}\ \ell n2.\ell n3=2(\ell n9)(\ell n8)\times 2\sqrt{2}\quad \Rightarrow k=24$$

12. Ans. (1)

Sol.

$y.\text{cosx}+x.\cos y=\pi;\quad y(0)=\pi$

$y'.\text{cosx}+y.(-\sin x)+x.(-\sin y).y^1+\cos y=0$

put, $x=0\Rightarrow y'+\cos y=0$

$y'=1$

$(\cos x-x.\sin y)y'=\text{sinx}.\text{y}-\cos y$

$\Rightarrow(\cos x-x.\sin y)y''+(-\text{sinx}.x-x\,\text{cosy}.y'-\sin y)=\cos x.y+\sin x.y'+\sin y.y'$

put, $x=0;$

$y''(0)=\pi$

13. Ans. (1)

Sol.

$f(x) = |x-1|$

$g(x) = \cos x$

$\phi(x) = \mathrm{f}(g(2\sin x)) - g(f(x)) = c|\cos(\sin x) - 1| - \cos|x-1|$

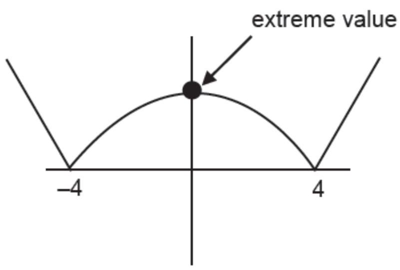
$1 - \cos(2\sin x) - \cos|x-1|$

differentiable at each point of R.

14. Ans. (1)

Sol.

$f(x) = |x^2 - 16|$



15. Ans. (3)

Sol.

$$I=\int\frac{e^{x}}{e^{4x}+e^{2x}+1}$$

$$I=\int\frac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx$$

$$=\int\frac{e^{3x}}{e^{4x}+e^{x}+1}dx$$

$$J-I=\int\frac{e^{3x}-e^{x}}{e^{4x}+e^{2x}+1}dx$$

$$J-I=\int\frac{e^{x}\left(e^{2x}-1\right)}{e^{4x}+e^{2x}+1}dx$$

$$e^{x}=t$$

$$J-I=\int\frac{t^{2}-1}{t^{4}+t^{2}+1}dt$$

$$J-I=\int\frac{1-\frac{1}{t^{2}}}{t^{2}+1-\frac{1}{t^{2}}}dt$$

$$J-I=\int\frac{\left(1-\frac{1}{t^{2}}\right)dt}{\left(t+\frac{1}{t}\right)^{2}-1}$$

$$t+\frac{1}{t}=z$$

$$\left(1-\frac{1}{t^{2}}\right)dt=dz$$

$$J - I = \int \frac{dz}{z^2 - 1}$$

$$= \frac{1}{2} \text{In} \left| \frac{z-1}{z+1} \right| + c$$

$$= \frac{1}{2} \text{In} \left| \frac{t + \frac{1}{t} - 1}{t + \frac{1}{t} + 1} \right| + c$$

$$= \frac{1}{2} \text{In} \left| \frac{t^2 - t + 1}{t^2 + t + 1} \right| + c$$

$$= \frac{1}{2} \text{In} \left| \frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1} \right| + c$$

16. Ans. (4)

Sol.

Use formula

$$= \int_0^{\pi/4} \frac{1 + \cos 4\theta}{2} d\theta + \int_0^{\pi/4} \frac{\cos 6\theta - 3\cos 2\theta}{4} d\theta$$

$$= \frac{\pi}{8} + \frac{1}{3} = m\pi + n$$

$$(m, n) = \left(\frac{1}{8}, \frac{1}{3} \right)$$

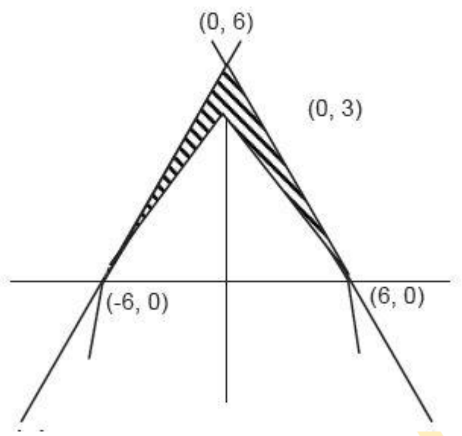
option (4)

