JEE MAIN - 2019

Mathematics

This paper contains 30 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

61: There are 5 girls and 7 boys. A team of 3 boys and 2 girls is to be formed such that no two specific boys are in the same team. Number of way to do so

(1) 400

(2) 250

(3) 200

(4) 300

62: The equation $x^2+2x+2=0$ has roots α and β. Then value of $\alpha^{15}+\beta^{15}$ is;

(1) 512

(2) 256

(3) –512

(4) –256

63: $\int_0^{\pi}|\cos x|^3 dx$ is equal to;

(1) $\frac{4}{3}$

(2) $\frac{2}{3}$

(3) 0

(4) $\frac{8}{3}$

64: If $x^2 \neq n\pi + 1,\ n \in \mathbb{N}$, then $\int x\sqrt{\dfrac{2\sin\left(x^2-1\right)-\sin 2\left(x^2-1\right)}{2\sin\left(x^2-1\right)+\sin 2\left(x^2-1\right)}}dx$ is equal to

(1) $\ln\left|\cos\left(\dfrac{x^2-1}{2}\right)\right|+c$

(2) $\dfrac{1}{2}\ln\left|\cos\left(\dfrac{x^2-1}{2}\right)\right|+c$

(3) $\ln\left|\sec\left(\dfrac{x^2-1}{2}\right)\right|+c$

(4) $\dfrac{1}{2}\ln\left|\sec\left(\dfrac{x^2-1}{2}\right)\right|+c$

65: If $\vec{a} = i - j$, $\vec{b} = i + j + k$ are two vectors, and c is another vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a} \cdot \vec{c} = 4$ then $|\vec{c}|^2 =$

(1) 9

(2) 8

(3) $\dfrac{19}{2}$

(4) $\dfrac{17}{2}$

66: If $f(x) = \begin{cases} 5 & ; \quad x \leq 1 \\ a + bx & ; \quad 1 < x < 3 \\ b + 5x & ; \quad 3 \leq x < 5 \\ 30 & ; \quad x \geq 5 \end{cases}$, then:

(1) $f(x)$ is discontinuous $\forall a \in \mathbb{R}, b \in \mathbb{R}$

(2) $f(x)$ is continuous if $a = 0$ and $b = 5$

(3) $f(x)$ is continuous if $a=5$ and $b=0$

(4) $f(x)$ is continuous if $a=-5$ and $b=10$

67: Average height and variance of 5 students in a class is 150cm and 18cm^2 respectively. A new student whose height is 156cm is added to the group. Find new variance (in cm^2).

(1) 20

(2) 22

(3) 16

(4) 14

68: a, b, c are in G.P. $a+b+c=bx$, then x cannot be;

a) 2

b) -2

c) 3

d) 4

69: $\left\{\frac{2^{403}}{15}\right\}=\frac{k}{15}$ then find k. (where $\{\ \}$ denotes fractional part function).

(1) 2

(2) 8

(3) 1

(4) 4

70: $\lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}=$

(1) $\frac{1}{4\sqrt{2}}$

(2) $\frac{1}{2\sqrt{2}}$

(3) $\frac{1}{2\sqrt{2}\left(1+\sqrt{2}\right)}$

(4) does not exist

71: There is a parabola having axis as x-axis, vertex is at a distance of 2 units from origin and focus is at $(4,\ 0)$. Which of the following point does not lie on the parabola?

(1) $(6,\ 8)$

(2) $(5,\ 2\sqrt{6})$

(3) $(8,\ 4\sqrt{3})$

(4) $(4,-4)$

72: Find sum of all possible values of θ in the interval $\left(-\frac{\pi}{2},\ \pi\right)$ for which $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely imaginary;

(1) $\frac{\pi}{3}$

(2) π

(3) $\frac{2\pi}{3}$

(4) $\frac{\pi}{2}$

73: Let $A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ find the value of A^{-50} at $\theta=\frac{\pi}{12}$.

(1) $\begin{bmatrix}-\frac{\sqrt{3}}{2} & -\frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$

(2) $\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$

(3) $\begin{bmatrix}-\frac{\sqrt{3}}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$

(4) $\begin{bmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$

74: If $(A\oplus B)\wedge(\sim A\otimes B)=A\wedge B$ what should be proper symbol in place of $\oplus$ and $\otimes$ to hold the equation

(1) $\wedge$ and $\vee$

(2) $\wedge$ and $\wedge$

(3) $\vee$ and $\vee$

(4) $\vee$ and $\wedge$

75: If $y(x)$ is solution of $x\left(\frac{dy}{dx}\right)+2y=x^2$, $y(1)=1$, then value of $y\left(\frac{1}{2}\right)$

(1) $-\frac{49}{16}$

(2) $\frac{49}{16}$

(3) $\frac{45}{8}$

(4) $-\frac{45}{8}$

76: From a well shuffled deck of cards, 2 cards are drawn with replacement. If x represent numbers of times ace coming, then value of $P(x=1)+P(x=2)$ is;

