JEE MAIN-2018

MATHEMATICS

31. Sol. (2)

$T.\text{at}\,(1,7)\,\text{for}\ x^2 = y-6\ \text{is}\ x.1 = \frac{1}{2}(y+7)-6$

$\Rightarrow\quad 2x = y+7-12 \quad\Rightarrow\quad 2x-y+5=0$

$\text{centre}(-8,-6)$

$\therefore$ foot of perpendicular is $\frac{x+8}{2} = \frac{y+6}{-1} = \frac{(-16+6+5)}{4+1}$

$\Rightarrow\quad \frac{x+8}{2} = \frac{y+6}{-1} = \frac{5}{5} = 1$

$x = 2-8 = -6, y = -6-1 = -7,$

$x^2 + y^2 + 16x + 12y + c = 0$

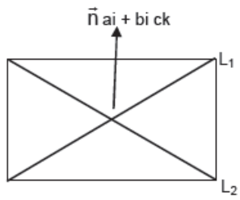
$36+49-96-84+c=0$

$c = 95$

32. Sol. (4)

$L_1 : L_1 : \begin{cases} 2x-2y+3z-2=0 \\ x-y+z+1=0 \end{cases}$

Let a point on $L_1(0,5,4)$ and dr; s of L_1 be a,b,c



$2a_1 + 2b_1 + 3c_1 = 0$

$$\frac{\begin{array}{l} a_1+b_1+c_1=0 \end{array}}{\frac{a_1}{1}=\frac{b_1}{1}=\frac{c_1}{0}}$$

so dr's of L_2 be a_2, b_2, c_2

dr's of L_2 can be $3, -5, -7$

so dr's of normal to the plane ca be

$$a+b+oc=0$$
$$3a-5b-7c=0$$
$$\frac{a}{-7}=\frac{b}{7}=\frac{c}{-8}$$

equation req. plane $7x-7(y-5)+8(z-4)=0$

$7x-7y+8z+3=0$

so req. distance $=\frac{3}{\sqrt{49+49+64}}=\frac{3}{\sqrt{162}}=\frac{3}{9\sqrt{2}}=\frac{1}{3\sqrt{2}}$

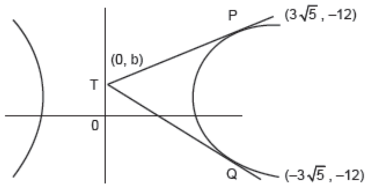
33. Sol. (1)

$$x^2-x+1=0$$
$$x=\frac{1\pm i\sqrt{3}}{2}$$ **(**let $-\omega$ and $-\omega^2$**)**

$$\alpha^{101}+\beta^{107}=(-\omega)^{101}+(-\omega^2)^{107}$$
$$=-\omega^{101ss}-\omega^{214} \quad =-\omega^2-\omega=1$$

34. Sol. (3)

$$\frac{x^2}{9}-\frac{y^2}{36}=1$$



Case-1

$$\frac{0\times x}{9}-\frac{3.y}{36}=1 \Rightarrow \frac{-y}{12}=1 \Rightarrow y=-12$$

$$\frac{22}{9}-\frac{144}{36}=1 \Rightarrow \frac{x^2}{9}=\frac{180}{36} \Rightarrow x=\pm 3\sqrt{5}$$

$$A=\frac{1}{2}\begin{vmatrix} 0 & 3 & 1 \\ 3\sqrt{5} & -12 & 1 \\ -3\sqrt{5} & -12 & 1 \end{vmatrix}$$

$$=\frac{1}{2}\left[-3\left(6\sqrt{5}\right)-36\sqrt{5}-36\sqrt{5}\right]=\frac{1}{2}\left[-18\sqrt{5}-36\sqrt{5}-36\sqrt{5}\right]$$

$$=\frac{1}{2}\times 90\sqrt{5}=45\sqrt{5}$$

35. Sol. (2)

$$y^2=6x \text{ and } 9x^2+by^2=16$$

$$2y\frac{dy}{dx}=6 \Rightarrow \frac{dy}{dx}=\frac{3}{y}$$

$$18x+2by\frac{dy}{dx}=0$$

$$9x+by\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=\frac{-9x}{by}$$

$$\frac{3}{y}\times\frac{-9x}{by}=-1$$

$$(b)6x=27x$$

$$b=\frac{27}{6}\Rightarrow b=\frac{9}{2}$$

36. Sol. (4)

$$D=0$$

$$\begin{vmatrix}1 & k & 3\\ 3 & k & -2\\ 2 & 4 & -3\end{vmatrix}=0$$

$$-3k+8-k(-9+4)+3(12-2k)=0$$

$$-3k+8-5k+36-6k=0$$

$$-4k=-44 \quad k=11$$

$$x+11y+3z=0$$

$$3x+11y+2z=0$$

$$2x+4y-3z=0$$

$$z=t$$

$$x+11y=-3t$$

$$3x+11y=2t$$

$$2x=5t$$

$$x=\frac{5t}{2}$$

$$y=\frac{-3z-x}{11}=\frac{-3t-\frac{5t}{2}}{11}=\frac{-11t}{2\times 11}=\frac{-t}{2}$$