17. Ans. (1)

Sol.

$$x^2 = 36 - 12y$$

$$x^2 = -12(y-3)$$



Required Area $=$

$$= 2\left(\frac{1}{2}\times 6\times 6 - \int_0^6 \frac{36-x^2}{12}dx\right) = 2\left(18 - \frac{1}{2}\left(36\times 6 - \frac{216}{3}\right)\right)$$

$$= 2\left(18 - \frac{1}{12}(216-72)\right) = 2(18-12) = 12$$

18. Ans. (3)

Sol.

$$x\ln x\frac{dy}{dx} + y = 3x\ln x$$

$$\frac{dy}{dx} + \frac{1}{x\ln x}y = 3$$

$$IF = e^{\int \frac{1}{x\ln x}dx}$$

$$= \ln x$$

$$y.\ln x=\int 3.\ln x\,dx+6$$

$$y.\ln x=3.x(\ln x-1)+6$$

$$y(e)=0$$

$$0=3e(0)+1$$

$$C=0$$

$$y.\ln x=3.x(\ln x-1)$$

$$y(e^2)=?$$

$$2y=3e^2(2-1)$$

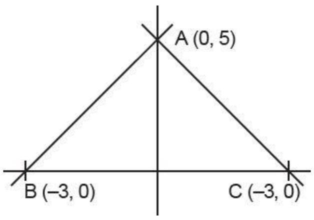
$$y=\frac{3}{2}e^2$$

19. Ans. (4)

Sol.

$$5x-3y+15=0$$

$$5x+3y-15=0$$



Circumcentre $(0,9/5)$

$$\text{Radius}=\frac{16}{5}$$

20. Ans. (3)

Sol.

$x^2+y^2-10x-10y=0$

$C(5,5)$

and Radius $=5\sqrt{2}$

Image of centre of circle in

$x+y+5=0$

$\frac{x-5}{1}=\frac{y-5}{1}=\frac{-2(15)}{2}$

$C'(-10,-10)$

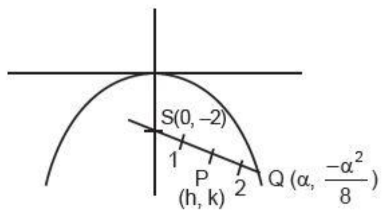
Eqn. $(x+10)^2+(y+10)^2=50$

Passing through $(-3,-11)$

21. Ans. (1)

Sol.

$x^2=-8y$



$$\frac{SP}{PQ}=\frac{1}{2} \quad \Rightarrow \quad h=\frac{\alpha}{3} \text{ and } k=\frac{\frac{-\alpha^2}{8}-4}{3}$$

$$\alpha=3h \text{ and } 3k\frac{-\alpha^2}{8}-4$$

$$3k=-\frac{9h^2}{8}-4$$

$$24k=-9h^2-32$$

$$9h^2+24k+32=0$$

$$9x^2+29y+32=0$$

22. Ans. (2)

Sol.

Hyperbola

$$\frac{x^2}{6}-\frac{y^2}{6\cos^2\theta}=1$$

$$\because \quad b^2=a^2\left(e_1^2-1\right)$$

$$6\cos^2\theta=6\left(e_1^2-1\right)$$

$$e_1=\sqrt{1+\cos^2\theta}$$

and ellipse

$$\frac{x^2}{30\cos^2\theta}+\frac{y^2}{30}=1$$

$\because \quad b>a$

$$30\cos^2\theta=30\left(1-e_2^2\right)$$

$$e_2=\sqrt{1-\cos^2\theta}$$

given $e_1=\sqrt{3}e_2$

$$\sqrt{1+\cos^2\theta}=\sqrt{3}\sqrt{1-\cos^2\theta}$$

Squaring

$$1+\cos^2\theta=3\left(1-\cos^2\theta\right)$$

$$1+\cos^2\theta=3-3\cos^2\theta$$

$$4\cos^2\theta=2$$

$$\cos^2\theta=\frac{1}{2} \quad \Rightarrow \quad \theta=\frac{\pi}{4}$$

23. Ans. (2)

Sol.

