JEE MAIN – 2019

MATHEMATICS

61: Sol. (4)

Total number of teams $= {}^{7}C_3 \times {}^{5}C_2$

$= 35 \times 10 = 350$

Let A, B be the specific boys.

Number of teams with these two boys in the same team $= {}^{5}C_1 \times {}^{5}C_2$

$= 5 \times 10 = 50$

Required number of ways $= 350 - 50 = 300$

62: Sol. (4)

$(x+1)^2 = -1$

$\Rightarrow x + 1 = \pm i$

$\Rightarrow x = -1 + i, \ -1 - i$

$\alpha = -1 + i, \ \beta = -1 - i$

$\alpha = \sqrt{2} e^{i\left(\frac{3\pi}{4}\right)}, \ \beta = \sqrt{2} e^{i\left(-\frac{3\pi}{4}\right)}$

$$\alpha^{15} + \beta^{15} = \left(\sqrt{2}\right)^{15}\left[e^{i\left(\frac{45\pi}{4}\right)} + e^{i\left(-\frac{45\pi}{4}\right)}\right]$$

$$= 2^{\frac{15}{2}} \times 2\cos\left(\frac{45\pi}{4}\right)$$

$$= 2^{\frac{15}{2}} \times 2\left(-\frac{1}{\sqrt{2}}\right)$$

$$= -2^8$$

$$= -256$$

63: Sol. (1)

$$\int_0^{\pi} |\cos x|^3\, dx = 2\int_0^{\frac{\pi}{2}} \cos^3 x dx$$

$$= 2\int_0^{\frac{\pi}{2}} \frac{3\cos x + \cos 3x}{4} dx$$

$$= \frac{1}{2}\left(3\sin x + \frac{\sin 3x}{3}\right)_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left(3 - \frac{1}{3}\right)$$

$$= \frac{1}{2} \times \frac{8}{3}$$

$$= \frac{4}{3}$$

64: Sol. (3)

Let $x^2 - 1 = t$

$\Rightarrow 2xdx = dt$

$\Rightarrow xdx = \dfrac{dt}{2}$

$$\int \sqrt{\frac{2\sin t - \sin 2t}{2\sin t + \sin 2t}}\frac{dt}{2} = \frac{1}{2}\int \sqrt{\frac{2\sin t(1-\cos t)}{2\sin t(1+\cos t)}}\, dt$$

$$= \frac{1}{2}\int \tan\frac{t}{2} dt$$

$$= \ln\left|\sec\frac{t}{2}\right| + c$$

$$= \ln\left|\sec\left(\frac{x^2-1}{2}\right)\right| + c$$

65: Sol. (3)

$\vec{a} \times \vec{c} + \vec{b} = \vec{0}$

$\Rightarrow \vec{a} \times \vec{c} = -\vec{b}$

$\Rightarrow \vec{a} \times (\vec{a} \times \vec{c}) = \vec{b} \times \vec{a}$

$\Rightarrow (\vec{a}\cdot\vec{c})\vec{a} - (\vec{a}\cdot\vec{a})\vec{c} = \vec{b} \times \vec{a} \qquad \ldots(1)$

$(\vec{a}\cdot\vec{c}) = 4$ and $(\vec{a}\cdot\vec{a}) = 2$ ...(2)

$$\vec{b}\times\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = \hat{i} + \hat{j} - 2\hat{k} \quad \ldots(3)$$

From $(1), (2)$ and , (3) we get

$$4\vec{a} - 2\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$$
$$\Rightarrow 2\vec{c} = \hat{i} + \hat{j} - 2\hat{k} - 4(\hat{i} - \hat{j})$$
$$\Rightarrow -2\vec{c} = -3\hat{i} + 5\hat{j} - 2\hat{k}$$
$$\Rightarrow 4|\vec{c}|^2 = 38$$
$$\Rightarrow |\vec{c}|^2 = \frac{19}{2}$$

66: Sol. (1)

$$f(1^-) = f(1^+)$$
$$\Rightarrow 5 = a + b \quad \ldots(1)$$

$$f(3^-) = f(3^+)$$
$$\Rightarrow a + 3b = b + 15 \quad \ldots(2)$$

$$f(5^-) = f(5^+)$$
$$\Rightarrow b + 25 = 30$$
$$\Rightarrow b = 5 \quad \ldots(3)$$