(1) $\frac{25}{169}$

(2) $\frac{24}{169}$

(3) $\frac{49}{169}$

(4) $\frac{23}{169}$

77: If eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta}-\frac{y^2}{\sin^2\theta}=1$ is more than 2 when $\theta\in\left(0,\ \frac{\pi}{2}\right)$ then values of length of latus rectum lies in the interval

(1) $(3,\ \infty)$

(2) $\left(1,\ \frac{3}{2}\right)$

(3) $(2,\ 3)$

(4) $(-3,\ -2)$

78: If slant height of a right circular cone is 3 cm then the maximum volume of cone is

(1) $2\sqrt{3}\pi\text{ cm}^3$

(2) $4\sqrt{3}\pi\text{ cm}^3$

(3) $\left(2+\sqrt{3}\right)\pi\text{ cm}^3$

(4) $\left(2-\sqrt{3}\right)\pi\text{ cm}^3$

79: If $\cos^{-1}\left(\frac{2}{3x}\right)+\cos^{-1}\left(\frac{3}{4x}\right)=\frac{\pi}{2},\ x>\frac{3}{4}$ then $x=$

(1) $\frac{\sqrt{145}}{11}$

(2) $\frac{\sqrt{145}}{12}$

(3) $\frac{\sqrt{146}}{10}$

(4) $\frac{\sqrt{146}}{11}$

80: If $px+qy+r=0$ represent a family of straight lines such that $3p+2q+4r=0$ then;

(1) All lines are parallel

(2) All lines are inconsistent

(3) All lines are concurrent at $\left(\frac{3}{4}, \frac{1}{2}\right)$

(4) All lines are concurrent at $(3,2)$

81: Consider the system of equations $x+y+z=1$, $2x+3y+2z=1$, $2x+3y+(a^2-1)z=a+1$ then

(1) system has a unique solution for $|a|=\sqrt{3}$

(2) system is inconsistence for $|a|=\sqrt{3}$

(3) system is inconsistence for $a=4$

(4) system is inconsistence for $a=3$

82: The value of $3(\cos\theta-\sin\theta)^4+6(\sin\theta+\cos\theta)^2+4\sin^6\theta$ is _____, where $\theta\in\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

(1) $13-4\cos^4\theta$

(2) $13-4\cos^6\theta$

(3) $13-4\cos^6\theta+2\sin^4\theta\cos^2\theta$

(4) $13-4\cos^4\theta+2\sin^4\theta\cos^2\theta$

83: Three circles of radii a, b, c $(a<b<c)$ touch each other externally and have x-axis as a common tangent then;

(1) a, b, c are in A.P.

(2) $\frac{1}{\sqrt{b}}=\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{c}}$

(3) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P.

(4) $\frac{1}{\sqrt{c}}+\frac{1}{\sqrt{b}}=\frac{1}{\sqrt{a}}$

84: If $f_1(x)=\frac{1}{x}$, $f_2(x)=1-x$, $f_3(x)=\frac{1}{1-x}$ then find $J(x)$ such that $f_2\circ J\circ f_1(x)=f_3(x)$

(1) $f_1(x)$

(2) $\frac{1}{x}f_3(x)$

(3) $f_3(x)$

(4) $f_2(x)$

85: Find the equation of line through $(-4,1,3)$ and parallel to the plane $x+y+z=3$ while the line intersects another line whose equation is $x+y-z=0=x+2y-3z+5$;

(1) $\frac{x+4}{-3}=\frac{y-1}{-2}=\frac{z-3}{1}$

(2) $\frac{x+4}{1}=\frac{y-1}{2}=\frac{z-3}{1}$

(3) $\frac{x+4}{-3}=\frac{y-1}{2}=\frac{z-3}{1}$

(4) $\frac{x+4}{-1}=\frac{y-1}{2}=\frac{z-3}{-3}$

86: Consider the curves $y=x^2+2$ and $y=10-x^2$. Let θ be the angle between both the curves at point of intersection, then find $|\tan\theta|$.

(1) $\frac{8}{15}$

(2) $\frac{5}{17}$

(3) $\frac{3}{17}$

(4) $\frac{8}{17}$

87: A plane parallel to y-axis passing through line of intersection of planes $x+y+z=1$ and $2x+3y-z-4=0$ which of the point lie on the plane.

(1) $(3,2,1)$

(2) $(-3,0,1)$

(3) $(-3,1,1)$

(4) $(3,1,-1)$

88: Find common tangent of the two curves $y^2=4x$ and $x^2+y^2-6x=0$.

(1) $y=\frac{x}{3}+3$

(2) $y=\frac{x}{\sqrt{3}}-\sqrt{3}$

(3) $y=\frac{x}{3}-3$

(4) $y=\frac{x}{\sqrt{3}}+\sqrt{3}$

89: The area bounded by the curve $y = x^2 - 1$, tangent to it at $(2,\ 3)$ and y-axis is;

(1) $\frac{2}{3}$

(2) $\frac{4}{3}$

(3) $\frac{8}{3}$

(4) 1

90. Let $a_1, a_2, ...a_{30}$ be an A.P, $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{2i-1}$. If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to

(1) 57

(2) 47

(3) 42

(4) 52