$\frac{x-1}{4}=\frac{y+3}{-2}=\frac{z+5}{1}$ lies in plane $2x+\ell y+mz=16$

$\because$ point $(1,-3,-5)$ lies in plane

$$2-3\ell-5m=16$$

$$3\ell+5m=-14 \qquad \ldots(1)$$

and

$$8-2\ell+m=0$$

$$2\ell-m=8 \qquad \ldots(2)$$

solving $\ell=2, m=-4$

$$\ell^2+m^2=20$$

24. Ans. (2)

Sol.

$$2x-3y+4z=1 \qquad \ldots(1)$$
$$x-y+4=0 \qquad \ldots(2)$$

not plane passes through line of intersection of plane (1) and plane (2)

$$(2x-3y+4z-1)+\lambda(x-y+4)=0$$
$$(2+\lambda)x-(3+\lambda)y+4z+(4\lambda-1)=0 \qquad \ldots(3)$$

plane (3) is perpendicular to plane

$$2x-y-z+4=0 \qquad \ldots(4)$$

so $2(2+\lambda)+(1)(3+\lambda)-4=0$

$$2\lambda+4+\lambda+3-4=0$$
$$3\lambda=-3$$
$$\lambda=-1$$

so required plane

$$x-2y+4z-5=0$$
$$\Rightarrow \vec{r}.\left(i-2j+4\hat{k}\right)=5$$

25. Ans. (2)

Sol.

$$|a|=|\hat{b}|=|C|=1$$

$\hat{b}$ is not parallel to $\hat{c}$

$$a\times\left(\hat{b}\times\hat{c}\right)=\frac{1}{2}\hat{b}+\frac{1}{2}\hat{c}\quad \left(a.\hat{c}\right)\hat{b}-\left(a.\hat{b}\right)\hat{c}=\frac{1}{2}\hat{b}+\frac{1}{2}\hat{c}$$

$$-\left(a.\hat{b}\right)=\frac{1}{2}\quad \text{let } a\wedge\hat{b}=\theta$$

$$-\left|a\right|\left|\hat{b}\right|\cos\theta=\frac{1}{2}\quad\Rightarrow\quad -\cos\theta=\frac{1}{2}$$

$$\cos\theta=-\frac{1}{2}\quad\Rightarrow\quad \theta=\frac{2\pi}{3}$$

26. Ans. (1)

Sol.

$$P\left(x=0\right)=\frac{{}^{2}C_{2}}{{}^{8}C_{2}}-\frac{1}{28}$$
$$P\left(x=1\right)=\frac{{}^{6}C_{1}\times{}^{2}C_{1}}{{}^{8}C_{2}}-\frac{12}{28}$$
$$P\left(x=2\right)=\frac{{}^{6}C_{2}}{{}^{8}C_{2}}-\frac{15}{28}$$
$$E\left(x\right)=\frac{\sum p_i x_i}{\sum p_i}=\frac{0+1.\frac{12}{28}+2.\frac{15}{28}}{1}=\frac{12+30}{28}=\frac{42}{28}=\frac{3}{2}$$

27. Ans. (2)

Sol.

$$P(\text{Even}) = 3P(\text{odd})$$
$$P(E) + P(0) = 1$$
$$3P(0) + P(0) = 1$$
$$P(0) = \frac{1}{4}$$
$$P(E) = \frac{3}{4}$$

$$\text{Now } P(\text{sum even}) = P(EE) + P(oo)$$

$$= \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} \times \frac{1}{4}$$

$$= \frac{10}{16} = \frac{5}{8} \ \text{Ans } 2$$

28. Ans. (3)

Sol.