From (1), $a = 0$ and from (2), $a = 5$

$f(x)$ is discontinuous $\forall a \in \mathbb{R}, b \in \mathbb{R}$

67: Sol. (1)

Let 5 students are x_1, x_2, x_3, x_4, x_5

Given $\bar{x} = \dfrac{\sum x_i}{5} = 150$

$$\sum x_i = 750 \quad \ldots(1)$$

$$\frac{\sum x_i^2}{5}-(\bar{x})^2=18$$
$$\Rightarrow \frac{\sum x_i^2}{5}-(150)^2=18$$
$$\Rightarrow \sum x_i^2=(22500+18)\times 5$$
$$\Rightarrow \sum x_i^2=112590 \qquad \ldots(2)$$

Height of new student $=156$ (let this be x_6)

Now $x_1+x_2+x_3+x_4+x_5+x_6=750+156$

$$\bar{x}_{new}=\left(\frac{x_1+x_2+x_3+x_4+x_5+x_6}{6}\right)=\frac{906}{6}=151 \qquad \ldots(3)$$

Variance (new) $=\dfrac{\sum x_i^2}{6}-(\bar{x}_{new})^2$

$$=\frac{x_1^{\,2}+x_2^{\,2}+x_3^{\,2}+x_4^{\,2}+x_5^{\,2}+x_6^{\,2}}{6}-(151)^2$$

From equation (2) and , (3) we get

Variance (new) $=\dfrac{112590+(156)^2}{6}-(151)^2$

$$=22821-22801$$
$$=20$$

68: Sol. (1)

Let the terms of G.P. be $\frac{a}{r},\ a,\ ar$

$$\frac{a}{r}+a+ar=ax$$
$$\Rightarrow x=\frac{1}{r}+1+r$$

But $r+\frac{1}{r}\geq 2$

Or $r+\frac{1}{r}\leq -2$ (using A.M., G.M. inequality)

$x-1\geq 2$ or $x-1\leq -2$

$x\geq 3$ or $x\leq -1$

So x cannot be 2.

69: Sol. (2)

$$2^4 \equiv 1 \pmod{15}$$
$$\Rightarrow 2^{400} \equiv 1 \pmod{15}$$
$$\Rightarrow 2^{403} \equiv 8 \pmod{15}$$
$$\Rightarrow \left\{\frac{2^{403}}{15}\right\} = \frac{8}{15}$$
$$\therefore k = 8$$

70: Sol. (1)

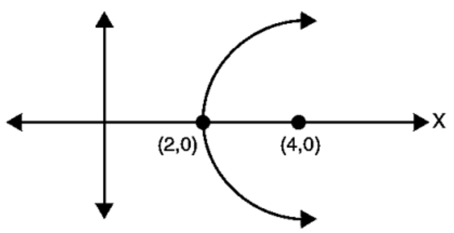
Rationalising numerator,

$$\lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$$
$$=\lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}\times\frac{\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}}{\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}}$$
$$=\lim_{y\to 0}\frac{\sqrt{1+y^4}-1}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}\times\frac{\sqrt{1+y^4}+1}{\sqrt{1+y^4}+1}$$
$$=\lim_{y\to 0}\frac{y^4}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$
$$=\frac{1}{4\sqrt{2}}$$

71: Sol. (1)

The equation of parabola is $y^2 = 8(x-2)$

$(6,\ 8)$ does not lie on this curve.