$$\frac{xz}{y^2}=\frac{\frac{5t}{2}\times t}{\frac{t^2}{4}}=\frac{5}{2}\times 4=10$$

37. Sol. (1)

$$2\left|\sqrt{x}-3\right|+\sqrt{x}\left(\sqrt{x}-6\right)+6=0$$

case I $\sqrt{x}\geq 3\Rightarrow 2\sqrt{x}-6+x-6\sqrt{x}+6=0$

$$\Rightarrow x-4\sqrt{x}=0$$
$$\Rightarrow \sqrt{x}=4 \quad \Rightarrow \sqrt{x}=16$$

case II

$$\sqrt{x}<3\Rightarrow -2\sqrt{x}+6+x-6\sqrt{x}+6=0$$
$$\Rightarrow x-8\sqrt{x}+12=0 \quad \Rightarrow\left(\sqrt{x}-6\right)\left(\sqrt{x}-2\right)=0\Rightarrow \sqrt{x}=2\Rightarrow x=4$$

38. Sol. (4)

$$8\cos x\left(\left(\cos^2\frac{\pi}{6}-\sin^2 x\right)-\frac{1}{2}\right)=1$$
$$8\cos x\left(\left(\frac{3}{4}-\sin^2 x\right)-\frac{1}{2}\right)=1$$

$$6\cos x-8\cos x\left(\sin^2 x\right)-4\cos x=1$$
$$6\cos x-8\cos x\left(1-\cos^2 x\right)-4\cos x-1=0$$
$$8\cos^3 x-6\cos x-1=0$$
$$2\left(4\cos^3 x-3\cos x\right)=1$$

$$\cos 3x=\frac{1}{2}$$
$$3x=2n\pi\pm\frac{\pi}{3}$$
$$x=(6n\pm 1)\frac{\pi}{9}$$
$$n=0, x=\frac{\pi}{9}$$
$$n=1, x=\frac{7\pi}{9},\frac{5\pi}{9}$$
$$s=\frac{13\pi}{9}$$
$$k=\frac{13}{9}$$

39. Sol. (4)

$$4R+6B=10$$

$$p=\frac{4}{10}.\frac{6}{12}+\frac{6}{10}.\frac{4}{12}$$

$$=\frac{24}{120}+\frac{24}{120}=\frac{2}{5}$$

40. Sol. (2)

$$f(x)x^2+\frac{1}{x^2},g(x)=x=\frac{1}{x}$$

$$h(x)=\frac{f(x)}{g(x)}=\frac{x^2+\frac{1}{x^2}}{x-\frac{1}{x}}=\frac{\left(x-\frac{1}{x}\right)^2+2}{\left(x-\frac{1}{x}\right)}$$

$$x-\frac{1}{x}=t$$

$$h(t)=\frac{t^2+2}{t}=t+\frac{2}{t}|t|\geq 2$$

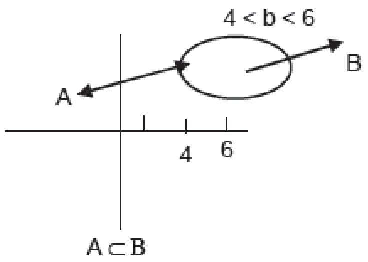
$$AM\geq GM.\frac{t+\frac{2}{t}}{2}\geq\sqrt{t.\frac{2}{t}}$$

$$t+\frac{2}{t}\geq 2\sqrt{2}$$

41. Sol. (4)

$-1 < a - 5 > 1$

$4 < a < 6$



$$\frac{(a-6)^2}{3^2} + \frac{(b-5)^2}{2^2} = 1$$

It passes through $(4,6) \Rightarrow \frac{16+9-36}{36} = \frac{25-36}{36} < 0$

42. Sol. (3)

$= \sim (p \vee q) \vee (\sim p \wedge q)$

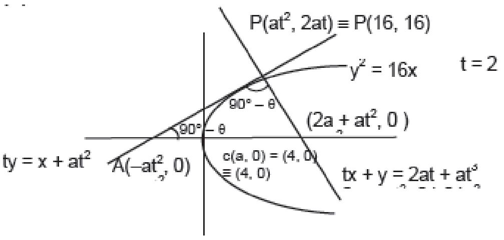
$= (\sim p \vee \sim q) \vee (\sim p \wedge q)$

