JEE MAIN-2018

MATHEMATICS

This section contains 30 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

31. If the tangent at $(1,7)$ to the curve $x^2 = y-6$ touches the circle $x^2+y^2+16x+12y+c=0$ then the value of c is :

(1) 85

(2) 95

(3) 195

(4) 185

32. If L_1 is the line of intersection of the planes $2x-2y+3z-2=0,\ x-y+z+1=0$ and L_2 is the line of intersection of the planes $x+2y-z-3=0,\ 3x-y+2z-1=0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is :

(1) $\frac{1}{2\sqrt{2}}$

(2) $\frac{1}{\sqrt{2}}$

(3) $\frac{1}{4\sqrt{2}}$

(4) $\frac{1}{3\sqrt{2}}$

33. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to:

(1) 1

(2) 2

(3) -1

(4) 0

34. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point $T(0,3)$ then the area (in sq. units) of ΔPTQ is :

(1) $60\sqrt{3}$

(2) $36\sqrt{5}$

(3) $45\sqrt{5}$

(4) $54\sqrt{3}$

35. If the curves $y^2 = 6x, 9x^2 + by^2 = 16$ intersect each other at right angels, then the value of b is :

(1) 4

(2) $\frac{9}{2}$

(3) 6

(4) $\frac{7}{2}$

36. If the system of linear equations

$$x + ky + 3z = 0$$
$$3x + ky - 2z = 0$$
$$2x + 4y - 3z = 0$$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to :

(1) –30

(2) 30

(3) –10

(4) 10

37. Let $S = \left\{x \in R : x \geq 0 \text{ and } 2\left|\sqrt{x} - 3\right| + \sqrt{x}\left(\sqrt{x} - 6\right) + 6 = 0\right\}$. Then S :

(1) contains exactly two elements.

(2) contains exactly four elements.

(3) is an empty set.

(4) contains exactly one element

38. If sum of all the solutions of the equation
$8\cos x\left(\cos\left(\frac{\pi}{6} + x\right).\cos\left(\frac{\pi}{6} - x\right) - \frac{1}{2}\right) = 1 \text{ in } [0, \pi] \text{ is } k\pi$. then k is equal to :

(1) $\frac{8}{9}$

(2) $\frac{20}{9}$

(3) $\frac{2}{3}$

(4) $\frac{13}{9}$

39. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is

(1) $\frac{1}{5}$

(2) $\frac{3}{4}$

(3) $\frac{3}{10}$

(4) $\frac{2}{5}$

40. Let $f(x)=x^2+\frac{1}{x^2}$ *and* $g(x)=x-\frac{1}{x}, x\in R-\{-1,0,1\}$. If $h(x)=\frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is :□

(1) $-2\sqrt{2}$

(2) $2\sqrt{2}$

(3) 3

(4) -3

41. Two sets A and B are as under $A=\{(a,b)\in R\times R:|a-5|<1 \text{and} |b-5|<1\}$;

$B=\{(a,b)\in R\times R:4(a-6)^2+9(b-5)^2\leq 36\}$. Then;

(1) $A\cap B=\phi$ (an empty set)

(2) Neither $A\subset B$ nor $B\subset A$

(3) $B\subset A$

(4) $A\subset B$

42. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to :

(1) q

(2) $\sim q$

(3) $\sim p$

(4) p

43. Tangent and normal are drawn at $P(16,16)$ on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is :

(1) 3

(2) $\frac{4}{3}$

(3) $\frac{1}{2}$

(4) 2

44. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$ then the ordered pair (A,B) is equal to :

(1) $(-4,5)$

(2) $(4,5)$

(3) $(-4,-5)$

(4) $(-4,3)$

45. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5, (x>1)$ is :

(1) 1

(2) 2

(3) -1

(4) 0

46. Let $a_1, a_2, a_3, \ldots, a_{49}$ be in $A.P.$ such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^2 + a_2^2 + \ldots + a = 140m$ then m is equal to :

(1) 34

(2) 33

(3) 66

(4) 68

47. A straight line through a fixed point $(2,3)$ intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle $OPRQ$ is completed, then the locus of R is