72: Sol. (3)

$$z = \frac{3+2i\sin\theta}{1-2i\sin\theta}\times\frac{1+2i\sin\theta}{1+2i\sin\theta}$$

$$= \frac{3-4\sin^2\theta+i(8\sin\theta)}{1+4\sin^2\theta}$$

For z to be purely imaginary, $\text{Re}(z)=0$

i.e. $\frac{3-4\sin^2\theta}{1+4\sin^2\theta}=0$

$\Rightarrow \sin^2\theta=\frac{3}{4}$

As $\theta\in\left(-\frac{\pi}{2},\pi\right)=\pm\frac{\pi}{3},\ \frac{2\pi}{3}$

Sum of all values of $\theta=\frac{2\pi}{3}$

73: Sol. (2)

$$A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

A is a rotation matrix.

$$A^n=\begin{bmatrix}\cos n\theta & -\sin n\theta\\ \sin n\theta & \cos n\theta\end{bmatrix}$$

$$A^{-50}=\begin{bmatrix}\cos 50\theta & \sin 50\theta\\ -\sin 50\theta & \cos 50\theta\end{bmatrix}$$

At $\theta=\frac{\pi}{2}$

$$A^{-50}=\begin{bmatrix}\cos\frac{25\pi}{6} & \sin\frac{25\pi}{6}\\ -\sin\frac{25\pi}{6} & \cos\frac{25\pi}{6}\end{bmatrix}$$

$$=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$$

74: Sol. (1)

By inspection $\oplus$ represents $\wedge$ and $\otimes$ represents $\vee$

A	B	$A \wedge B$	$\sim A$	$\sim A \vee B$	$(A \wedge B) \wedge (\sim A \vee B)$
T	T	T	F	T	T
T	F	F	F	F	F
F	T	F	T	T	F
F	F	F	T	T	F

$(A \wedge B) \wedge (\sim A \vee B) = A \wedge B$

75: Sol. (2)

$$x\left(\frac{dy}{dx}\right) + 2y = x^2$$

$$\Rightarrow \frac{dy}{dx} + \frac{2y}{x} = x$$

This is linear differential equation.

$$\text{I.F.} = e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$$

solution is $x^2 y = \int x^3 dx$

$$x^2 y = \frac{x^4}{4} + c$$

$$y(1) = 1 \times c = \frac{3}{4}$$

at $x = \frac{1}{2}$ we get

$$\frac{1}{4}y = \frac{1}{64} + \frac{3}{4}$$

$$\Rightarrow y = \frac{49}{16}$$

76: Sol. (1)

$$P(x=1)={}^2C_1\times\frac{4}{52}\times\frac{48}{52}=\frac{24}{169}$$

$$P(x=2)={}^2C_2\times\left(\frac{4}{52}\right)^2=\frac{1}{169}$$

$$P(x=1)+P(x=2)=\frac{25}{169}$$

77: Sol. (1)

For hyperbola, $e^2=1+\frac{b^2}{a^2}$

$$e^2=1+\tan^2\theta$$
$$=\sec^2\theta$$

Now $e>2\Rightarrow\sec\theta>2$

$$\Rightarrow\theta\in\left(\frac{\pi}{3},\ \frac{\pi}{2}\right)$$

Length of latus rectum $=\frac{2b^2}{a}$

$$=2\tan\theta\sin\theta$$
$$>2\times\sqrt{3}\times\frac{\sqrt{3}}{2}$$
$$=3$$

Therefore, the length of latus rectum lies in the interval $(3,\ \infty)$.

78: Sol. (1)

$$l=3$$
$$r^2+h^2=9$$
$$r^2=9-h^2$$

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi h\left(9-h^2\right) \\ &= 3\pi h - \frac{1}{3}\pi h^3 \end{aligned}$$

For maximum volume, $\frac{dv}{dh} = 0$

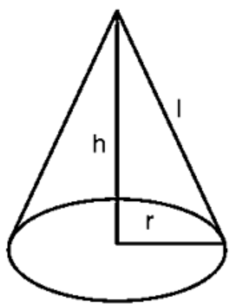
$3\pi - \pi h^2 = 0$

$h^2 = 3$

$h = \sqrt{3}$

$r^2 = 6$

$\therefore V = \frac{1}{3}\pi \ (6)\left(\sqrt{3}\right) = 2\sqrt{3}\pi \text{ cm}^3$



79: Sol. (2)

$\cos^{-1}\left(\frac{2}{3x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{3}{4x}\right)$

$\Rightarrow \cos^{-1}\left(\frac{2}{3x}\right) = \cos^{-1}\left(\frac{\sqrt{16x^2-9}}{4x}\right)$