$= \sim \text{p} \wedge (\sim q \vee q)$

$= \sim p \wedge (t)$

$= \sim p$

43. Sol. (4)



$\angle CPB = \theta$

Hence $\angle APC = 90 - \theta \Rightarrow \quad \angle PAC = 90 - \theta$

now tangent slope $= \tan(90 - \theta) = \frac{1}{t} \Rightarrow \tan\theta = 2$

44. Sol. (1)

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$$

$$\Rightarrow \begin{vmatrix} 5x-4 & 2x & 2x \\ 5x-4 & x-4 & 2x \\ 5x-4 & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$$

$$\Rightarrow (5x-4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x-4 & 2x \\ 1 & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$$

$$\Rightarrow (5x-4)\begin{vmatrix} 1 & 2x & 2x \\ 0 & -x-4 & 2x \\ 1 & 2x & -x-4 \end{vmatrix} = (A+Bx)(x-A)^2$$

$$(5x-4)(x+4)^2 = (A+Bx)(x-A)^2$$

$$A=-4, B=5$$

45. Sol. (2)

$$\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$$

$$=(T_1+T_2+T_3+T_4+T_5+T_6)+(T_1-T_2+T_3-T_4+T_5-T_6)$$

$$=2(T_1+T_3+T_5)$$

$$=2\left({}^5C_0(x)^5+{}^5C_2(x)^3\left(\sqrt{x^3-1}\right)^2+{}^5C_4(x)^1\left(\sqrt{x^3-1}\right)^4\right.$$

$$=2\left(x^5+10x^3\left(x^3-1\right)+5x\left(x^6+1-2x^3\right)\right]$$

$$=2\left(x^5+10x^6-10x^3+5x^7+5x-10x^4\right)$$

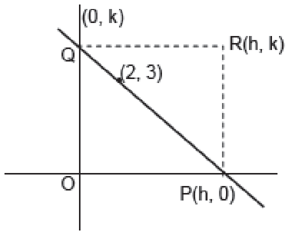
$$=2\left(5x^7+10x^6+x^5-10x^4-10x^3+5x\right)$$

sum of odd degree terms $=10+2-20+10=2$

46. Sol. (1)

$$a_1 + a_5 + a_9 + a_{13} + \ldots + a_{49} = 416$$
$$a_1 + (a_1 + 4d) + (a_1 + 8d) + (a_1 + 12d) + \ldots + (a_1 + 48d) = 416$$
$$13a_1 + 4d(1 + 2 + 3 + \ldots + 12) = 416$$
$$13a_1 + \frac{4d \times 12 \times 13}{2} = 416$$
$$13a_1 + 24 \times 13d = 416$$
$$a_1 + 24d = 32$$
$$a_9 + a_{43} = 66$$
$$a_1 + 8d + a_1 + 42d = 66$$
$$2a_1 + 50d = 66$$
$$a_1 + 25d = 33$$
$$d = 1$$
$$a_1 = 8$$
$$a_1^2 + (a_1 + d)^2 + (a_1 + 2d)^2 + \ldots\ldots + (a_1 + 16d)^2 = 140m$$
$$17a_1^2 + d^2(1^2 + 2^2 + \ldots + 16^2) + 2a_1 d(1 + 2 + \ldots + 16) = 140m$$
$$17 \times 64 + \frac{16 \times 17 \times 33}{6} + \frac{2 \times 8 \times 1 \times 16 \times 17}{2} = 140m$$
$$17 \times 64 + 8 \times 11 \times 17 + 8 \times 11 \times 17 = 140m$$
$$17 \times 16 + 22 \times 17 + 2 \times 16 \times 17 = 35m$$
$$272 + 374 + 544 = 35m$$
$$1190 = 35m \quad \Rightarrow m = 34$$

47. Sol. (1)



$$\begin{vmatrix} 0 & k & 1 \\ 2 & 3 & 1 \\ h & 0 & 1 \end{vmatrix} = 0$$
$$-(2 - h) + 1(-3h) = 0$$
$$-2y + xy - 3x = 0$$
$$3x + 2 = xy \text{ Ans.}$$