(1) $3x+2y=xy$

(2) $3x+2y=6xy$

(3) $3x+2y=6$

(4) $2x+3y=xy$

48. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is :

(1) 4π

(2) $\frac{\pi}{4}$

(3) $\frac{\pi}{8}$

(4) $\frac{\pi}{2}$

49. Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$, and $\alpha, \beta (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha, x = \beta$ and $y = 0$, is

(1) $\frac{1}{2}(\sqrt{3} - \sqrt{2})$

(2) $\frac{1}{2}(\sqrt{2} - 1)$

(3) $\frac{1}{2}(\sqrt{3} - 1)$

(4) $\frac{1}{2}(\sqrt{3} + 1)$

50. For each $t \in R$ let $[t]$ be the greatest integer less than or equal to t. Then

$$\lim_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \cdots + \left[\frac{15}{x}\right]\right)$$

(1) is equal to 120

(2) does not exist (in R)

(3) is equal to 0

(4) is equal to 15

51. If $\sum_{i=1}^{9}(x_i-5)=9$ and $\sum_{i=1}^{9}(x_i-5)^2=45$, then the standard deviation of the 9 items $x_1, x_2, \ldots, x_9$ is :

(1) 2

(2) 3

(3) 9

(4) 4

52. The integral $\int \frac{\sin^2 x \cos^2 x}{\left(\sin^5 x+\cos^3 x \sin^2 x+\sin^3 x \cos^2 x+\cos^5 x\right)^2} dx$ is equal to :

(1) $\frac{1}{1+\cot^3 x}+C$

(2) $\frac{-1}{1+\cot^3 x}+C$

(3) $\frac{1}{3\left(1+\tan^3 x\right)}+C$

(4) $\frac{-1}{3\left(1+\tan^3 x\right)}+C$

(where C is a constant of integration)

53. Let $S=\left\{t \in R: f(x)=|x-\pi| .\left(e^{|x|}-1\right) \sin |x| \text{ is not differentiable at } t.\right\}$ Then the set S is equal to :

(1) $\{\pi\}$

(2) $\{0, \pi\}$

(3) $\phi(\text{an empty set})$

(4) $\{0\}$

54. Let $y = y(x)$ be the solution of the differential equation

$\sin\frac{dy}{dx} + y\cos x = 4x, x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to

(1) $-\frac{8}{9}\pi^2$

(2) $-\frac{4}{9}\pi^2$

(3) $\frac{4}{9\sqrt{3}}\pi^2$

(4) $\frac{-8}{9\sqrt{3}}\pi^2$

55. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}.\vec{b} = 24$, then $|\vec{u}|^2$ is equal to :

(1) 256

(2) 84

(3) 336

(4) 315

56. The length of the projection of the line segment joining the points $(5,-1,4)$ and $(4,-1,3)$ on the plane, $x + y + z = 7$ is :

(1) $\frac{1}{3}$

(2) $\sqrt{\frac{2}{3}}$

(3) $\frac{2}{\sqrt{3}}$

(4) $\frac{2}{3}$

57. PQR is a triangular park with $PQ = PR = 200\ m$. A $T.V.$ tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively $45^\circ, 30^\circ$ and 30°, then the height of the tower (in m) is :

(1) $100\sqrt{3}$

(2) $50\sqrt{2}$

(3) 100

(4) 50

58. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is :

(1) at least 500 but less than 750

(2) at least 750 but less than 1000

(3) at least 1000

(4) less than 500

59. Let A be the sum of the first 20 terms and B be sum of the first 40 terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots$ If $B - 2A = 100\lambda$, then λ is equal to :

(1) 464

(2) 496

(3) 232

(4) 248

60. Let the orthocenter and centroid of a triangle be $A(-3,5)$ and $B(3,3)$ respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segement AC as diameter, is :

(1) $3\sqrt{\frac{5}{2}}$

(2) $\frac{3\sqrt{5}}{2}$

(3) $\sqrt{10}$

(4) $2\sqrt{10}$