$$x \to [0, 2\pi]$$

$$4(\cos^{10} x + \sin^2 x) = 4 + \sin^6 x \sin^2 2x$$
$$4\cos^{10} x - 4\cos^2 x = \sin^6 x(4\sin^2 x \cos^2 x)$$
$$\cos^{10} x - \cos^2 x - \sin^6 x \cos^2 x = 0$$
$$\cos^2 x[\cos^8 x - \sin^8 x - 1] = 0$$
$$\cos^2 x[\cos^8 x - (\sin^8 x + 1)] = 0$$
$$\cos^2 x = 0 \quad \cos^8 x = \sin^8 x + 1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \cos^8 x = 1 \quad \text{and} \sin^8 x = 0$$
$$x = 0, \pi, 2\pi \quad x = 0, \pi, 2\pi$$

so number of solutions $= 5$.

29. Ans. (1)

Sol.

$$\tan\left(\frac{1}{2}\left(\sin^{-1}\frac{4}{5}+\cos^{-1}\frac{15}{17}\right)\right)$$

$$\tan\left(\frac{1}{2}\left(\tan^{-1}\frac{4}{5}+\tan^{-1}\frac{8}{15}\right)\right)$$

$$\tan\left(\frac{1}{2}\tan^{-1}\frac{\left(\frac{4}{3}+\frac{8}{15}\right)}{1-\frac{4}{3}\times\frac{8}{15}}\right)$$

$$\tan\left(\frac{1}{2}\tan^{-1}\left(\frac{60+24}{45-32}\right)\right)$$

$$\tan\left(\frac{1}{2}\tan^{-1}\left(\frac{18}{13}\right)\right)$$

$$\tan(0)$$

$$=\frac{6}{7}$$

Ans. 1

Let $\frac{1}{2}\tan^{-1}\frac{84}{13}=\theta$

$$\tan^{-1}\frac{84}{13}=2\theta$$

$$\tan 2\theta=\frac{84}{13}$$

$$\frac{2\tan\theta}{1-\tan^2\theta}=\frac{84}{13}$$

$$13\tan\theta=42\left(1-\tan^2\theta\right)$$

$$42\tan^2\theta + 13\tan\theta - 42 = 0$$

$$(7\tan\theta - 6)(6\tan\theta + 7) = 0$$

$$\tan\theta = \frac{6}{7} \text{ or } -\frac{7}{6}$$

(Not possible)

30. Ans. (2)

Sol.

q	$\sim q$	$p \wedge q$	$(\sim q) v p$	$(p \wedge q) v (\sim q) v p$
T	F	T	T	T
F	T	F	T	T
T	F	F	F	F
F	T	F	T	T

column V and VI are identical. So option (2) correction

Part-II Aptitute Test

31. Ans. (2)

32. Ans. (3)

33. Ans. (4)

34. Ans. (2)

35. Ans. (2)

36. Ans. (3)

37. Ans. (1)

38. Ans. (4)

39. Ans. (3)

40. Ans. (1)

41. Ans. (4)

42. Ans. (2)

43. Ans. (1)

44. Ans. (3)

45. Ans. (1)

46. Ans. (1)

47. Ans. (2)

48. Ans. (1)

49. Ans. (2)

50. Ans. (4)

51. Ans. (2)

52. Ans. (4)

53. Ans. (2)

54. (In this answer are doubtful (2) OR Bonus)

55. Ans. (1)

56. Ans. (3)

57. Ans. (2)

58. Ans. (3)

59. Ans. (2)

60. Ans. (4)

61. Ans. (2)

62. Ans. (1)

63. Ans. (4)

64. Ans. (1)

65. Ans. (2)

66. Ans. (2)

67. Ans. (3)

68. Ans. (3)

69. Ans. (2)

70. Ans. (1)

71. Ans. (4)

72. Ans. (3)

73. Ans. (3)

74. Ans. (2)

75. Ans. (2)

76. Ans. (2)

77. Ans. (3)

78. Ans. (4)

79. Ans. (3)

80. Ans. (2)