48. Sol. (2)

$$I=\int_{+\pi/2}^{+\pi/2}\frac{\sin^2 x}{1+2^x}dx$$

$$I=\int_{0}^{+\pi/2}\left(\frac{\sin^2 x}{1+2^x}+\frac{\sin^2 x}{1+2^{-x}}\right)dx$$

property $\int_{-9}^{+9} f(x)dx=\int_{0}^{+9}(f(x)+(x))dx$

$$I=\int_0^{\pi/2}\sin^2 x \quad dx=\int_0^{\pi/2}\frac{(1-\cos(2x))}{2}dx=\left(\frac{1}{2}\left(x-\frac{\sin(2x)}{2}\right)_0^{\pi/2}\right)=\frac{1}{2}[\pi/2-0]=\pi/4$$

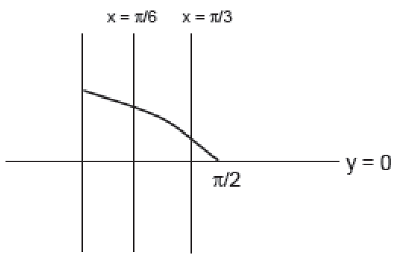
49. Sol. (3)

$g(x)=\cos x^2,\ f(x)=\sqrt{x}$

$18x^2-9\pi x+\pi^2=0 \begin{cases}\alpha\\ \beta\end{cases}$ $(\alpha+\beta)=(6x-\pi)(3x-\pi)=0 \Rightarrow x=\pi/6,\ \pi/3$

$y=(gof)(x)=g\left(f\left(\sqrt{x}\right)\right)=g\left(\sqrt{x}\right)=\cos(x)$

$\alpha=\pi/6,\quad \beta=\pi/3,\quad x=\pi/6,\quad x=\pi/3$



$$A=\int_{\pi/6}^{\pi/3}\cos x dx=(\sin x)_{\pi/6}^{\pi/3}=\frac{\sqrt{3}}{2}-\frac{1}{2}=\frac{\sqrt{3}-1}{2}$$

50. Sol. (1)

$$\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\cdots+\left[\frac{15}{x}\right]\right)$$

$$\lim_{x\to 0^+}\left(x\left[\frac{1}{x}\right]+x\left[\frac{2}{x}\right]+\cdots+x\left[\frac{15}{x}\right]\right)$$

$$=1+2+3+\ldots\ldots+15$$

$$=\frac{15}{2}(15+1)=120$$

51. Sol. (1)

SD is independent of shifting of origin

So $S.D.=+\sqrt{\text{var}(x_i-5)}=\sqrt{\frac{1}{9}(45)-\frac{9}{9}}=2$

52. Sol. (4)

$$I=\frac{\tan^2 x.\sec^2 x}{\left(\tan^5 x+\tan^2 x+\tan^3 x+1\right)^2}dx=\frac{\tan^2 x\sec^6 x}{\left(\tan^2 x+1\right)^2\left(\tan^3 x+1\right)^2}dx=\frac{\tan^2 x\sec^2 x}{\left(1+\tan^3 x\right)^2}dx$$

let $1+\tan^3 x=t$

$$3\tan^2 x\sec^2 x dx=dt=\frac{1}{3}\int\frac{1}{t^2}dt=-\frac{1}{3\left(1+\tan^3 x\right)}+C$$

53. Sol. (3)

$f(x)=|x-\pi|.\left(e^{|x|}-1\right)\sin|x|$

According to given options we have to check only at $x=0$ and π

at $x=0, f(0)=0$

$LHD=\lim_{h\to 0^+}\frac{f(0-h)-f(0)}{-h}$

$=\lim_{h\to 0^+}\frac{(\pi+h).\left(e^{h}-1\right)\sinh}{-h}$

$=RHD=\lim_{h\to 0^+}\frac{(\pi-h).\left(e^{h}-1\right)\sinh}{h}$

$=0\Rightarrow \text{diff.at } x=0$

Now at $X=\pi(x=\pi)$

$f(\pi)=0$

$LHD=\lim_{h\to 0^+}\frac{f(\pi-h)-f(\pi)}{-h}=\lim_{h\to 0^+}\frac{k.\left(e^{\pi-h}-1\right).\sinh}{h}=0$

$RHD=\lim_{h\to 0^+}\frac{f(\pi+h)-f(\pi)}{h}=\lim_{h\to 0^+}\frac{-h\left(e^{\pi+h}-1\right)\sinh}{h}=0$

differential at $x=\pi$ also, hence answer is (3)