$\Rightarrow \frac{2}{3x} = \frac{\sqrt{16x^2-9}}{4x}$

$\Rightarrow \sqrt{16x^2-9} = \frac{8}{3}$

$\Rightarrow 16x^2 - 9 = \frac{64}{9}$

$$\Rightarrow 16x^2 = \frac{145}{9}$$

$$\Rightarrow x^2 = \frac{145}{9\times16}$$

$$\Rightarrow x = \frac{\sqrt{145}}{12}$$

80. Sol. (3)

$px + qy + r = 0 \qquad \ldots(1)$

$3p + 2q + 4r = 0$

$\frac{3p}{4} + \frac{q}{2} + r = 0 \qquad \ldots(2)$

(1) and (2) are identical.

$$\frac{x}{\frac{3}{4}} = \frac{y}{\frac{1}{2}} = 1$$

$x = \frac{3}{4}$ and $y = \frac{1}{2}$

81: Sol. (2)

$x + y + z = 1 \qquad \ldots(1)$

$2x + 3y + 2z = 1 \qquad \ldots(2)$

$2x + 3y + (a^2 - 1)z = a + 1 \qquad \ldots(3)$

By observation, when $a^2 - 1 = 2$

LHS of (2) and (3) are same but RHS different

Hence $a^2 = 3 \Rightarrow |a| = \sqrt{3}$

For $|a| = \sqrt{3}$, the system is inconsistent.

82: Sol. (2)

$$3(\cos\theta-\sin\theta)^4+6(\sin\theta+\cos\theta)^2+4\sin^6\theta$$

$$=3(\cos^2\theta+\sin^2\theta-\sin 2\theta)^2+6(\sin^2\theta+\cos^2\theta+\sin 2\theta)+4\sin^6\theta$$

$$=3(1+\sin^2 2\theta-2\sin 2\theta)+6+6\sin 2\theta+4\sin^6\theta$$

$$=9+3\sin^2 2\theta+4\sin^6\theta$$

$$=9+3(4\sin^2\theta\cos^2\theta)+4(1-\cos^2\theta)^3$$

$$=9+12\cos^2\theta\sin^2\theta+4(1-\cos^6\theta-3\cos^2\theta\sin^2\theta)$$

$$=9+12\sin^2\theta\cos^2\theta+4-4\cos^6\theta-12\sin^2\theta\cos^2\theta$$

$$=13-4\cos^6\theta$$

83: Sol. (4)

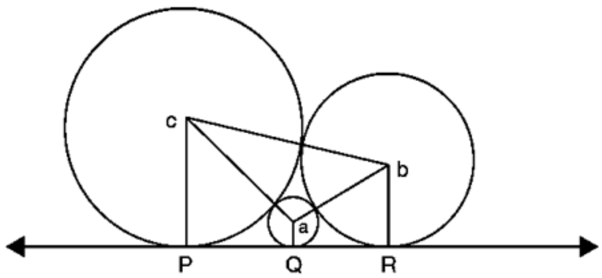
$$PQ+QR=PR$$

$$\Rightarrow\sqrt{(c+a)^2-(c-a)^2}+\sqrt{(b+a)^2-(b-a)^2}=\sqrt{(b+c)^2-(c-b)^2}$$

$$\Rightarrow\sqrt{4ac}+\sqrt{4ab}=\sqrt{4bc}$$

Dividing with $\sqrt{4abc}$

$$\frac{1}{\sqrt{b}}+\frac{1}{\sqrt{c}}=\frac{1}{\sqrt{a}}$$



84: Sol. (3)

$$f_2\left(J\left(f_1(x)\right)\right)=f_3(x)$$

$$\Rightarrow f_2\left(J\left(\frac{1}{x}\right)\right)=\frac{1}{1-x}$$

$$\Rightarrow 1-J\left(\frac{1}{x}\right)=\frac{1}{1-x}$$

$$\Rightarrow J\left(\frac{1}{x}\right)=1-\frac{1}{1-x}=\frac{x}{x-1}$$

$$\Rightarrow J(x)=\frac{1}{1-x}=f_3(x)$$

85: Sol. (3)