54. Sol. (1)

$$\frac{dy}{dx}+\cot x\, y = 4x\operatorname{cosec} x$$

$$\text{I.F.} = e^{\int \cot x dx} = \sin x$$

$$y(\sin x) = \int 4x \operatorname{cosec} x.\sin x dx + C$$

$$y\sin x = 2x^2 + C$$

$$\because\ y\left(\frac{\pi}{2}\right)=0$$

$$C=\frac{\pi^2}{2}$$

$$y\sin x = 2x^2 - \frac{\pi^2}{2}$$

$$\text{so } y\left(\frac{\pi}{6}\right)=2\left(\frac{2\pi^2}{36}-\frac{\pi^2}{2}\right)=2\pi^2\left(\frac{1}{18}-\frac{1}{2}\right)=-\frac{8\pi^2}{9}$$

55. Sol. (3)

$$\vec{u}=x\hat{i}+y\hat{j}+z\hat{k}$$
$$\vec{u}.\vec{a}=0 \Rightarrow 2x+3y-z=0 \quad \ldots(\text{i})$$
$$\vec{u}.\vec{b}=24 \Rightarrow y+z=24 \quad \ldots(\text{ii})$$

$$\left[\vec{u}\vec{a}\vec{b}\right]=0 \Rightarrow \begin{vmatrix} x & y & z \\ 2 & 3 & -1 \\ 0 & 1 & 1 \end{vmatrix}=0$$

$$\Rightarrow 4x-y+2z=0$$
$$2x-y+z=0 \quad \ldots(\text{iii})$$
$$(2)+(3)$$
$$2x+2z=24$$
$$x+z=12 \quad \ldots(\text{iv})$$

Now $24-2z+3(24-z)-z=0$

$96=6z$

$z=16 \Rightarrow x=-4 \Rightarrow y=8$

$\vec{u}=-4\vec{i}+8\vec{j}+16\hat{k}$

$|\vec{u}|=16+64+256=336$

56. Sol. (2)

$A(5,-1,4)$

$B(4,-1,3)$

$AB=\sqrt{2}$

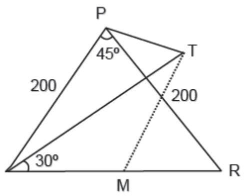
Direction ratio of $AB <1,0,1>$

Angle between line AB and plane is $\theta \Rightarrow \sin\theta=\frac{2}{\sqrt{6}}$

$\Rightarrow \cos\theta=\frac{1}{\sqrt{3}}$

Projection of AB on plane $=AB\cos\theta=\sqrt{\frac{2}{3}}$

57. Sol. (3)



Let height of the tower is $TM=h$ and $QM=MR=x$

$PM=\sqrt{40000-x^2}$

$\Rightarrow \tan 45^\circ=\frac{TM}{PM}=\frac{h}{\sqrt{40000-x^2}}\Rightarrow h^2=40000-x^2$

$\Rightarrow h^2+x^2=40000 \quad \ldots(\text{i})$

$\tan 30^\circ=\frac{TM}{QM}\Rightarrow x=\sqrt{3}h. \ \ldots(\text{ii})$

by (i) and (2) $4h^2=40000\Rightarrow h=100.m$

58. Sol. (3)

Number of ways $x={}^6C_4\times {}^3C_1\times 4!=15\times 3\times 24=1080$

59. Sol. (4)

$1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+\ldots$

$A=1^2+2.2^2+\ldots+2.20^2$

$=\left(1^2+2^2+\ldots+20^2\right)+\left(2^2+4^2+\ldots+20^2\right)$

$=\frac{20.21.41}{6}+.\frac{10.11.21}{6}=\frac{20.21}{6}(41+22)=70\times 63=4410$

$B=1^2+2.2^2+\ldots\ldots+2.40^2$

$=\left(1^2+2^2+\ldots\ldots+40^2\right)+\left(2^2+4^2+\ldots 40^2\right)$

$=\frac{40.41.81}{6}+\frac{4.20.21.41}{6}=\frac{40.41}{6}(81+42)=\frac{40.41}{6}\times 123$

$=20(41)^2=33620$

$B-2A=100\lambda\Rightarrow\lambda=\frac{33620-8820}{100}=\frac{24800}{100}=248$

60. Sol. (1)

$$3 = \frac{2x-3}{3} \Rightarrow x = 6$$

$$3 = \frac{2y+5}{3} \Rightarrow y = 2$$

$$\frac{AC}{2} = \frac{1}{2}\sqrt{81+9} = \frac{1}{2}\sqrt{90} = \frac{3}{2}\sqrt{10}$$

$$r = 3\sqrt{\frac{5}{2}}$$