Family of planes containing the line of intersection of planes is $\pi_1+\lambda\pi_2=0$

i.e. $(x+y-z)+\lambda(x+2y-3z+5)=0$

This is passing through $(-4,1,3)$

$$\lambda=-1$$

Hence the equation of plane is $y-2z+5=0$

Required line is lie in this plane and is parallel to $x+y+z=5$

direction of required line $=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 0 & 1 & -2\end{vmatrix}$

Required line is $\frac{x+4}{-3}=\frac{y-1}{2}=\frac{z-3}{1}$

86: Sol. (1)

$x^2+2=10-x^2$

$\Rightarrow x=\pm 2$ and $y=6$

Point of intersection of curves $=(\pm 2,\ 6)$

Let's find the slopes.

$y=x^2+2 \quad \Rightarrow \frac{dy}{dx}=m_1=2x$

$y=10-x^2 \quad \Rightarrow \frac{dy}{dx}=m_2=-2x$

At $(\pm 2,\ 6)$, $m_1=\pm 4$ and $m_1=\mp 4$

Now $|\tan\theta|=\left|\frac{4-(-4)}{1+4(-4)}\right|=\frac{8}{15}$

87: Sol. (4)

Required plane is $\pi_1 + \lambda\pi_2 = 0$

$$(x+y+z-1)+\lambda(2x+3y-z-4)=0$$
$$\Rightarrow (1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z-(1+4\lambda)=0$$

This is parallel to y-axis $\Rightarrow \lambda = -\frac{1}{3}$

Required plane is

$$\left(1-\frac{2}{3}\right)x+(1-1)y+\left(1+\frac{1}{3}\right)z-\left(1-\frac{4}{3}\right)=0$$
$$\Rightarrow \frac{1}{3}x+\frac{4}{3}z+\frac{1}{3}=0$$
$$\Rightarrow x+4z+1=0$$

By inspection, $(3,1,-1)$ lie in this plane.

88: Sol. (4)

Equation of tangent to the parabola $y^2 = 4x$ is

$$y = mx + \frac{1}{m}$$
$$\Rightarrow m^2x - my + 1 = 0$$

This is also tangent to $x^2 + y^2 - 6x = 0$

i.e. $\left|\frac{3m^2+1}{\sqrt{m^4+m^2}}\right| = 3$

$$\Rightarrow 9m^4+1+6m^2 = 9m^4+9m^2$$
$$\Rightarrow 3m^2 = 1$$
$$\Rightarrow m = \pm\frac{1}{\sqrt{3}}$$

The common tangent is $y = \pm\left(\frac{x}{\sqrt{3}}+\sqrt{3}\right)$.

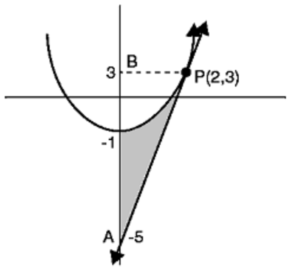
89: Sol. (3)

Equation of tangent at $(2,\ 3)$ is

$$\frac{y+3}{2}=2x-1$$
$$\Rightarrow y+3=4x-2$$
$$\Rightarrow 4x-y=5$$

Required area $=\text{Ar}(\Delta PAB)-\int_{-1}^{3}x_{\text{parabola}}\,dy$

$$=\frac{1}{2}\times 2\times 8-\int_{-1}^{3}\sqrt{y+1}\,dy$$
$$=8-\frac{2}{3}\left[(y+1)^{\frac{3}{2}}\right]_{-1}^{3}$$
$$=\ 8-\frac{16}{3}$$
$$=\ \frac{8}{3}$$



90: Sol. (4)

$a_1,a_2,.....a_{30}$ is an A.P. Let d be the common difference.

$S=\sum_{i=1}^{30}a_i$ and $T=\sum_{i=1}^{15}a_{2i-1}$

$a_5=27\Rightarrow a_1+4d=27 \quad .....(1)$

$S-2T=75$

$\Rightarrow(a_2-a_1)+(a_4-a_3)+........+(a_{30}-a_{29})=75$

$\Rightarrow 15d=75$

$\Rightarrow d=5$

From equation (1), we have, $a_1=7$.

$\therefore a_{10}=a_1+9d=7+45=52$.